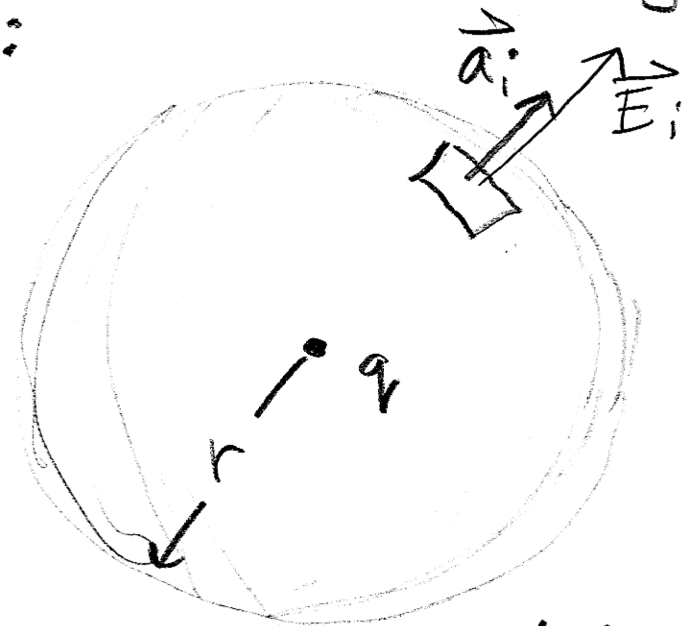
Gauss's Law

Put a spherical surface around a charge, centered on the charge — compute flux of $\vec{E}$ through that surface:



$\vec{E}_i$ is parallel to $\vec{a}_i$

$|\vec{E}|$ is constant, $= \frac{q}{r^2}$ everywhere on the surface

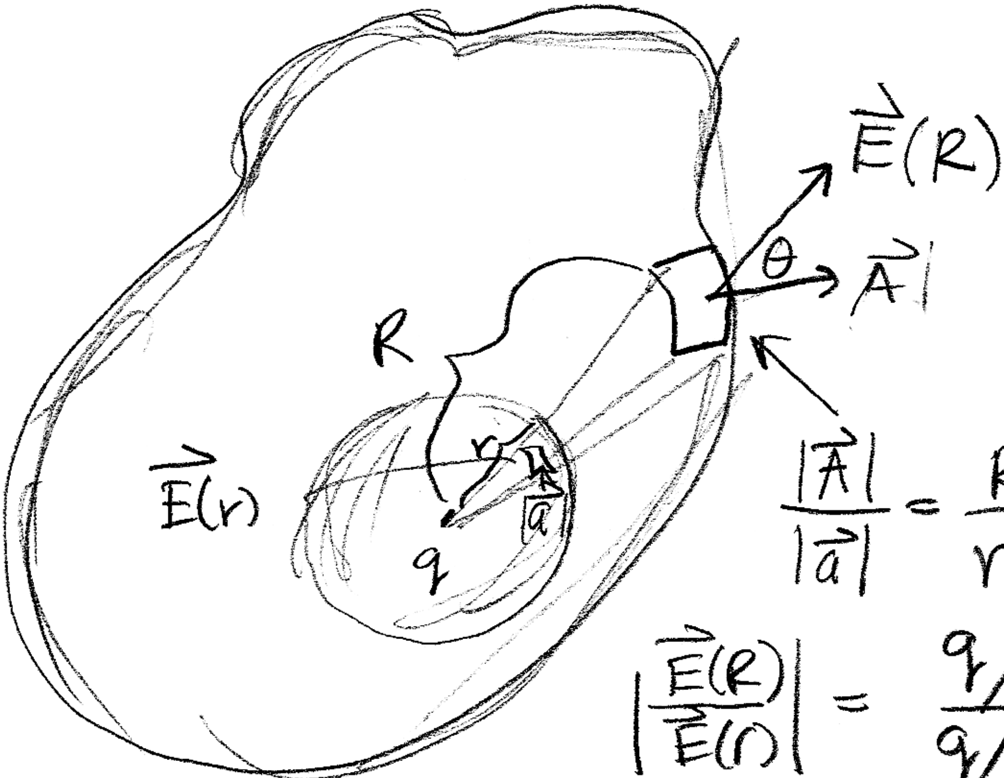
so $$\int_{\text{surface}} \vec{E} \cdot d\vec{a} = \frac{q}{r^2} \int_{\text{surface}} da$$

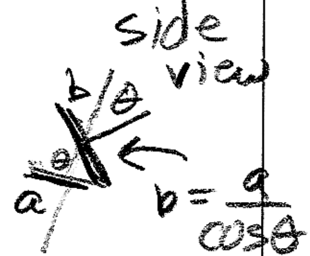
$$= 4\pi r^2 \cdot \frac{q}{r^2} = 4\pi q$$

Size of sphere (r) didn't matter!

