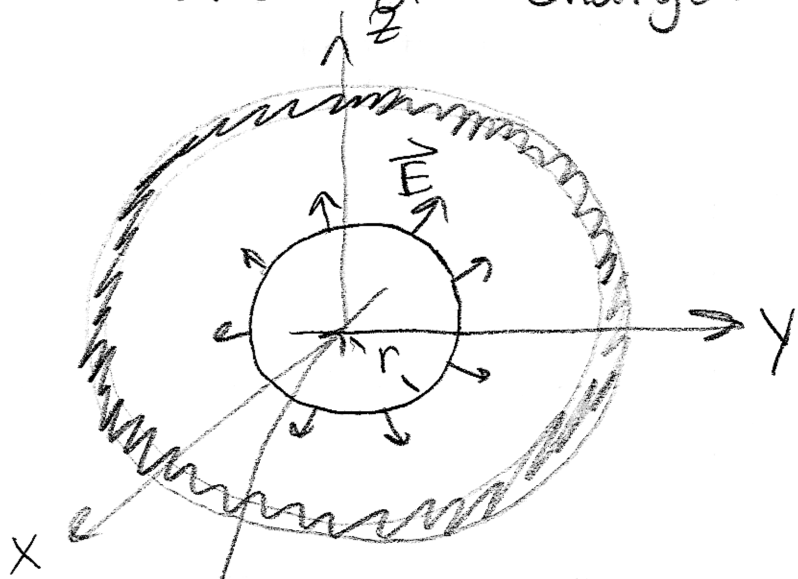
No field inside a spherically symmetric shell of charge:



- no enclosed charge
- by symmetry, only radial component of $\vec{E}$ possible (imagine rotating the ball)

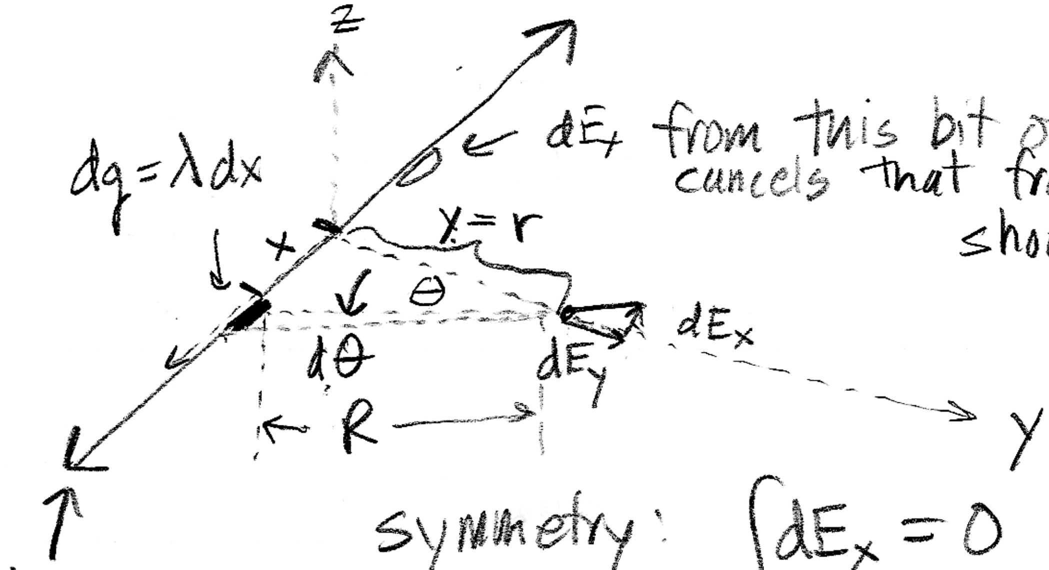
$$E_r = \frac{0}{r^2} \leftarrow \text{charge enclosed}$$

$\therefore$ $\vec{E} = 0$ inside... recall gravity!

Newton proved this in a very different (and more tedius) manner.

