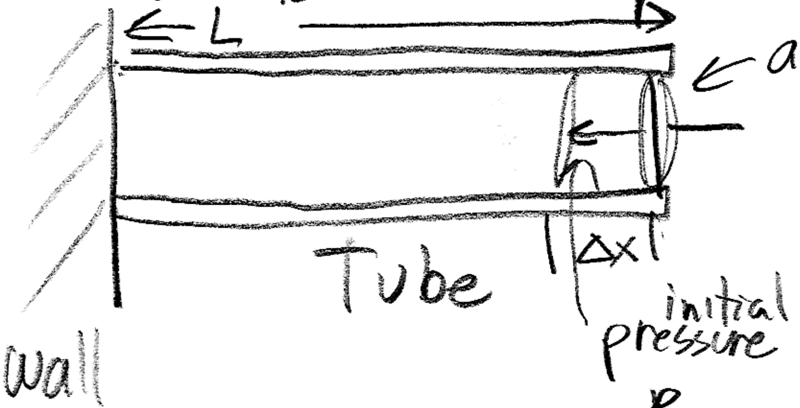
Ideal Gases

What is the equivalent of Y?



$V = A \cdot L$

What is Δp? $\Delta V = A \Delta x$

↑ depends!

$\gamma = \frac{c_p}{c_v} = \frac{f+2}{f}$

isothermal (rare) | adiabatic (usual)

$pV = \text{constant} = NkT$ | $pV^\gamma = \text{constant}$

$p\Delta V + \Delta p V = 0$ | $p \gamma V^{\gamma-1} \Delta V + \Delta p V^\gamma = 0$

