Physics 23 Problem Set 4

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Due Monday, October 17

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

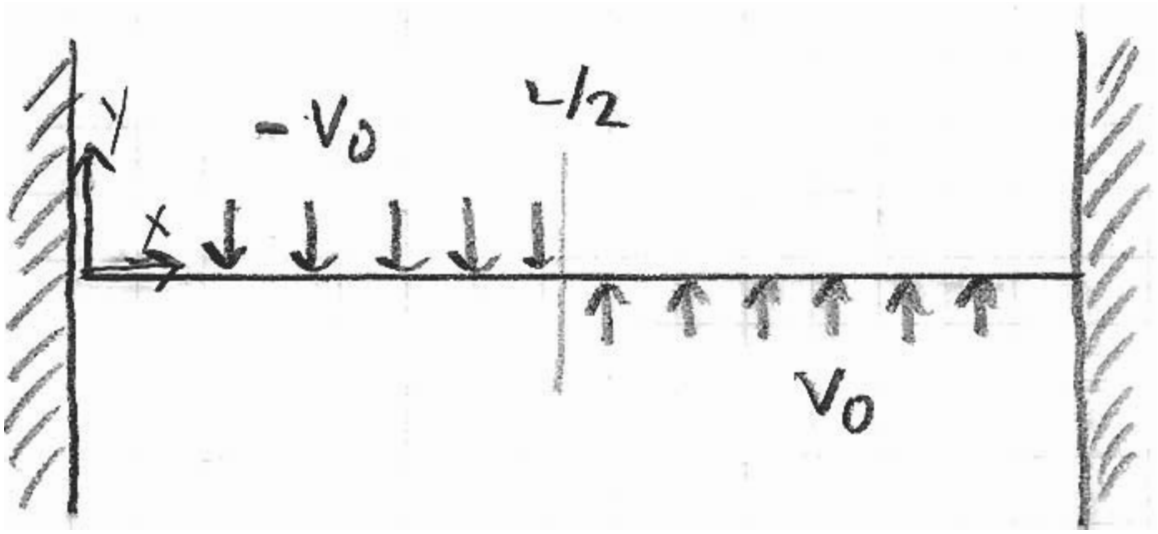

Figure 1: For use in Problem 1.

1. A string of length L at $t = 0$ is shown in Fig. 1, at which time $y(x, 0) = 0$, that is, it is undisplaced. However, parts of the string have velocities: for $y < L/2$, all points on the string are moving downward with velocity (y-component) $-v_0$, and for $y > L/2$ all points on the string are moving upward, so the velocity (y-component) is v_0. Describe the motion $y(x, t)$ of the string by a fourier series:

$$y(x,t) = \sum_{n=1}^{\infty} B_n \sin\left[\frac{n\pi}{L}x\right] g_n(t)$$

   (a) Specify the functions $g_n(t)$... note the initial conditions, and remember that $y(x, t)$ must satisfy the wave equation.

   (b) Solve for the B_n... it should help to recall that the initial velocity is $\partial y/\partial t$, evaluated at $t = 0$. Make sure to consider parity and similar concepts to evaluate B_n is zero for certain n.

   (c) Plot $y(x, t)$ when it hits its maximum value.

2. Imagine two instruments are playing with frequencies $f_1 = 99.5\,\text{Hz}$ and $f_2 = 100.5\,\text{Hz}$. Make plots of the total amplitude versus time, showing the envelope of the amplitude (don't worry about portraying the carrier freqency) for the following cases:

   (a) $\cos(2\pi f_1 t) + \cos(2\pi f_2 t)$

   (b) $\cos(2\pi f_1 t) + \cos(2\pi f_2 t + \pi/4)$

   (c) $\cos(2\pi f_1 t) + \sin(2\pi f_2 t)$

   (d) $3\cos(2\pi f_1 t) + \cos(2\pi f_2 t)$

3. Two students each play recorders at a frequency of 523.2 Hz, which is C_5. One student is on a car, moving at 90 miles per hour away from the other student. Each student hears beats, due to the combination of the sound from their own recorder and the sound from the other student's recorder. How many beats (which are the times of *maximum* amplitude) does each student hear in 50 seconds? It may (or may not) be that each student hears a different number of beats; you must think that through.

4. A guitar has string length $L = 63.5\,\text{cm}$ and strings of phosphor-bronze (density $\rho = 8\,\text{gm/cm}^3$). The strings are of various diameters and tuned to various notes as shown in the table below, with the tension shown. Each string is made to vibrate in the fundamental normal mode, with amplitude of 1 mm. Find the total energy in each guitar string, and fill in the blank in the table below.

Note	D **(inches)**	ν **(Hz)**	T **(newtons)**	Energy **(Joules)**
$\mathbf{E_2}$	0.056	81	134	
$\mathbf{A_2}$	0.046	108	162	
$\mathbf{D_3}$	0.036	144	174	
$\mathbf{G_3}$	0.026	192	163	
$\mathbf{B_3}$	0.017	243	111	
$\mathbf{E_4}$	0.013	324	116	