$$E_y = 2\pi a^2 \frac{y}{x^2+y^2}$$

$$\frac{\partial E_x}{\partial x} = 2\pi a^2 \left[\frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2}\right]$$

$$= 2\pi a^2 \left[\frac{x^2+y^2-2x^2}{(x^2+y^2)^2}\right] = 2\pi a^2 \frac{y^2-x^2}{x^2+y^2}$$

$$\frac{\partial E_y}{\partial y} = 2\pi a^2 \left[\frac{1}{x^2+y^2} - \frac{2y^2}{(x^2+y^2)^2}\right]$$

$$= 2\pi a^2 \left[\frac{x^2+y^2-2y^2}{(x^2+y^2)^2}\right] = -2\pi a^2 \frac{y^2-x^2}{x^2+y^2}$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0$$

Laplacian:

$$\vec{E} = -\vec{\nabla}\phi = -\left(\hat{x}\frac{\partial \phi}{\partial x} + \hat{y}\frac{\partial \phi}{\partial y} + \hat{z}\frac{\partial \phi}{\partial z}\right)$$

$$\vec{\nabla}\cdot\vec{E} = -\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right)$$

$$= -\underbrace{\nabla^2}_{\uparrow}\phi$$

"Laplacian"

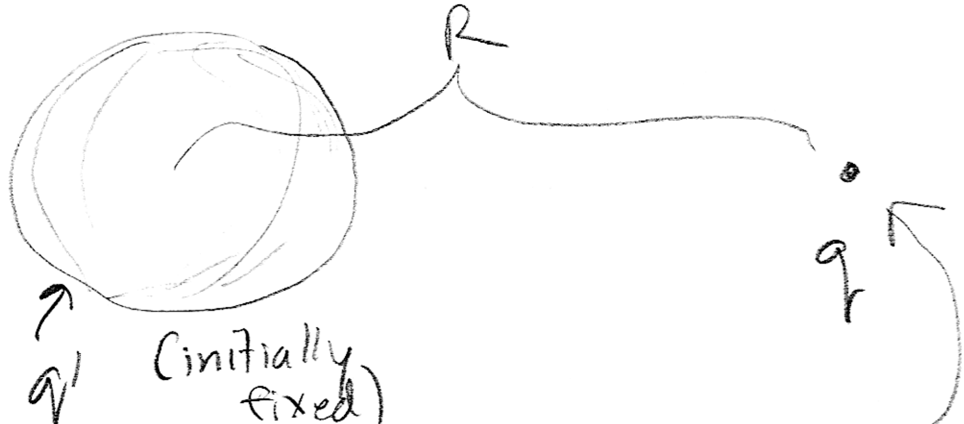
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -4\pi\rho$$

When no charge,

$$\vec{\nabla}^2 \phi = 0$$

solutions $\phi(x,y,z)$ have a fascinating property:

- find $\phi(x,y,z)$ on a sphere.
- take average of ϕ, using all points on the sphere.
- that averaage will be equal to the value of ϕ at center.

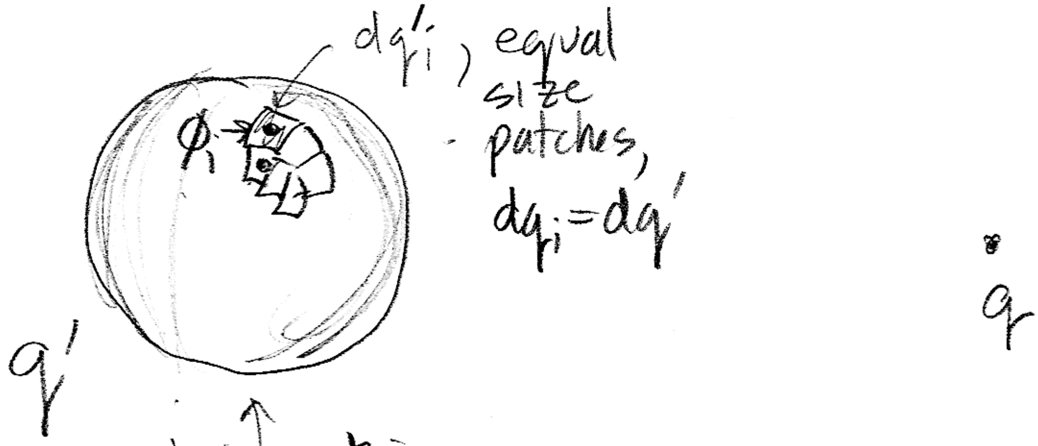


spread evenly over spherical shell.

