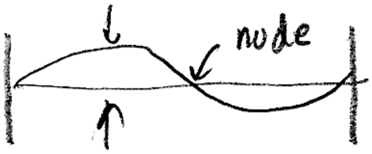
Summary

$k L$ ⟶ "Normal Mode" : Time $\quad \nu = \frac{\omega}{2\pi}$

$n=1$ 
$A \cdot \sin\left(\frac{\pi}{L}\cdot x\right) \times \sin(\omega_1 t) \qquad \sqrt{\frac{T}{\mu}}\cdot\frac{1}{2L} = \frac{v}{2L}$

C_4 A antinode $\qquad k_1$

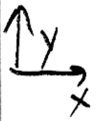
2 
$A \cdot \sin\left(\frac{2\pi}{L} x\right) \times \sin(\omega_2 t) \qquad \frac{v}{2L}\times 2$

node

A octave C_5 $\qquad 2k_1 \qquad \omega_2 = 2\cdot\omega_1$

3 
$A \cdot \sin\left(\frac{3\pi}{L} x\right) \times \sin(\omega_3 t) \qquad \frac{v}{2L}\times 3$

A octave + fifth G_5 $\qquad 3k_1 \qquad \omega_3 = 3\omega_1$



all satisfy $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$

(1) $A\sin(k_n x)\sin(\omega_n t + \delta)$ works too

what about $\qquad k_n = \frac{\omega_n}{v}$

$A\sin(k_n x + \delta)\sin(\omega_n t)$?

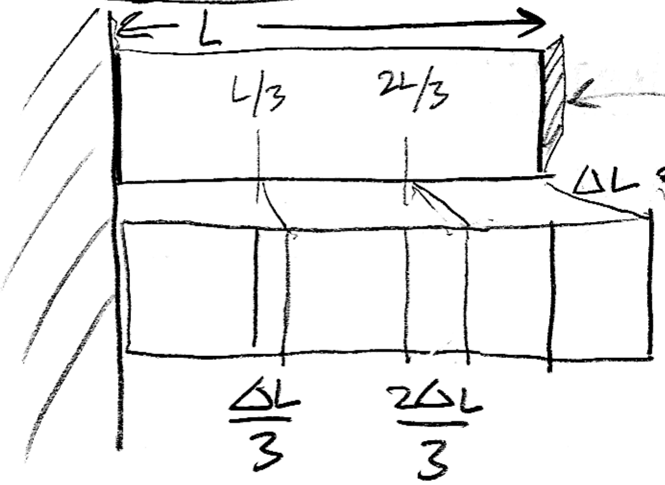
Physical meaning :

(2) $A_n \sin(k_n x)\sin(\omega_n t) + A_m \sin(k_m x)\sin(\omega_m t)$

also a solution

→ linear superposition.

Longitudinal waves



bar, length L
cross section (shape not important)
area A

stretch by ΔL

what is "constant" throughout bar is $\dfrac{\frac{\Delta L}{3}}{\frac{L}{3}}, \dfrac{\frac{2\Delta L}{3}}{\frac{2L}{3}}, \dfrac{\Delta L}{L}$

strain $= \dfrac{\Delta L}{L}$ (note: cross sectional area does get a little smaller... not much).

dimensionless

To achieve a strain, apply a certain force, per unit area

"stress" $= \dfrac{F}{A}$ ← F ∥ to displacement, ← A ⊥ to dislacement.

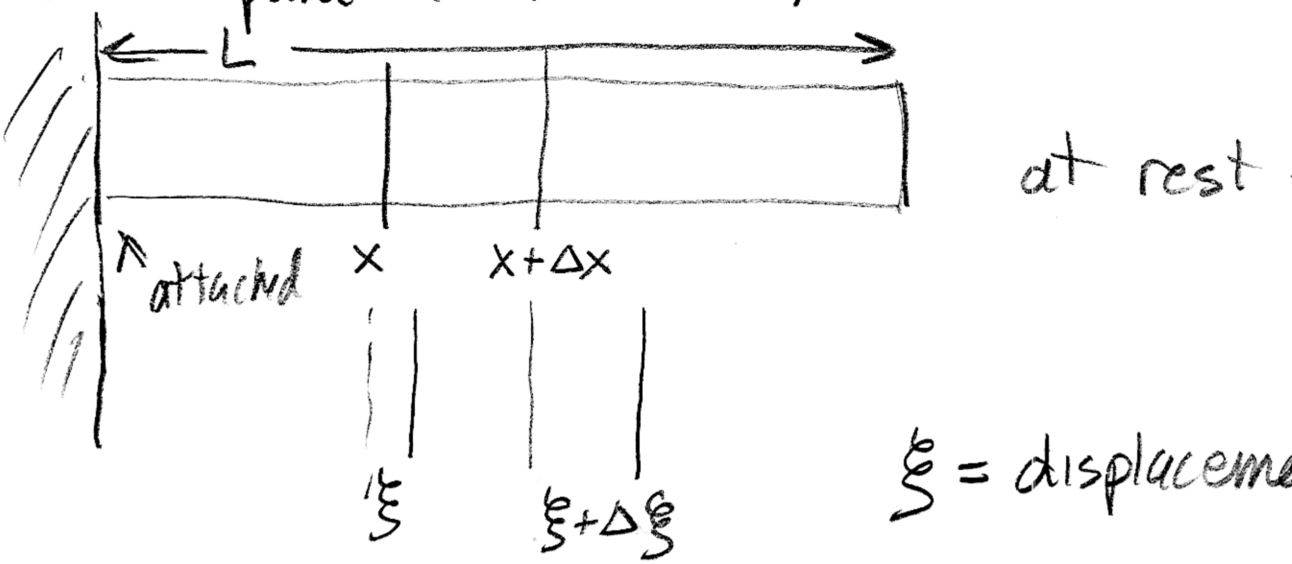
Rod fights back with stress $-\dfrac{F}{A}$ ...

