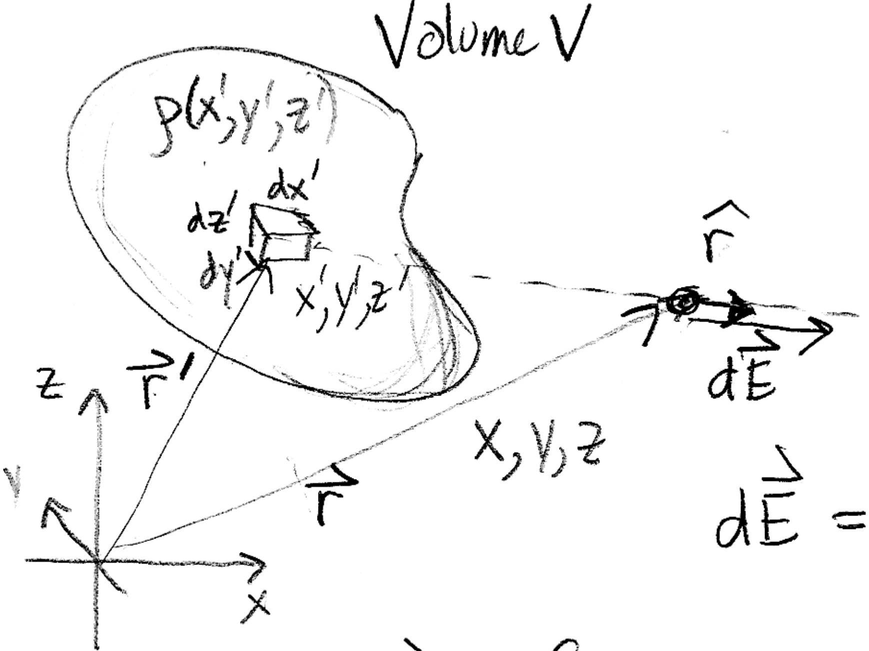
Then, imagine a region of space filled with a charge distribution:

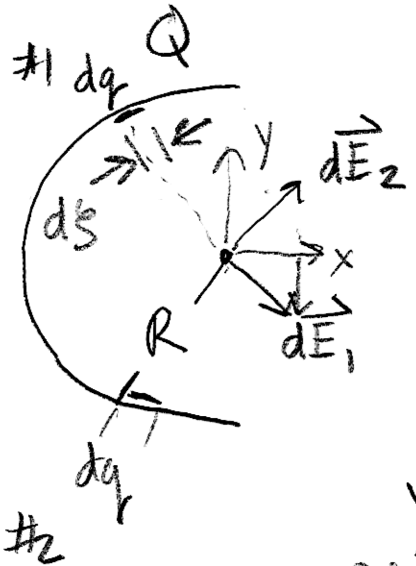


$$d\vec{E} = \frac{\hat{r}\,\rho(x', y', z')\,dx'dy'dz'}{|\vec{r} - r'|^2}$$

$$\vec{E} = \int_V \frac{\rho(x', y', z')\,dx'dy'dz'}{|\vec{r} - \vec{r}\,'|^2}\,\hat{r}$$

This is easier than it looks.... often, full 3-d not really needed.

Example:



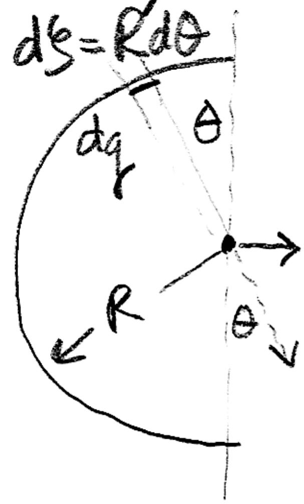
Q put evenly on plastic rod bent in semicircle of radius R

Find $\vec{E}$ field at center

$$\lambda = \frac{Q}{\pi R} \Leftarrow \tfrac{1}{2} \text{ circumference}$$

$$dq = \lambda\, ds$$

y-components of symmetric points (#1 + #2) cancel



$$|dE| = \frac{dq}{R^2}$$

$$dE_x = \frac{dq}{R^2} \cdot \sin\theta$$

$$= \frac{1}{R^2}\left(\frac{Q}{\pi R}\right) \cdot R\,d\theta \cdot \sin\theta$$

$$E_x = \frac{Q}{\pi R^2} \int_0^{\pi} d\theta \sin\theta$$

$$E_x = \frac{Q}{\pi R^2}\left(-\cos\theta \Big|_0^{\pi}\right)$$

$$= \frac{Q}{\pi R^2}\left(-(-1) - (-1)\right)$$

$$\boxed{E_x = \frac{2}{\pi}\frac{Q}{R^2} \quad (\text{gCGS}) \qquad = \frac{1}{4\pi\varepsilon_0}\frac{2}{\pi}\frac{Q}{R^2} \quad (\text{SI})}$$