$\vec{E}$ outside same as a point charge at origin

bring this charge in from infinity.

$$U = \frac{q q'}{R}$$



bring this one up ...

$$U = \sum dq'_i \phi_i$$

$\phi_i \to$ due to q

$dq'_i \to$ all same

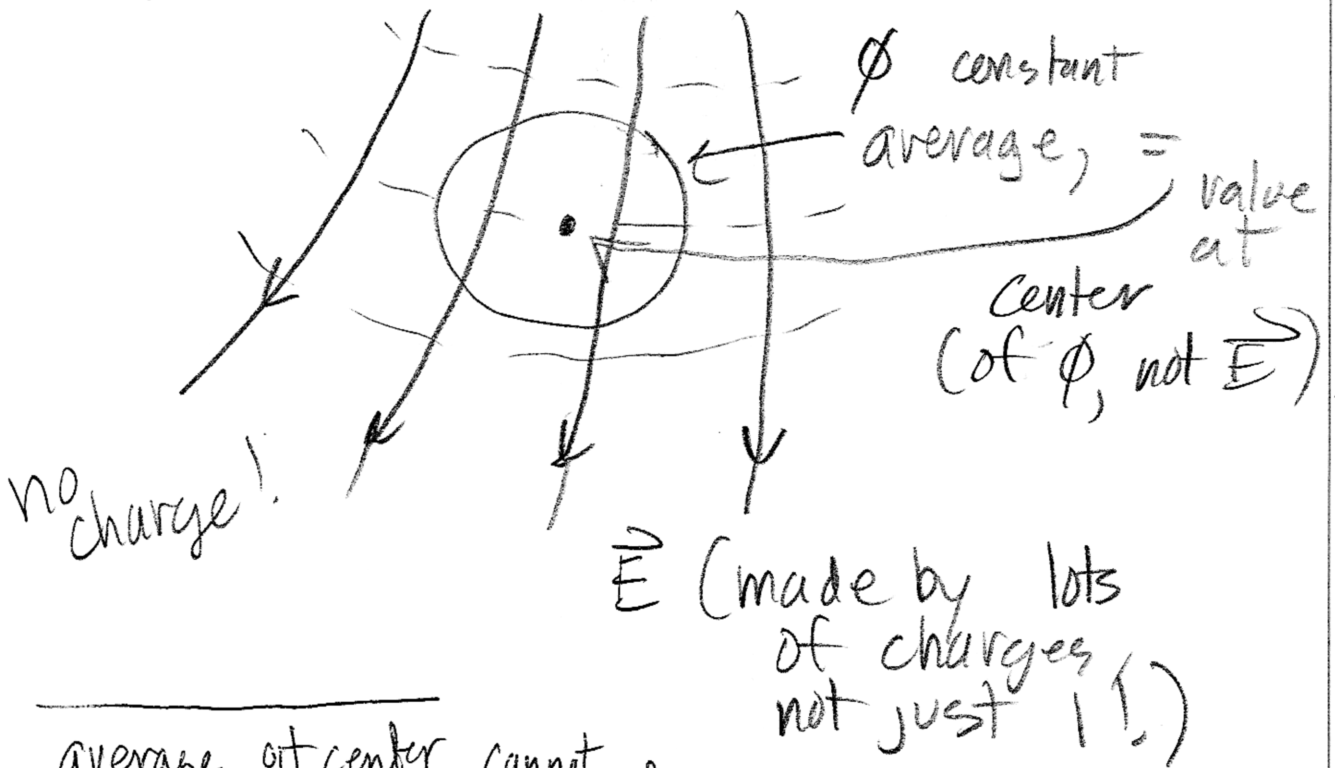
(potential/energy assembling sphere itself neglected.

