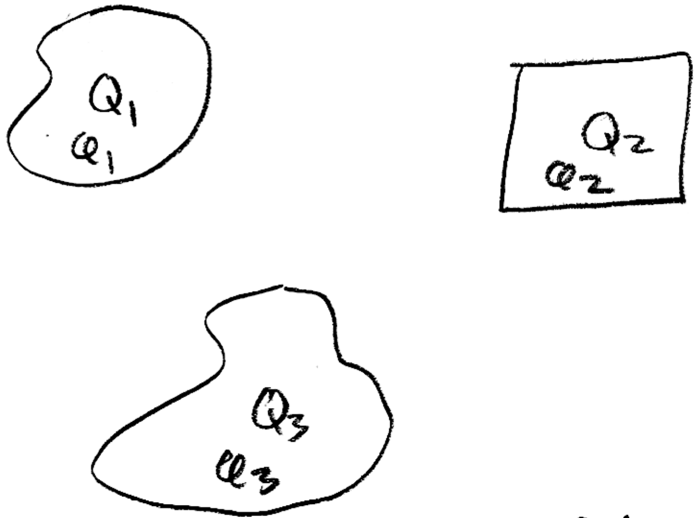
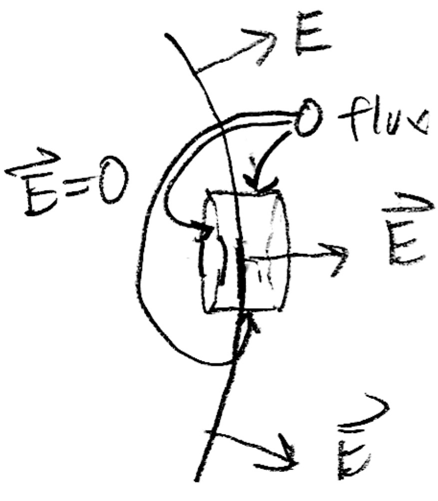


- Three perfect conductors

Put charge Q_1, Q_2, Q_3 on them, none = 0

- Electric fields will eminate from the charges
- No $\vec{E}$ field <u>inside</u> → inside at one electric potential (no work needed to move charge around inside)
- $\vec{E}$ must be $\perp$ to the surface at the surface ... $\vec{E}$ always $\perp$ to equipotential



