Capacitance

Consider an isolated conductor; its electric potential, relative to infinity, is ϕ_0. The charge on the conductor is Q.

Should not be too surprising that:

$$\phi_0 \propto Q$$

argument: $\phi_0 \propto \int_{\infty}^{\text{surface}} \vec{E} \cdot d\vec{s}, \quad \vec{E} \propto Q$

Example: sphere:

$$\phi_0 = \frac{Q}{a}$$

Constant of proportionality is called ... 1/capacitance

$$\phi_0 = \left(\frac{1}{a}\right) \cdot Q$$

Memorize "Q = C·V"

or $Q = a \cdot \phi_0 \equiv C\phi_0$

gCGS: units are centimeters.

SI/mks: spherical shell:

$$\phi_0 = \frac{Q}{4\pi\varepsilon_0 a}$$

$$Q = \underbrace{(4\pi\varepsilon_0 a)}_{\text{has units of Farads}} \cdot \phi_0$$

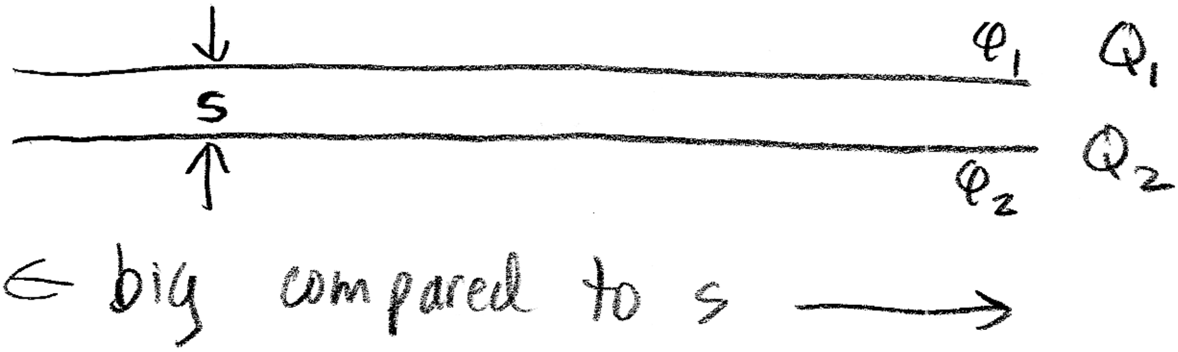
$$\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{Farads}}{\text{meter}}.$$

$Q = C\phi_0$ ← set this, like, with a battery.

↑ more capacitance, more charge!

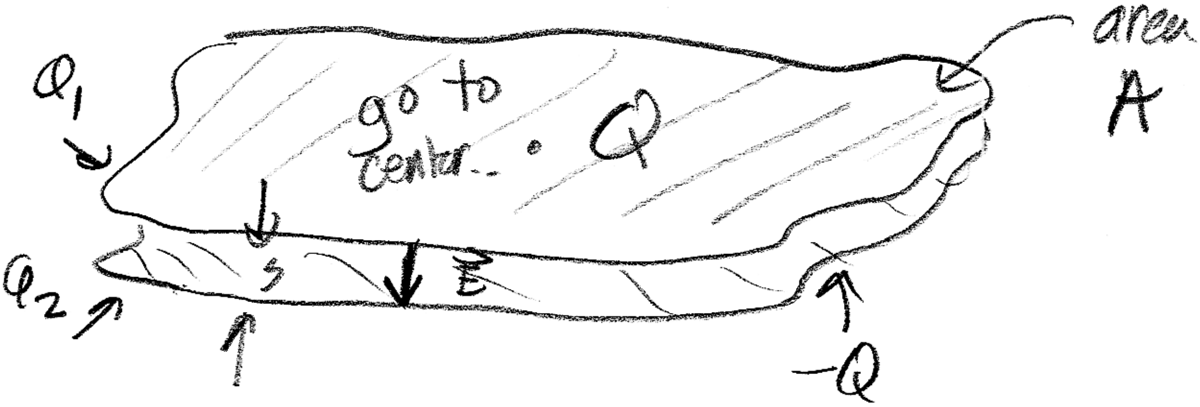
more charge, more energy...

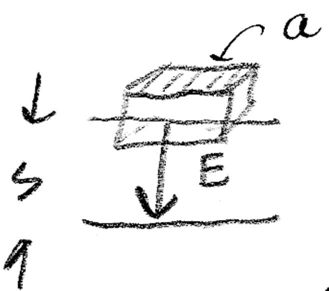
Parallel Plate Conductor



- want smallest field as possible as one goes far away...

$$Q_1 = -Q_2 = Q$$ "balanced" usually true.





$$E \cdot \cancel{a} = 4\pi\sigma \cdot \cancel{a}$$

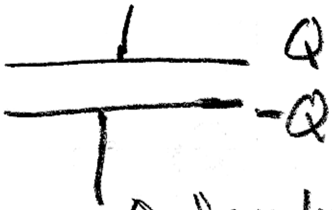
$$E = 4\pi\sigma$$

$$\varphi_1 - \varphi_2 = E s = 4\pi \cdot \overbrace{\frac{Q}{A}}^{\sigma} \cdot s$$

true when width/depth much bigger than s.

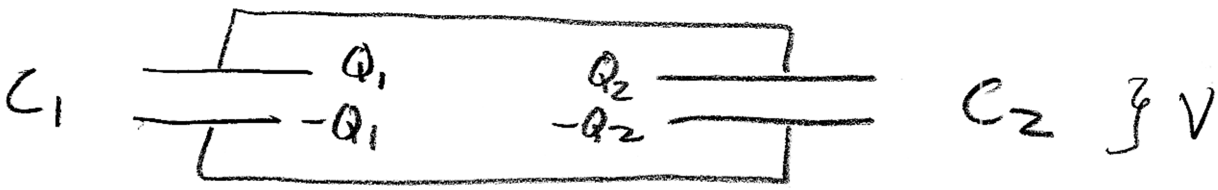
so, $$Q \simeq \underbrace{\frac{A}{4\pi s}}_{\text{usually called "the" capacitance.}} \cdot (\varphi_1 - \varphi_2)$$

Symbol:



↖ "conducting wire.

Parallel Connection:



$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$Q_1 + Q_2 = Q = (C_1 + C_2)\cdot V = CV$$

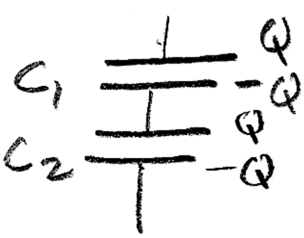
$$\boxed{C = C_1 + C_2}$$

$$\frac{Q_1}{Q} = \frac{C_1}{C_1 + C_2}$$ (more capacity, more charge.

$$= \frac{\frac{A_1}{s_1}}{\frac{A_1}{s_1} + \frac{A_2}{s_2}}$$

← bigger A, more cap.
← smaller s, more cap

Series Connection:



V_1 $\qquad Q = C_1 V_1 \qquad Q = C_2 V_2$

V_2 $\qquad V = V_1 + V_2 = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q$