$$= dq' \sum \phi_i = q' \bar{\phi}_i = \frac{q q'}{R}$$

$$\bar{\phi}_i = \frac{q}{R} = \text{value at center.}$$

Generally,

average = sum over average of each charge .. still true.



average at center cannot exceed

Chapter 3

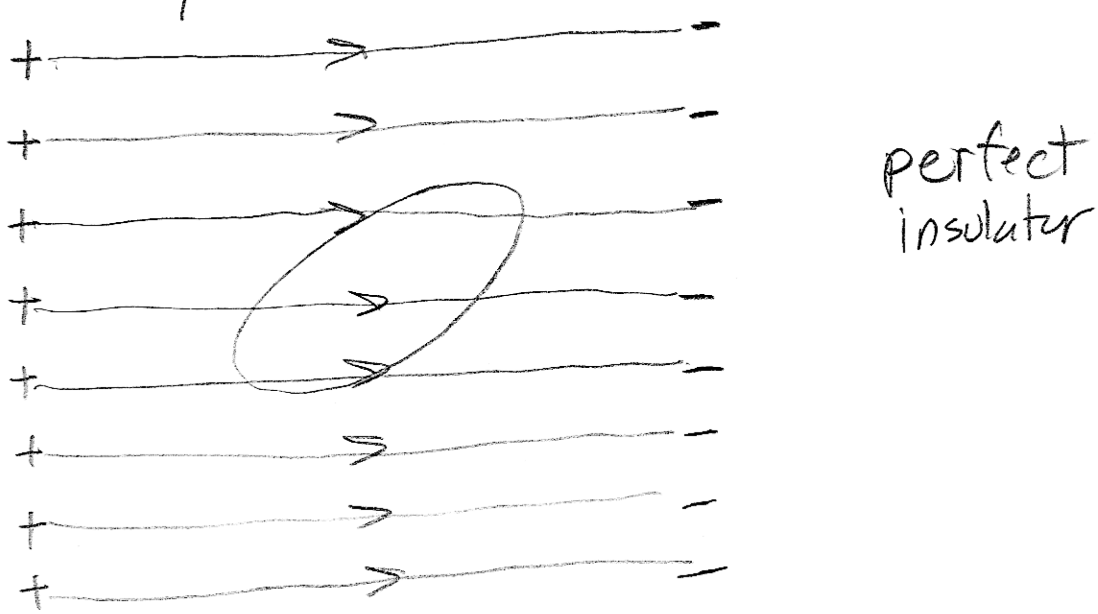
Conductors:
- usually metals
- equal numbers (usually) of + + − charges, but e^- (electrons) can easily move

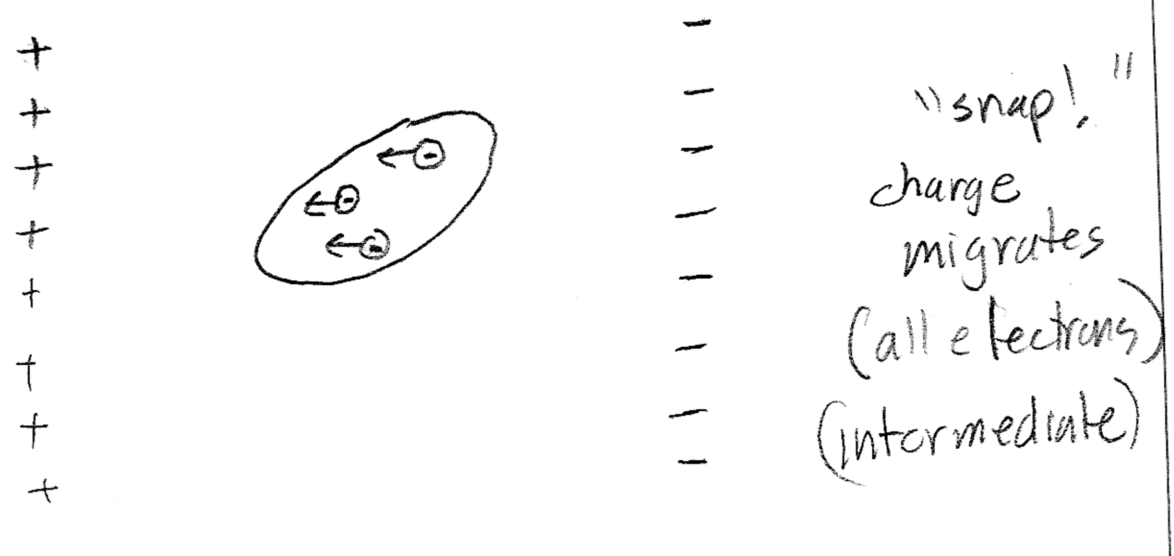
ease of "movement" is quantified by a parameter called the "conductivity"