"Hooke's Law" → $-\dfrac{F}{A} = Y \dfrac{\Delta L}{L}$

dimensions of pressure   no dimensions

Material	Y
Aluminum	$60 \cdot 10^9$ ($Pa = \frac{N}{m^2}$)
Steel	$200 \cdot 10^9$ "

Bars vibrate like strings, only now the displacement is hard to visualize, it is parallel to bar!



ξ = displacement

in interval Δx, average strain is

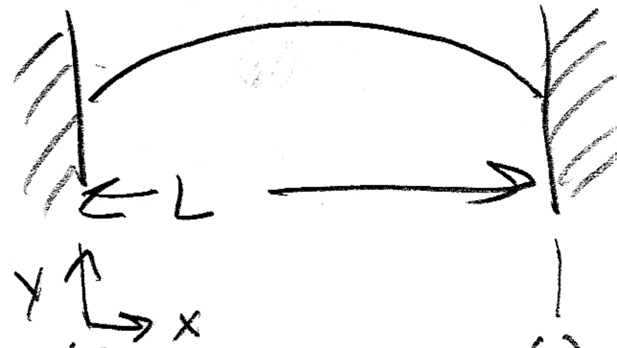
$$\frac{\Delta \xi}{\Delta x} \rightarrow \frac{\partial \xi}{\partial x} \ldots$$ need $\frac{\partial^2 \xi}{\partial x^2}$ for waves,

N2 + algebra $\Rightarrow$ $$\frac{\partial^2 \xi}{\partial x^2} = \frac{\rho}{Y} \frac{\partial^2 \xi}{\partial t^2}$$

$\mu \rightarrow \rho \qquad T \rightarrow Y$

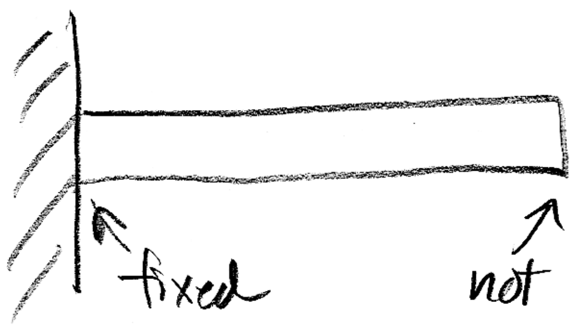
Boundary Conditions

String:



$y(0)=0$ $\qquad$ $y(L)=0$

Bar, Fixed One end



$\xi(0)=0$ $\qquad$ stress $\propto$ strain

$$\propto \frac{\partial \xi}{\partial x} = 0$$

Fundamental: $\frac{1}{4}$ wavelength

Normal Mode Fundamental: $\xi(x) \propto \sin\left(\frac{2\pi}{4L} \times x\right)$

$$\propto \sin\left(\frac{\pi}{2L} x\right)$$

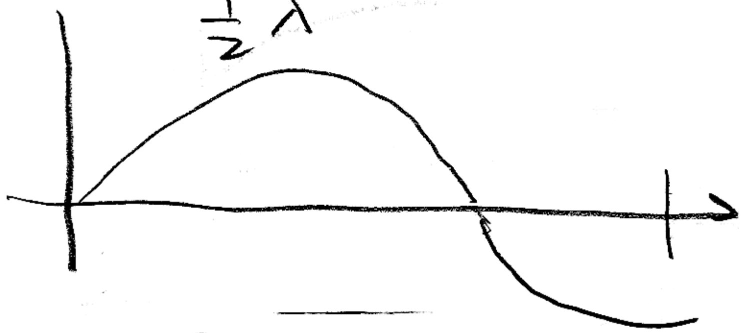
$$\omega_1 = \frac{\pi}{2L} \times v \qquad v = \sqrt{\frac{Y}{\rho}}$$

Overtones

$n=1$ $\lambda = 4L$

$n=2$

$\frac{1}{2}\lambda$



$\frac{1}{4}\lambda$

$L = \frac{3}{4}\lambda$

$\lambda = \frac{4}{3}L$

$n=3$

$\leftarrow 1\lambda \rightarrow \frac{1}{4}\lambda$

$L = \frac{5}{4}\lambda$

$\lambda = \frac{4}{5}L$

$$\boxed{\lambda = \frac{4}{2n-1}L}$$

$$\xi(x) = \sin\left(\frac{2\pi}{\left(\frac{4}{2n-1}\right)L}x\right)$$

$$= \sin\left(\frac{\left(n-\frac{1}{2}\right)\pi}{L}x\right)$$

$$\boxed{\omega_n = \frac{\left(n-\frac{1}{2}\right)\pi}{L}\cdot v \qquad v = \sqrt{\frac{Y}{\rho}}}$$