Physics 23 Problem Set 8

Harry Nelson... extra office hours Thursday, 11/10 3pm-5pm

Due Monday, November 14

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. Let's investigate whether the gradient of a function $f(x, y)$ is really the direction that maximizes the change in f. Imagine taking a step, starting at position $x_0\hat{\boldsymbol{\imath}} + y_0\hat{\boldsymbol{\jmath}}$, of length $\Delta\xi$, and in direction θ with respect to the x axis.

   (a) Describe the step by a vector of the form $\Delta x\hat{\boldsymbol{\imath}} + \Delta y\hat{\boldsymbol{\jmath}}$, and express Δx and Δy in terms of $\Delta\xi$ and θ. Make a graph in the $x - y$ plane showing the starting position $x_0\hat{\boldsymbol{\imath}} + y_0\hat{\boldsymbol{\jmath}}$ and the vector that describes the step (you must assume illustrative values for all the quantities to make the graph).

   (b) What is the change in the value of f as one steps from $x_0\hat{\boldsymbol{\imath}} + y_0\hat{\boldsymbol{\jmath}}$ to the final postion?

   (c) Now find the extrema in the change in the value of f as a function of θ. In particular, what values of $\tan\theta$ correspond to the extrema? What do you conclude about the direction of the step that leads to the maximum and minimum changes in f?

   (d) Just for fun, find the direction that corresponds to *no* change in f... this is the direction of an 'iso-f' line, like a line of constant altitude on a topo map. What is the direction of the 'iso-f' line relative to that of the gradient?

2. Purcell 2.2

3. Purcell 2.4

4. Purcell 2.7

5. Purcell 2.12
