	equal	pythag	Δf
C_4	261.6		0
D_4	293.7	294.3	-0.66
E_4	329.6	331.1	-1.5
F_4	349.2	348.8	0.39 Hz
G_4	392.0	392.4	-0.4 Hz
A_4	440.0	441.5	-1.5 Hz
B_4	493.9	496.7	-2.8 Hz
C_5	523.3	523.3	0
	Hz	Hz	

Beats

- neglect space dependence.

$$A(x,t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$$

amplitude

assume $A_1 = A_2$

$$= \frac{A_1}{2} \mathrm{Re}\left[e^{i\omega_1 t} + e^{i\omega_2 t}\right]$$

factor out $\bar{\omega} \equiv \frac{1}{2}(\omega_1 + \omega_2)$ "carrier freq"
exponent with

leave $\Delta\omega = \omega_1 - \omega_2$ "envelope"

$$\bar{\omega} + \tfrac{1}{2}\Delta\omega = \omega_1$$

$$\bar{\omega} - \tfrac{1}{2}\Delta\omega = \omega_2$$

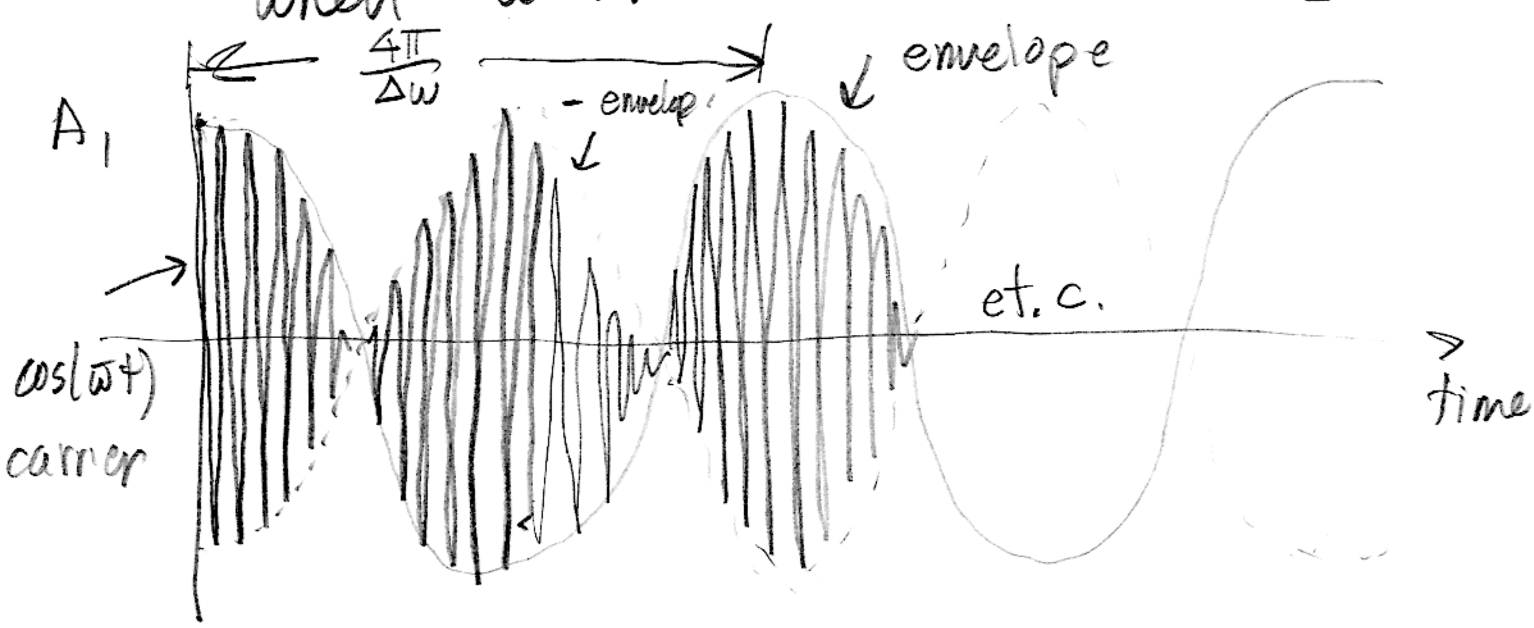
$$A(x,t) = \frac{A_1}{2}\,\mathrm{Re}\left[e^{i\bar{\omega}t}\left(e^{i\frac{1}{2}\Delta\omega t} + e^{-i\frac{1}{2}\Delta\omega t}\right)\right]$$

$$= A_1\,\mathrm{Re}\left[e^{i\bar{\omega}t}\cos\left(\tfrac{1}{2}\Delta\omega t\right)\right]$$