<u>Also</u>:
- didn't matter that charge was at center
- shape of surface didn't matter.

That surface <u>closed</u> and <u>around</u> charge did





$$\frac{|\vec{A}|}{|\vec{a}|} = \frac{R^2}{r^2} \cdot \frac{1}{\cos\theta}$$

$$\left|\frac{\vec{E}(R)}{\vec{E}(r)}\right| = \frac{q/R^2}{q/r^2} = \frac{r^2}{R^2}$$

$$\frac{\vec{E}(R)\cdot\vec{A}}{\vec{E}(r)\cdot\vec{a}} = \underbrace{\cancel{\frac{r^2}{R^2}}}_{\text{fields}} \cdot \underbrace{\cancel{\frac{R^2}{r^2}}}_{\text{areas}} \cdot \cancel{\frac{1}{\cos\theta}} \cdot \underbrace{\cancel{\cos\theta}}_{\text{dot product}}$$

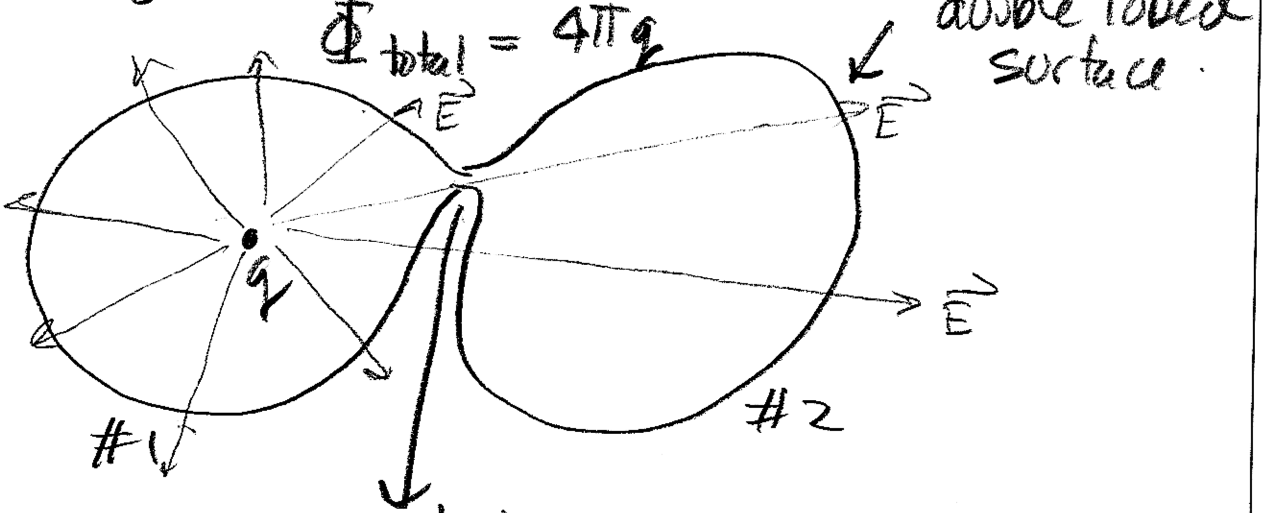
$= \underline{1}$

Flux through the first little patch equals the flux through the outer, bigger patch!, Sum up...

$$\int_{\text{closed surface}} \vec{E}\cdot d\vec{a} = 4\pi \sum_i q_i$$

only the charges in side..

Charges Outside?

