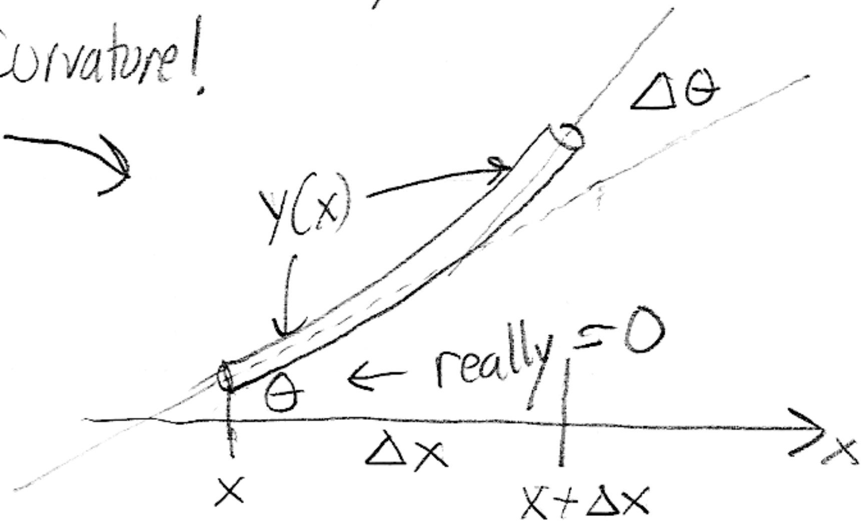
Need Curvature!

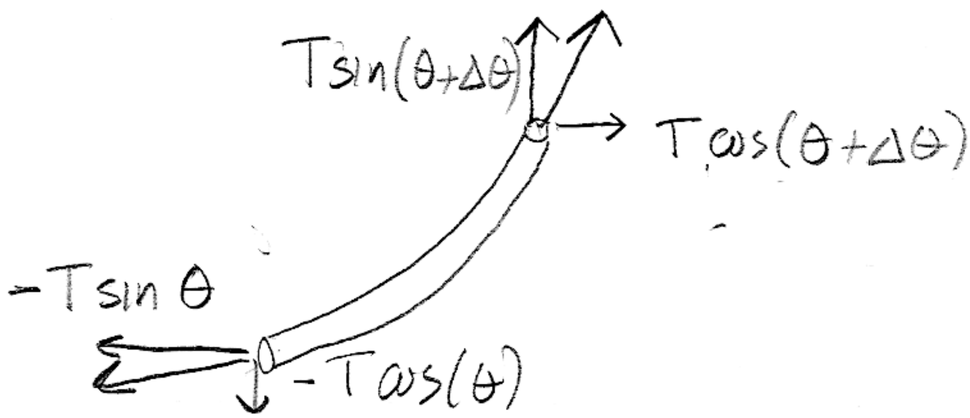



$$\tan\theta(x) = \frac{\partial y}{\partial x}$$

$$\theta(x) \simeq \frac{\partial y}{\partial x}$$

$$\theta(x+\Delta x) \cong \cancel{\frac{\partial y}{\partial x}} + \Delta x \frac{\partial^2 y}{\partial x^2} \simeq \cancel{\theta} + \Delta\theta$$

$$\boxed{\Delta\theta = \Delta x \frac{\partial^2 y}{\partial x^2}}$$




assume $\theta$ very small (unlike diagram!)

$$\cos(\theta+\Delta\theta) = \cos\theta\cos\Delta\theta - \sin\theta\sin\Delta\theta$$

$$\cong \underbrace{\cancel{\cos\theta}}_{1} - \cancel{\theta\Delta\theta}$$

second order

$$\sin(\theta+\Delta\theta) = \sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta$$

$$\sin(\theta+\Delta\theta) \simeq \sin\theta + \Delta\theta \underbrace{\cos\theta}_{\approx 1}$$

$$\simeq \sin\theta + \Delta\theta \simeq \theta + \Delta\theta$$

Net Force:

$$\sum F_x = T\cdot 1 - T\cdot 1 \cong 0$$

$$\sum F_y = T(\theta + \Delta\theta) - T\theta$$

$$= T\Delta\theta \simeq T\Delta x \frac{\partial^2 y}{\partial x^2}$$

$\boxed{\text{Newton II}}$

$$\underbrace{(\mu\cdot\cancel{\Delta x})}_{\text{mass}}\underbrace{\frac{\partial^2 y}{\partial t^2}}_{\text{accel.}} = T\cancel{\Delta x}\frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}}$$

- wave equation
- $$\left[\frac{\mu}{T}\right] = \frac{\frac{\cancel{kg}}{m}}{\frac{\cancel{kg}\cdot m}{s^2}} = \frac{s^2}{m^2} = \left(\frac{1}{\text{velocity}}\right)^2$$
- $$\frac{1}{v^2} = \frac{\mu}{T}, \quad v^2 = \frac{T}{\mu}$$

wave speed

more tension, faster

more density, slower

"Pure Tones" $\longleftrightarrow$ "Normal Mode"

one frequency
time

spatial pattern
yields pure tone

$$y_n(x,t) = A \cdot \sin(\omega t) \cdot \sin\left(\frac{n\pi}{L} x\right)$$ (not unique!)

must satisfy

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial y_n}{\partial x} = A \cdot \left(\frac{n\pi}{L}\right) \sin(\omega t) \cos\left(\frac{n\pi}{L} x\right)$$

$$\frac{\partial^2 y_n}{\partial x^2} = -A \left(\frac{n\pi}{L}\right)^2 \sin(\omega t) \sin\left(\frac{n\pi}{L} x\right)$$

$$= -\underbrace{\left(\frac{n\pi}{L}\right)^2}_{} \cdot y_n(x) = -k_n^2\, y_n(x)$$

$k_n \equiv \frac{n\pi}{L}$ <u>wave-number</u>

$$\frac{\partial^2 y_n}{\partial t^2} = -\omega^2 y_n$$

$$-k_n^2\, \cancel{y_n(x)} = -\frac{\omega^2}{v^2} \cancel{y_n(x)}$$

$$\boxed{\omega_n = v \cdot k_n = \sqrt{\frac{T}{\mu}} \cdot n \frac{\pi}{L}}$$

$$\boxed{\nu_n = \frac{\omega_n}{2\pi} = \sqrt{\frac{T}{\mu}} \cdot \frac{n}{2L}}$$ (6-8) p. 165 French

$$\omega_n = n\omega_1 \qquad f_n = nf_1$$

fundamental frequency

often, excitation methods for any vibrating object (musical instrument, building in earthquake, etc) don't excite only one frequency...

Note, wave equation is linear

$$f(y) = a y_m(x,t) + b y_n(x,t) \quad y_m(x,t) = y_n(x,t)$$

and $$\frac{\partial^2 y_m}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_m}{\partial t^2} \qquad \frac{\partial^2 y_n}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_n}{\partial t^2}$$

then (you show)

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

so,

$$a_1 y_1(x,t) + a_2 y_2(x,t) + a_3 y_3(x,t) + \ldots$$

$$\parallel$$

$$a_1 \sin(\omega_1 t)\sin(k_1 x) + a_2 \sin(2\omega_1 t)\sin(2k_1 x) + a_3 \sin(3\omega_1 t)\sin(3k_1 x)$$

$$\omega_1 = \sqrt{\frac{T}{\mu}}\frac{\pi}{L} \quad k_1 = \frac{\pi}{L}$$

fundamental first overtone second overtone