$$A(x,t) = A_1 \underbrace{\cos(\bar{\omega}t)}_{\text{fast}} \underbrace{\cos\left(\tfrac{1}{2}\Delta\omega t\right)}_{\text{slow}}$$

$$T_{env} = \frac{2\pi}{\frac{1}{2}\Delta\omega} = \frac{4\pi}{\Delta\omega}$$

when $\bar{\omega} \gg \Delta\omega$



$$f(x) = -\tfrac{1}{2}\hat{k}(x-L)^2 + \alpha \quad \text{when} > 0$$

$\alpha > 0$

$$= 0 \quad \text{other wise}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$f''(x \pm vt) = \frac{v^2}{v^2} f''(x \pm vt)$$

because of the -1 available to the $\cos(\bar{\omega} t)$, the "beat" period is:

$$T_{beat} = \frac{1}{2} T_{env} = \frac{2\pi}{\Delta\omega}$$

$$\boxed{f_{beat} = \frac{1}{T_{beat}} = \frac{\Delta\omega}{2\pi} = \Delta f}$$

Radio.... sonic frequencies <u>way</u> lower than radio waves

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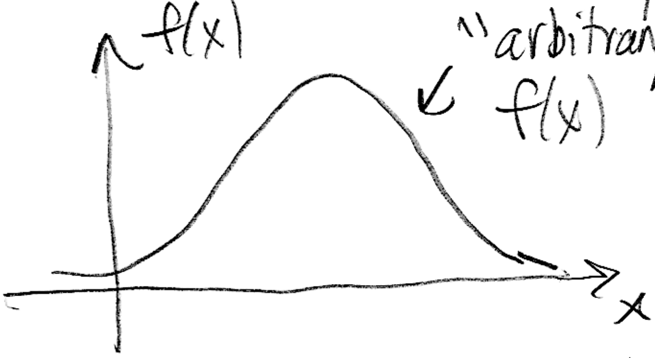
Solution of wave equation, $L \to \infty$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

when no boundary conditions, super easy!



← can make a solution.

change x to $x \pm vt$

$$f(x) = -\frac{1}{2}k(x-L)^2 + \alpha \quad \text{when } > 0$$

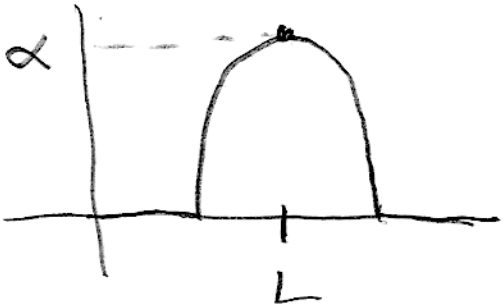
$\uparrow$ $\alpha > 0$

$$= 0 \quad \text{otherwise.}$$

$$\frac{\partial^2 y}{\partial x^2} \stackrel{?}{=} \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

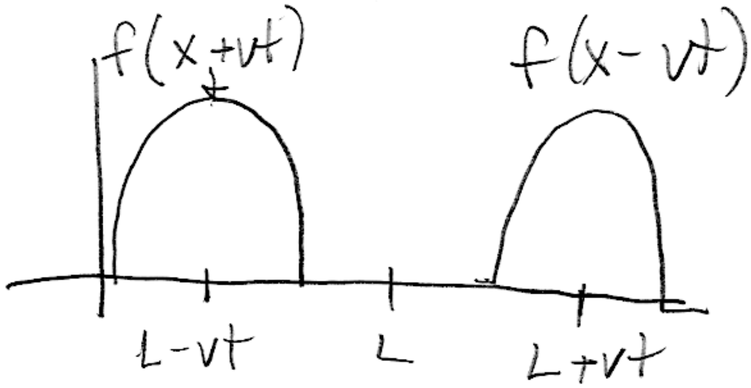
$$f''(x \pm vt) = \frac{v^2}{v^2} f''(x \pm vt)$$

✓ yes!



$$f(x \pm vt) = -\frac{1}{2}k(x \pm vt - L)^2 + \alpha$$

at time t later.



a moving "pulse"

$$\frac{\partial^2 y}{\partial x^2} = f''(x \pm vt)$$

$$\frac{\partial y}{\partial t} = f'(x \pm vt)\cdot(\pm v)$$

$$\frac{\partial^2 y}{\partial t^2} = f''(x \pm vt)\, v^2$$

$$\frac{\partial^2 y}{\partial x^2} \stackrel{?}{=} \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$f''(x \pm vt) = \frac{\cancel{v^2}}{\cancel{v^2}} f''(x \pm vt)$$

✓ yes!