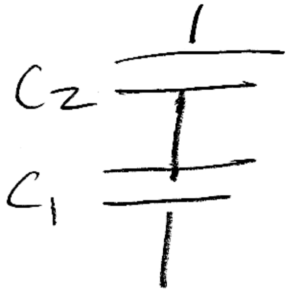


$$\boxed{\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2} \quad \text{Series}}$$

Energy

$\delta W \simeq V\cdot\delta Q$
$\simeq 0\cdot\delta Q$

$Q_i=0$

$V=0$

$Q_i=0$

initial

$\xrightarrow{\text{first}}$ step: take δQ, move it

δQ

$-\delta Q$

no work in first step.

intermediate step.

Q

$V=\dfrac{Q}{C}$

$-Q$

move $\xrightarrow{\quad} \delta Q$

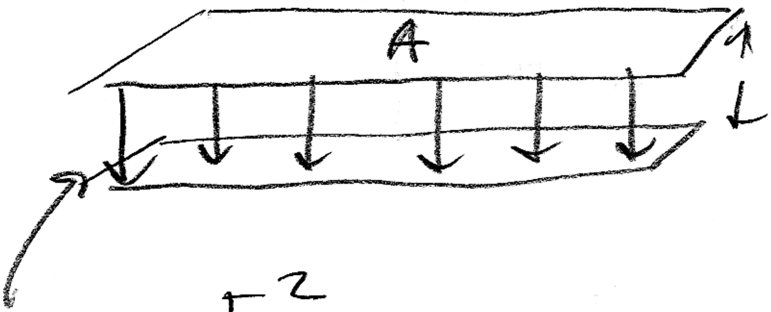
$Q+\delta Q$

$-Q-\delta Q$

$\delta W \simeq \delta Q\cdot V$
$\simeq \dfrac{Q}{C}\,\delta Q$

$$\boxed{W=\frac{1}{C}\int_0^{Q_f} Q\,dQ = \frac{Q_f^2}{2C}}$$

$\propto$ charge^2



another way to look at it

$$U \cong \frac{E^2}{8\pi} \times \underbrace{A \times s}_{\text{volume}}$$ (p. 32)

$$E = 4\pi\sigma = \frac{4\pi Q_f}{A}$$

$$U = \frac{16\pi^2 Q_f^2}{8\pi A^2} \cdot A \cdot s$$

$$= \frac{2\pi \cdot s}{A} \cdot Q_f^2 = \frac{1}{2} \cdot \frac{4\pi s}{A} Q_f^2$$

$$U = \frac{1}{2} \frac{Q_f^2}{\left(\frac{A}{4\pi s}\right)} \leftarrow C \quad = \frac{1}{2} \frac{Q_f^2}{C}$$

How to get this energy back?

$$C = \frac{A}{4\pi s}$$

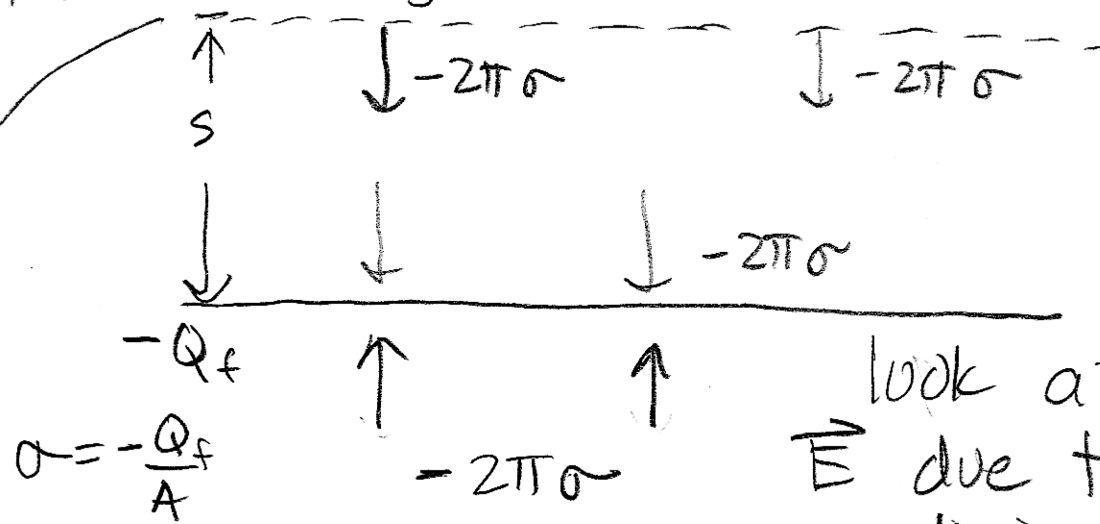
$$U = \frac{2\pi s}{A} Q_f^2$$

Holding Q_f constant...

$$F = -\frac{\partial}{\partial s}\left(\frac{2\pi s}{A} Q_f^2\right) = -\frac{2\pi}{A} Q_f^2$$

can that be reasoned out/checked?

→ where $+Q_f$ goes



look at $\vec{E}$ due to this side only!

$$F = Q_f \cdot E = -2\pi \sigma Q_f$$

$$\boxed{F = \frac{-2\pi Q_f^2}{A}} \checkmark$$

Including the field due to the other plate gives the total field...

$+Q_f$ ↓↑ $\vec{E} = 0$

↓↓ $4\pi\sigma$

$-Q_f$ ↑↓ $\vec{E} = 0$

What if V is held constant?

then, $Q = CV$ $C = \frac{A}{4\pi s}$ depends on s!

$$U = \frac{1}{2}\frac{Q_f^2}{C} = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}\frac{A}{4\pi s}V^2$$

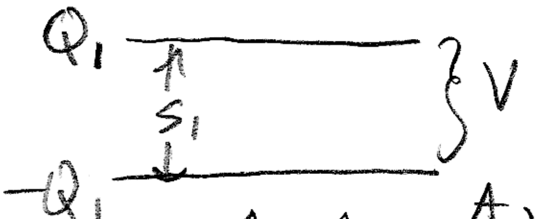
$$-\frac{\partial U}{\partial s} = -\frac{1}{2}\frac{A}{4\pi}\left(-\frac{1}{s^2}\right)V^2$$

$$= +\frac{1}{s}\cdot\frac{1}{2}CV^2 = +\frac{1}{s}\frac{1}{2}\frac{Q_f^2}{C}$$

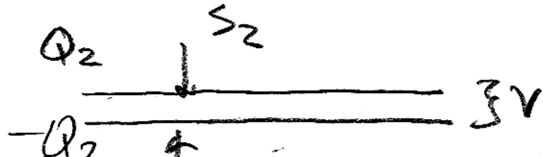
$$= +\frac{2\pi}{A}Q_f^2$$

in this case, plates repel!!!

Why? because some entity (battery) does work to change the charge to keep the voltage the same!



$Q_1 = CV = \frac{A}{4\pi s_1}V$



$Q_2 = \frac{A}{4\pi s_2}V \gg Q_1$

much more energy!