$\Delta p = -p \frac{\Delta V}{V}$ | $\Delta p = -\gamma p \frac{\Delta V}{V}$

$= -p \frac{\Delta x}{L}$ strain | $= -p \frac{\Delta x}{L}$ strain

$Y \rightarrow p$

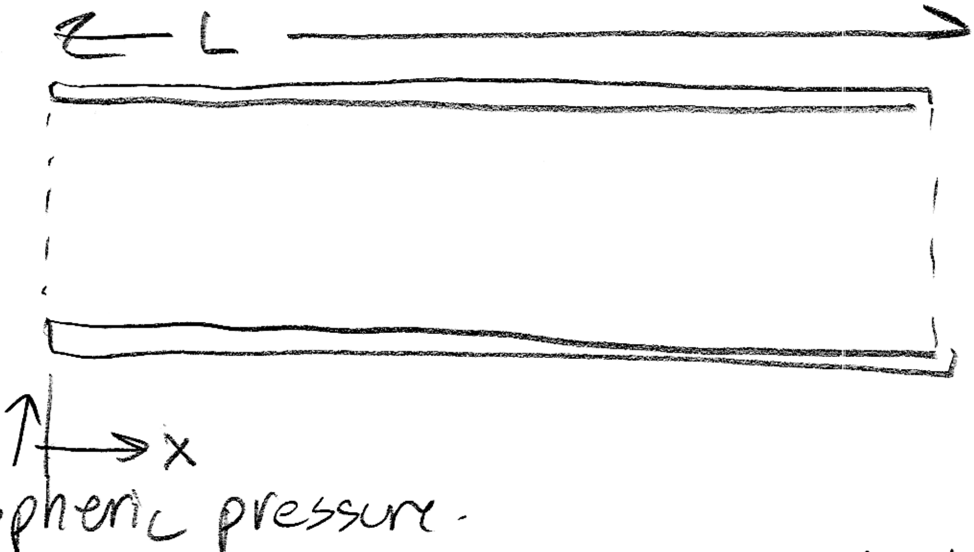
$$Y \rightarrow \gamma p$$

real air

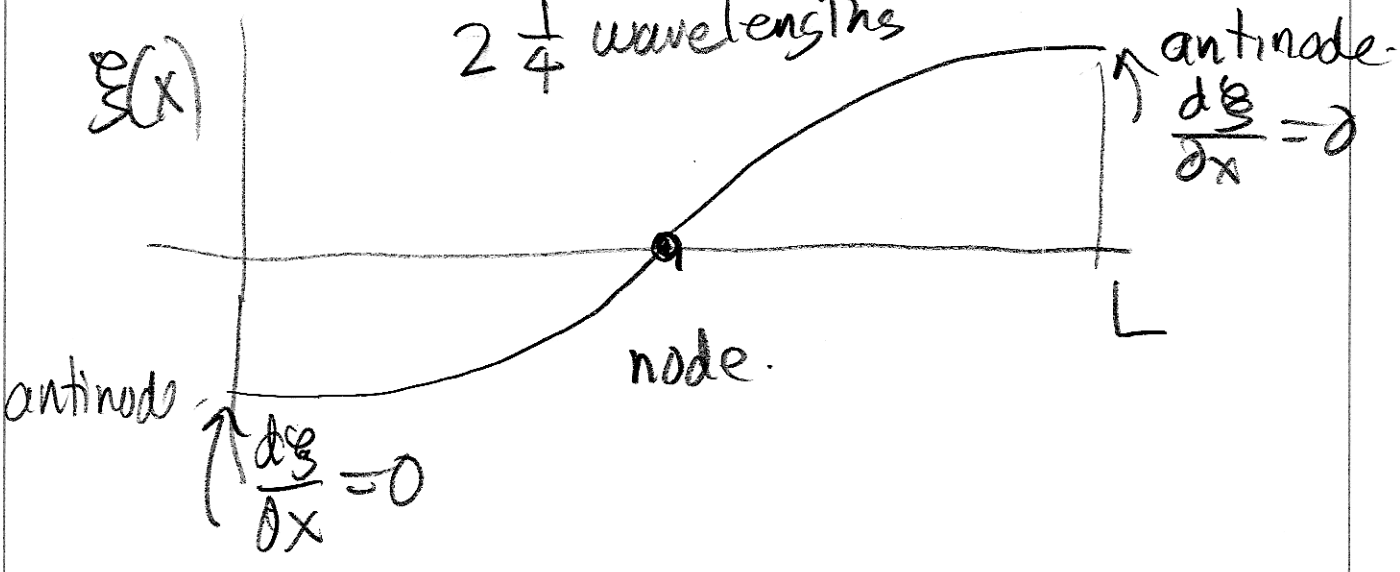
But now...

stress → pressure increment $\propto \frac{\partial \xi}{\partial x}$ (ξ ← longitudinal displacement)

Double-Open-Ended Tube



$$\frac{\partial \xi}{\partial x}(x=0)=0 \qquad \frac{\partial \xi}{\partial x}(x=L)=0$$



nth mode: $\cos\left(\frac{n\pi}{L}\right)\sin(\omega_n t)$

where now: $\omega_n = \frac{n\pi}{L} \cdot v$

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

for an ideal gas.....

$$pV = nRT$$

n ↑ # moles; R ↖ 8.3 J/K/mole.

$$M = n \cdot A_W \qquad A_W = \text{molecular Weight}$$

$$\rho = \frac{M}{V} = \frac{n A_W}{V} = \frac{pV \cdot A_W}{V \cdot RT} = \frac{p A_W}{RT}$$

$$\boxed{v = \sqrt{\frac{\gamma \cdot p}{\left(\frac{p A_W}{RT}\right)}} = \sqrt{\frac{\gamma \cdot R \cdot T}{A_W}}}$$

← only depends on T!

$\gamma = \frac{5}{3}$ (monatomic gas)

$= \frac{7}{5}$ (diatomic)

$T \rightarrow$ absolute temp.

Air : mostly nitrogen

N_2 $A_W \simeq 28$ (78%)

O_2 $A_W \simeq 32$ (21%)

$\bar{A}_W$ 28.6 gm/mole

+ other gases : $\boxed{\text{Air } \bar{A}_W \simeq 29 \frac{\text{gm}}{\text{mole}}}$

$= 0.029 \frac{\text{kg}}{\text{mole}}$.