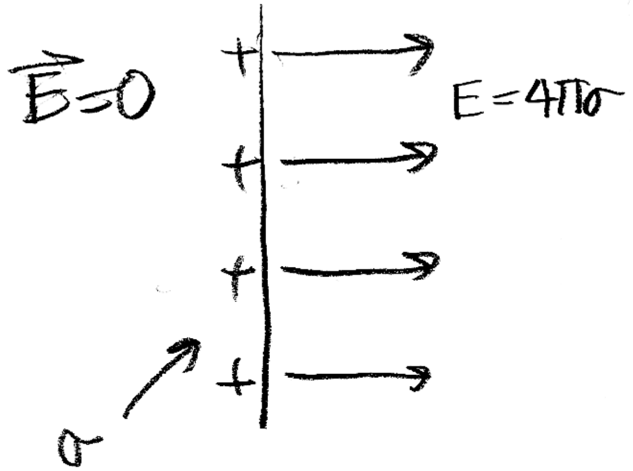
- gauss's law

$$E \cdot A = 4\pi\sigma \cdot A$$

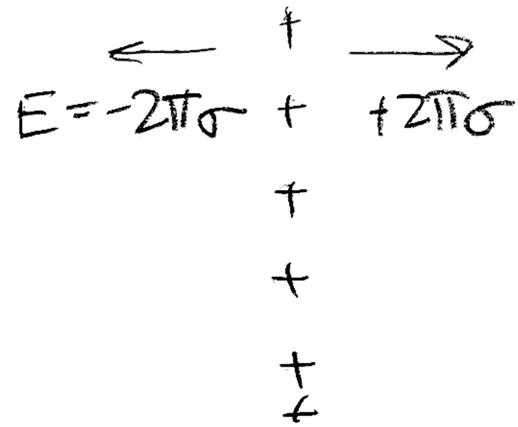
σ: depends on where you are

$$E = 4\pi\sigma$$

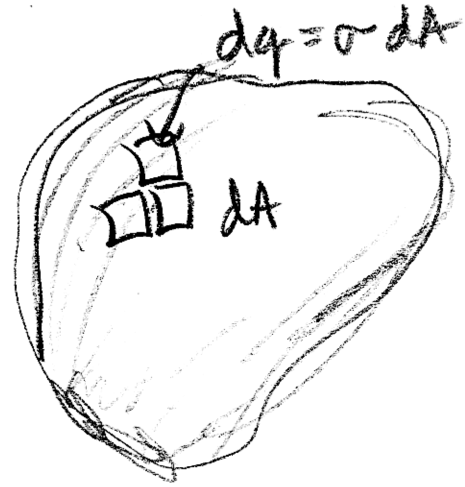
Conductor



Sheet Charge



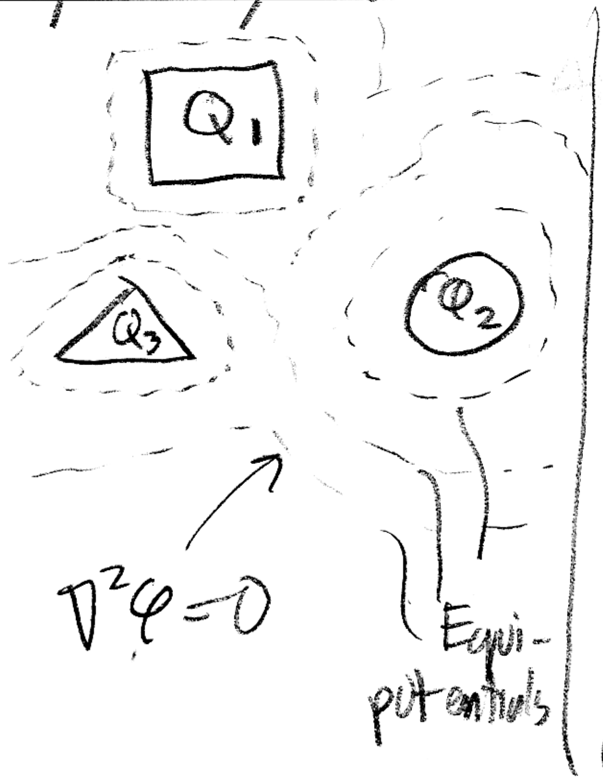
Change in $\vec{E}$ due to σ is $4\pi\sigma$ in horizontal direction



$$\int_{\text{surface}} \sigma \, dA = Q$$

General Electrostatic Problem Outside a System of Conductors

$$\nabla^2 \varphi = 0 = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}$$



"Boundary Condition"

① shapes of the conductors

② ⓐ Potenials of them (or some of them)

ⓑ Charge on them (or some of them)

③ in between them $\nabla^2\varphi=0$ ④ $\phi \to 0$ as $r\to\infty$

How many solutions?

(A) ~~none~~ ← physics

(B) exactly one ← That is it.

(C) more than one

<u>"Uniqueness"</u>: $\varphi(x,y,z)$

<u>suppose</u> two solutions $\psi(x,y,z)$

Both Satisfy boundary conditions... <u>at</u> the conductors and

$\nabla^2\varphi=0$, $\nabla^2\psi=0$ outside

Consider

$$W(x,y,z) = \varphi(x,y,z) - \psi(x,y,z)$$

① At the conductors, $W(\underbrace{x_c, y_c, z_c}_{\text{points on conductors}}) = 0$

because

$$\varphi(x_c, y_c, z_c) = \psi(x_c, y_c, z_c)$$

(same boundary conditions)

② $\nabla^2 W =$

$$\frac{\partial^2}{\partial x^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$+ \frac{\partial^2}{\partial y^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$+ \frac{\partial^2}{\partial z^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$= \frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi$$

$$+ \frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial y^2}\psi + \frac{\partial^2}{\partial z^2}\psi$$

$$= \nabla^2\varphi + \nabla^2\psi = 0$$

(3) $W = 0$ everywhere ...