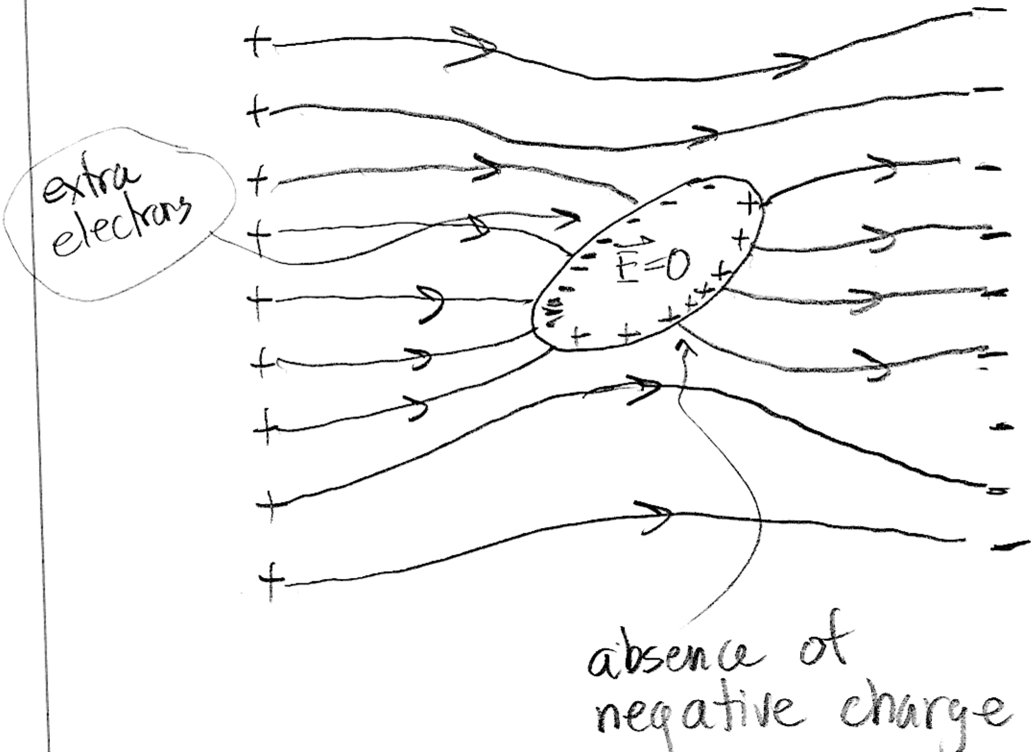
Varies in materials by ~20 (or more) orders of magnitude

<u>Insulators</u> : charges in there, they just cannot move

Key point about <u>conductors</u>: charge will rearrange itself <u>on the surface</u> of the conductor to "protect" the interior of conductor from feeling any electric field. In pictures, imagine a material that is initially an insulator, and then can be suddenly turned into a conductor






extra electrons
$\vec{E}=0$
absence of negative charge