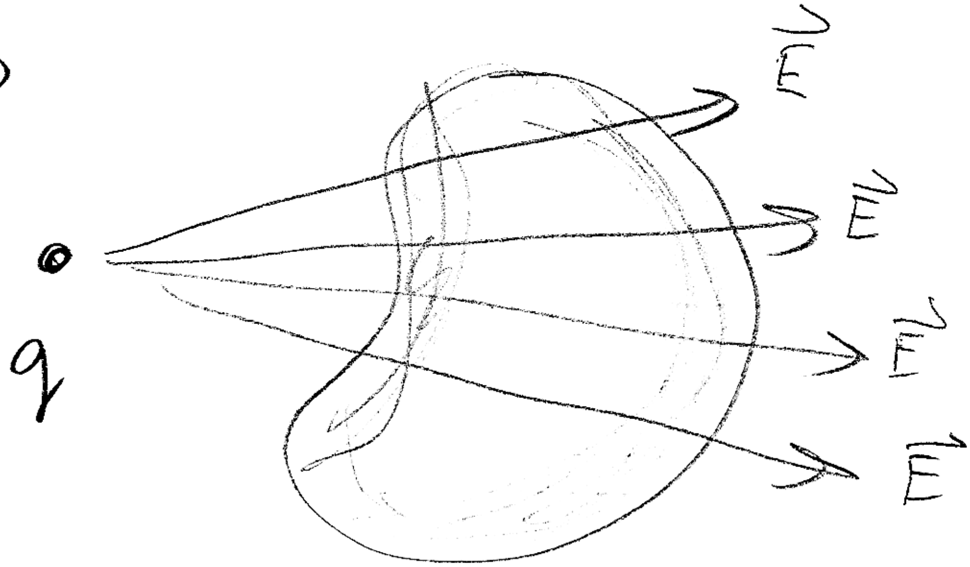


as this size $\to 0$

$$\Phi_1 \to 4\pi q$$

$$\Phi_2 = \Phi_{total} - \Phi_1 \to 0$$

so



$$\Phi = 0!$$

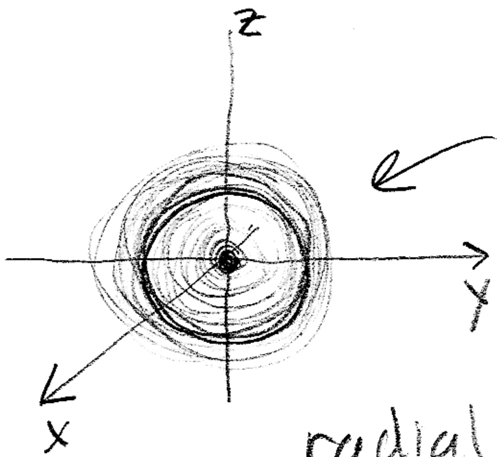
This is not true for all vector functions...

For example, what if $\vec{D} = \frac{q}{R^3}\hat{r}$?

then $\Phi_D = \int \vec{E}\cdot d\vec{a} = \left(\frac{q}{R^3}\right)\cdot 4\pi R = \frac{4\pi q}{R}$

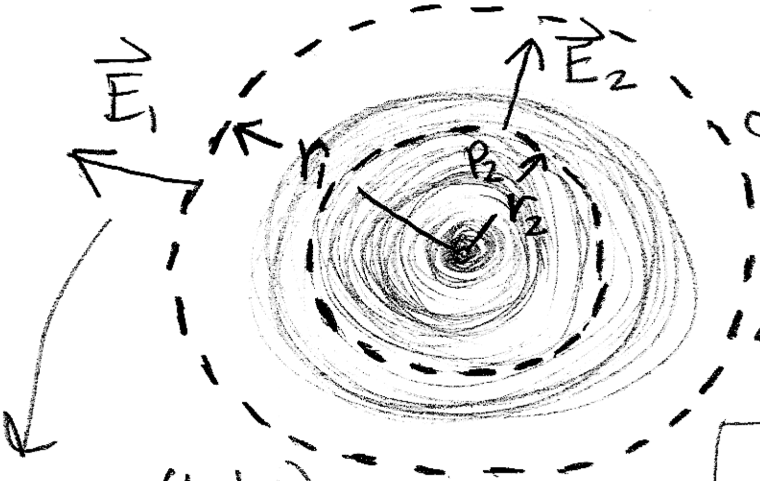
which is <u>not</u> independent of R.