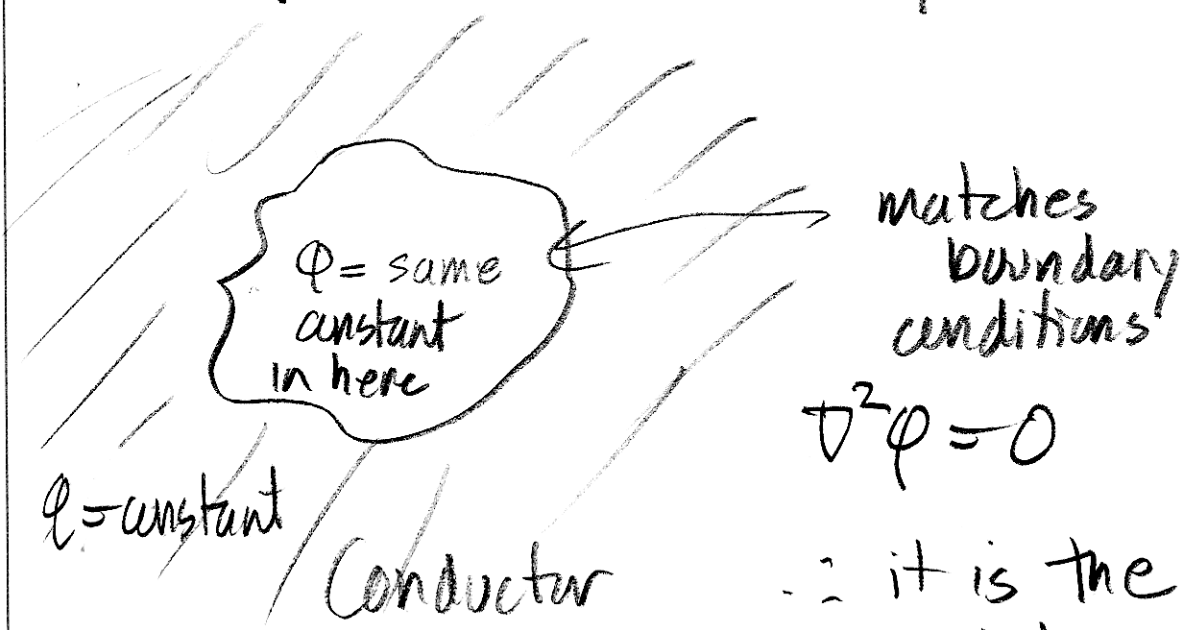
$W = 0$ on conductors

$W = 0$ as $r \to \infty$

if $W \neq 0$ everywhere, must be a maximum (or minimum)... cannot be true.