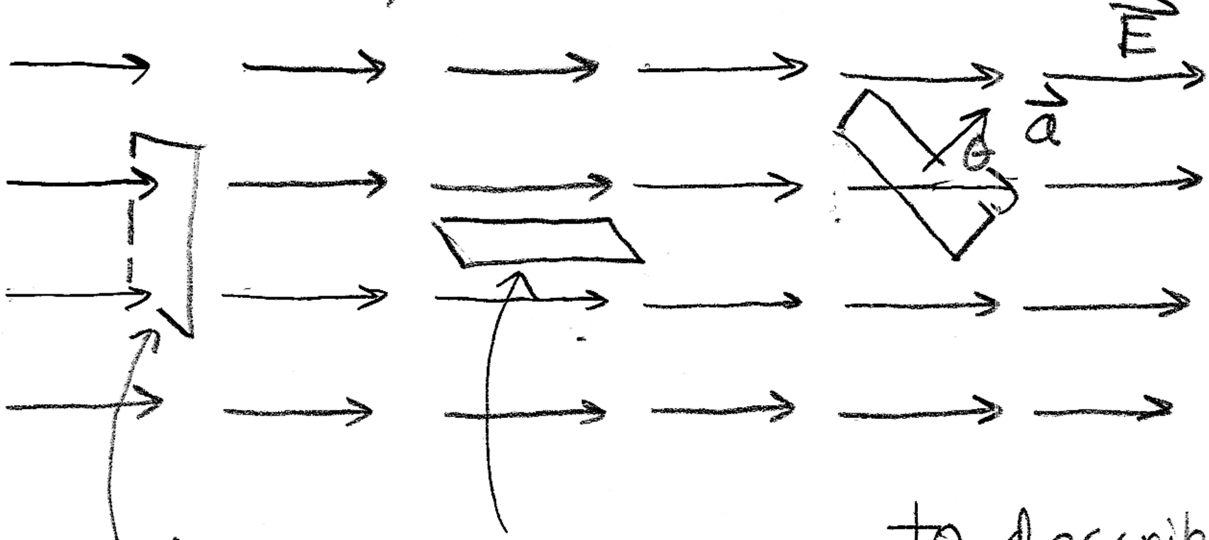
Flux

given: • a vector that is a function of location in space.

• an area A

the concept of flux quantifies how much of the vector goes through the area... so orientation of the area with respect to the vector field

Easiest example : vector field that is constant, always in x-direction



area A $\perp$ to constant $\vec{E}$
this case
$\Phi = E \cdot A$

area A $\parallel$ to constant $\vec{E}$
this case
$\Phi = 0$

to describe flux ... make a vector that describes A, that is $\perp$ to the surface
$|\vec{a}| = A$
direction $\perp$ to surface.