Application of Gauss's Law

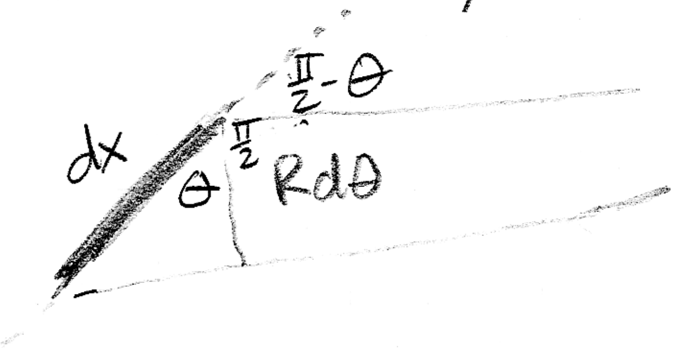
Uniform Line Charge, ∞ long



symmetry: $\int dE_x = 0$

λ $\frac{esu}{cm}$

$$dE_y = dE\cos\theta = \frac{\lambda dx}{R^2}\cos\theta$$



$$\cos\theta = \frac{Rd\theta}{dx}$$

$$dx = \frac{R}{\cos\theta}d\theta$$

$$\cos\theta = \frac{y}{R} = \frac{r}{R} \quad \text{(defines } r\text{)}$$

$$R = \frac{r}{\cos\theta}$$

$$dx = \left(\frac{r}{\cos\theta}\right)\cdot\frac{1}{\cos\theta}d\theta = \frac{r}{\cos^2\theta}d\theta$$

$$\text{so } dE_y = \frac{\lambda \cdot \frac{r}{\cos^2\theta}d\theta}{\left(\frac{r}{\cos\theta}\right)^2}\cos\theta = \frac{\lambda}{r}\cos\theta\, d\theta$$

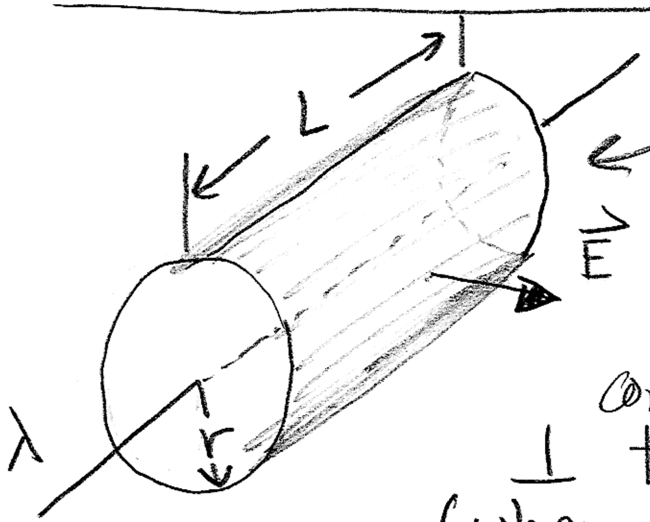
when $x = -\infty$, $\theta = -\pi/2$

$x = +\infty$, $\theta = \pi/2$

$$E_y = \frac{\lambda}{r}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = \frac{\lambda}{r} \sin\theta \Big|_{-\pi/2}^{\pi/2} = \frac{\lambda}{r}(1-(-1))$$

$$\boxed{E_y = \frac{2\lambda}{r}}$$

Gauss's Law Version



gaussian surface

by symmetry (imagine rotating about line charge) $\vec{E}$ only has component along direction $\perp$ to wire.

(when wire ∞ long)

Gauss: $\int \vec{E} \cdot d\vec{a} = 4\pi q$ (enclosed)

endcaps: no flux, $\vec{E} \parallel$ to them

sides: $E \cdot (2\pi r \cdot L) = 4\pi q$

$$\boxed{E = \frac{2}{r} \cdot \frac{q}{L} = \frac{2\lambda}{r}}$$