Spherical Charge Distribution



$\rho(x,y,z)$ depends only on $r = \sqrt{x^2+y^2+z^2}$

$\vec{E}$ must <u>only</u> have radial component; Gauss's law really useful here:



at P_2

$$\underbrace{\int E_2\, da}_{\text{spherical symmetry}} = 4\pi\, q(\text{inside})$$

$$4\pi r_2^2 E_2 = 4\pi\, q(\text{inside})$$

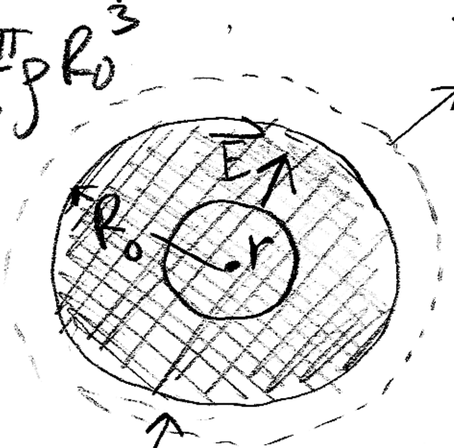
$$\boxed{E_2 = \frac{q(\text{inside})}{r_2^2}}$$

$$E_1 = \frac{q(\text{total})}{r_1^2}$$

same as if q concentrated at center!

Example:

$Q = \frac{4\pi}{3}\rho R_0^3$



charge uniformly distributed, density ρ

Inside

$$E(r) = \frac{\overbrace{\frac{4\pi}{3}\rho r^3}^{q(\text{inside})}}{r^2}$$

$$\boxed{E(r) = \frac{4\pi}{3}\rho r \quad r<R_0 \text{ inside}}$$

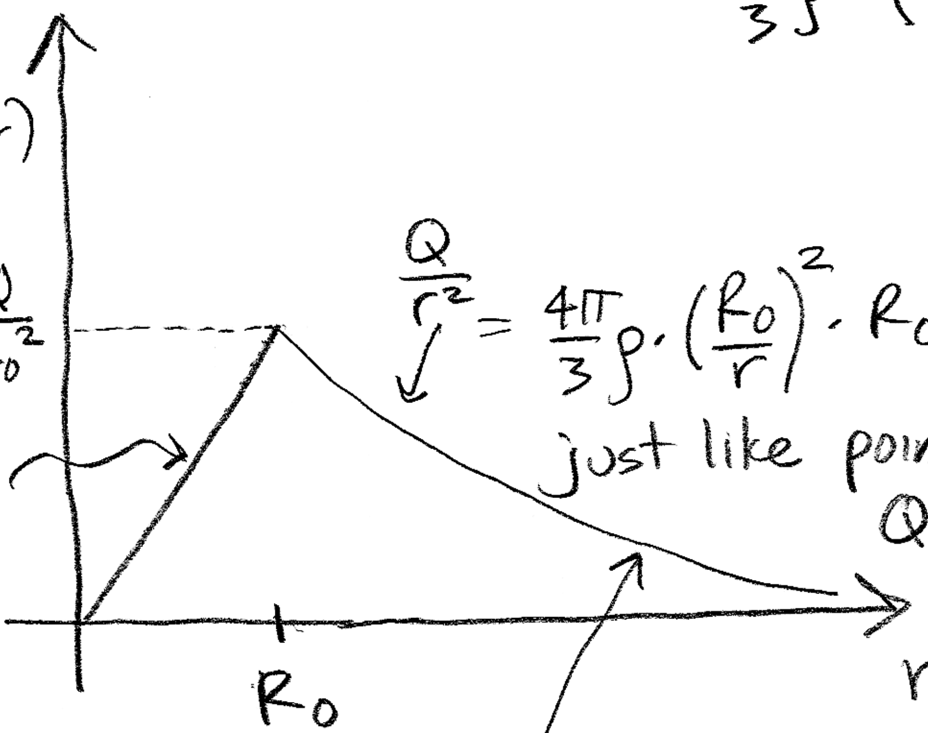
Outside

$$E(r) = \frac{\overbrace{\frac{4\pi}{3}\rho R_0^3}^{Q}}{r^2}$$

$$= \frac{Q}{r^2}$$

also

$$= \frac{4\pi}{3}\rho \cdot \left(\frac{R_0}{r}\right)^2 \cdot R_0$$



$\frac{4\pi}{3}\rho R_0 = \frac{Q}{R_0^2}$

slope $\frac{4\pi}{3}\rho r$

$\frac{Q}{r^2} = \frac{4\pi}{3}\rho \cdot \left(\frac{R_0}{r}\right)^2 \cdot R_0$

just like point charge Q!

not true for a cubical charge distribution