in this simple case

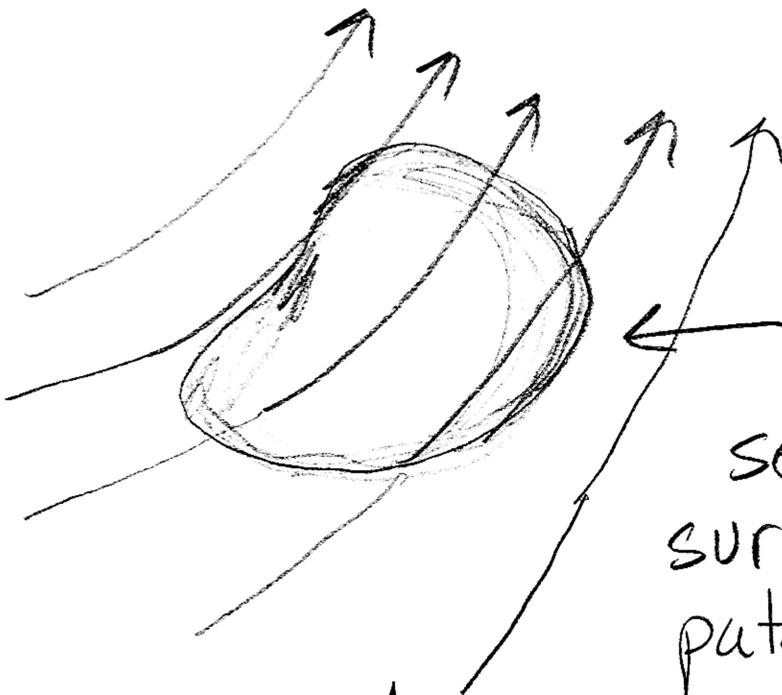
$$\Phi = \vec{E} \cdot \vec{a}$$

$$= E A \cos\theta$$

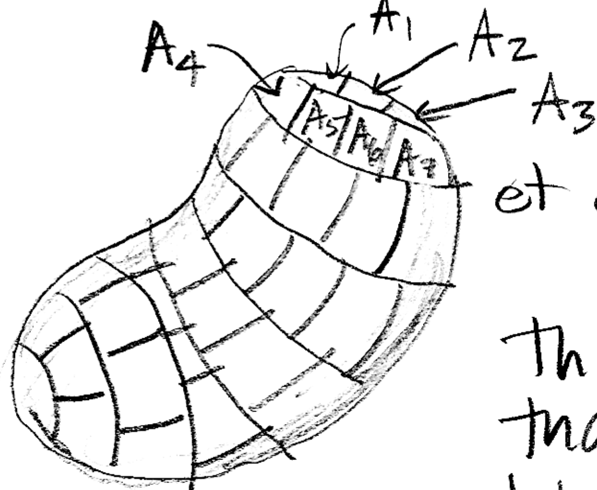
A $\perp$ to $\vec{E}$: $\theta = 0$, $\Phi = EA \cdot (\cos 0 = 1)$

$\parallel$ to $\vec{E}$: $\theta = \frac{\pi}{2}$, $\Phi = E \cdot A \cdot (\cos\frac{\pi}{2} = 0)$

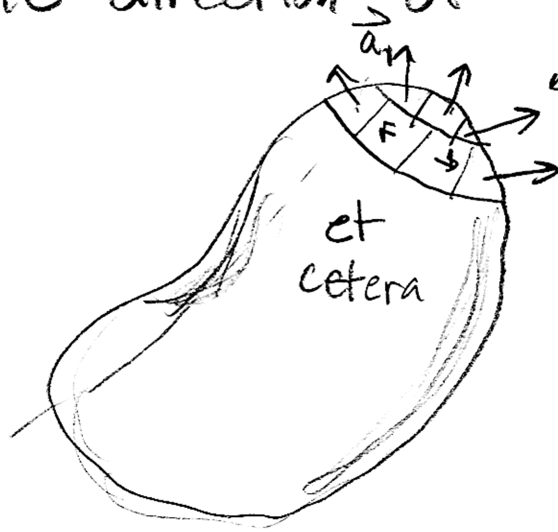
Surface Integral for Flux



segment surface into patches.

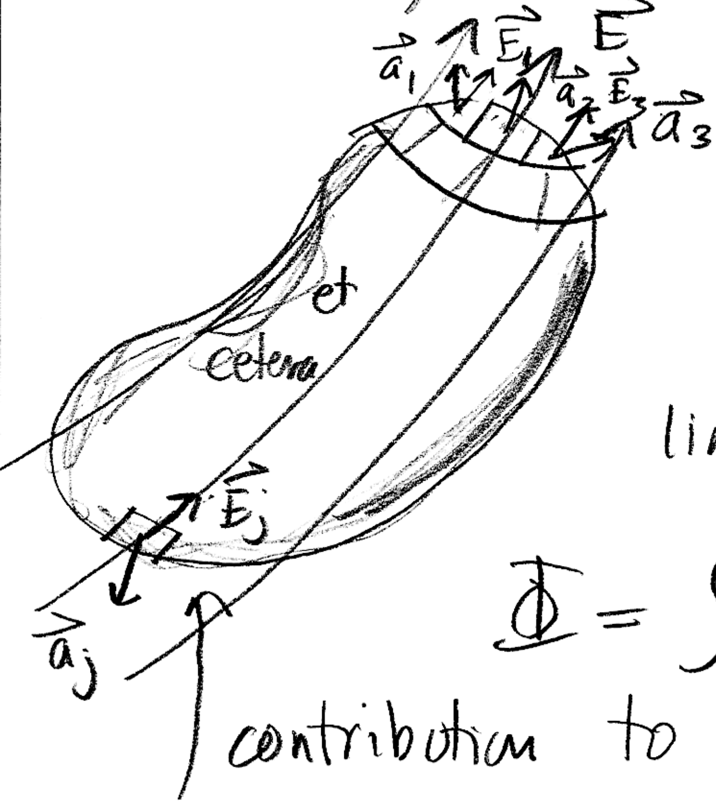


Then... make a direction that is $\perp$ to each patch.. that will be the direction of the normal vector



$\vec{a}_i = A_i \cdot$ (unit vector $\perp$ to surface)

Then... at the center of each patch, determine the value (magnitude + direction) of the vector field...



contributions to $\Phi > 0$

$$\underbrace{\Phi = \sum_{\text{all } j} \vec{E}_j \cdot \vec{a}_i}_{\text{limit as } \infty \text{ patches } \frac{1}{\infty} \text{ big}}$$

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{a}$$

contribution to flux < 0

<u>Intuition</u>: imagine changing $\vec{E}$ to the <u>velocity</u>, $\vec{v}$ of some fluid. Then $\Phi_v = \int \vec{v} \cdot d\vec{a}$ is proportional to the net mass of fluid going into the/out of the surface. For a closed surface that does not have a sink or a source of fluid, that will be zero!