$\gamma = \frac{7}{5} = \frac{5+2}{5} = 1.4$

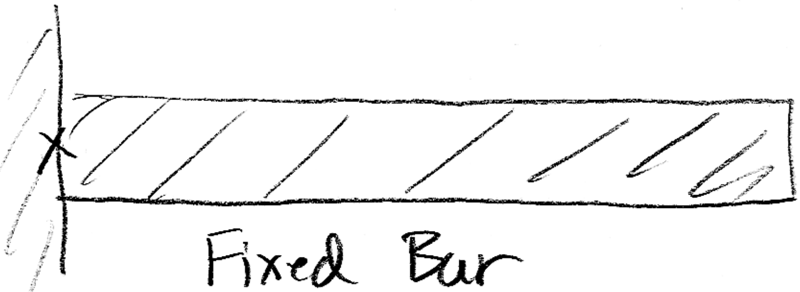
$R = 8.31$

$T = 293\ ^\circ K$ ($\simeq 20^\circ C$, room Temp)

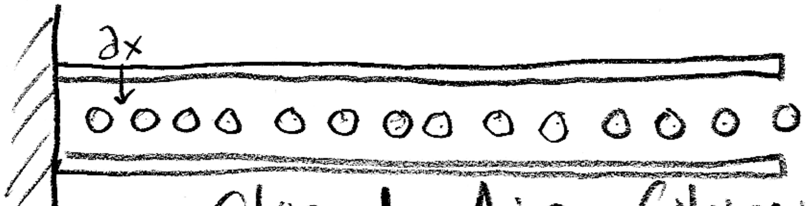
$v = \sqrt{\frac{1.4 \cdot 8.31 \cdot 293}{0.029}}$

$\boxed{v = 343\ \text{m/s}}$

Visualization of Stress, Strain, + Pressure Differential in Rods, Columns

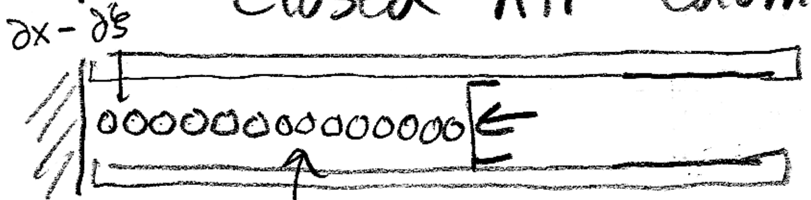


Fixed Bar



$$\frac{d\xi}{dx} = 0$$

Closed Air Column

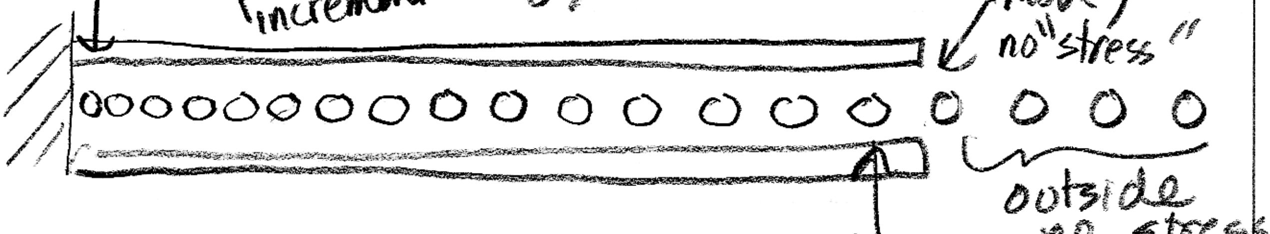


higher pressure, molecules closer

$$\frac{\partial \xi}{\partial x} = \text{constant} = \text{big!}$$

large $\frac{d\xi}{dx} \propto \delta p$ pressure increment

$$\frac{\partial^2 \xi}{\partial x^2} = 0$$



$\frac{d\xi}{dx}$ falls to 0