(4) $\varphi(x,y,z) = \psi(x,y,z)$

Consequence: Cavity

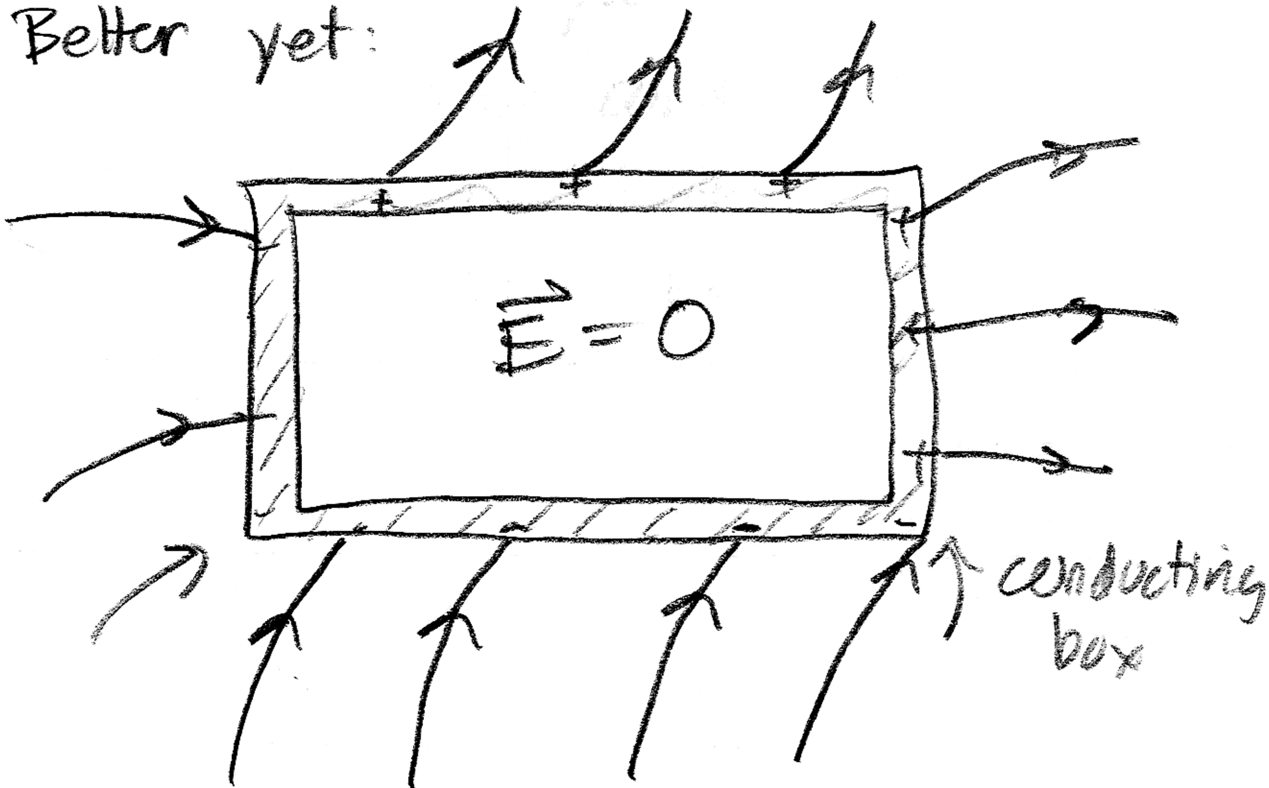


$\nabla^2 \varphi = 0$

$\therefore$ it is the solution

$\vec{E} = 0$ inside then, since $-\vec{\nabla}\phi = 0$

Better yet:



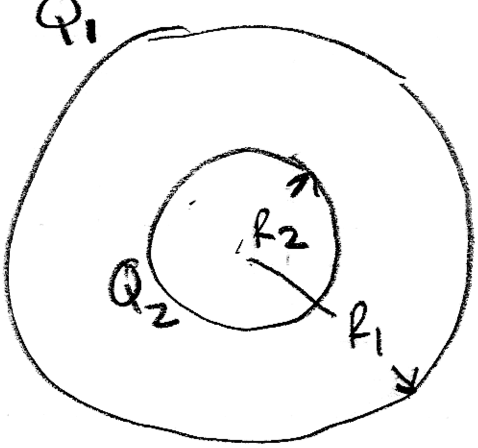
$\vec{E} \neq 0$ outside

$\vec{E} = 0$ inside!

→ perfect conductor

→ electrostatic limit (moving $\vec{E}$ field can "penetrate")

Conductors



$R_1 < r \qquad \varphi = \frac{Q_1 + Q_2}{r}$

$r = R_1 \qquad \varphi = \frac{Q_1}{R_1} + \frac{Q_2}{R_2}$

$R_2 < r < R_1 \qquad \varphi = \frac{Q_1}{R_1} + \frac{Q_2}{r}$

$r < R_2 \qquad \varphi = \frac{Q_1}{R_1} + \frac{Q_2}{R_2}$