Physics 23 Problem Set 2

Harry Nelson

Due Monday, October 3

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

---

1. The function $y_n(x,t) = A\sin(k_n x)\sin(\omega_n t)$ satisfies the 1-d wave equation:

$$\frac{\partial^2 y_n}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_n}{\partial t^2}$$

because the wave number k_n and the frequency ω_n are related by $k_n = \omega_n/v$.

   (a) Plot $y_n(x, l\pi/\omega_n)$, where l is any integer.

   (b) Show that $z_n(x,t) = A\sin(k_n x)\sin(\omega_n t + \delta)$ is also a solution to the wave equation, where δ is a constant. Qualitatively, how will a plot of $z_n(x, 2\pi/\omega_n)$ look for $\delta \neq 0$? Why could one say a non-zero δ 'shifts the time origin'?

   (c) Show that $a_n y_n(x,t) + a_m y_m(x,t)$, for $n \neq m$, and wher a_n and a_m are constants, is also a solution to the wave equation.

2. This is a problem about the actual frequencies on a guitar. The length of the strings on the guitar is 63.5 cm, and the strings are made of phosphor-bronze, which has a mass density of $\rho = 8\,\mathrm{gm/cm^3}$. The notes on the guitar, the diameter D of the strings, and the desired frequencies ν are in the following table: fill in the table for the tension T both in newtons and pounds. What is the total force on the ends of the guitar? (Remember to convert quantities to standard, Meters-Kilograms-Seconds units; also note that the frequencies here are customized to a particular guitar).

Note	D **(inches)**	ν **(Hz)**	T **(newtons)**	T **(pounds)**
$\mathbf{E_2}$	0.056	81		
$\mathbf{A_2}$	0.046	108		
$\mathbf{D_3}$	0.036	144		
$\mathbf{G_3}$	0.026	192		
$\mathbf{B_3}$	0.017	243		
$\mathbf{E_4}$	0.013	324		

3. As discussed in class, the first overtone of the fundamental frequency sounds one octave higher. So, if a string is tuned to middle C, which we'll denote by C_4 with a frequency of 261.63 Hz, then the frequency of $2 \times 261.63 = 523.25$ Hz is high C, or C_5.

   The chromatic scale splits the frequencies in an octave into 12 steps. Let's label the notes in the octave between C_4 and C_5 by an index i, and let's call C_4 $i = 0$ and C_5 $i = 12$. The index i can be called the 'step' of the note. In modern times, something called 'equal temperament' is used to find the frequencies of the notes:

$$\nu_i = 2^{(i/12)} \times \nu_0$$

where ν_0 is the frequency of the lowest note in the octave, which for this problem is 261.63 Hz.

Long ago, people did not know how to easily calculate the $2^{(i/12)}$ factor. Also, there is an aesthetic problem with equal temperament: the interval beween G_4 and C_4, where G_4 has index $i = 7$, sounds a little off to people with good ears. This is because $2^{(7/12)} = 1.4983$, while people with good ears would like the ratio of ν_7/ν_0 to be exactly 1.5. If the ratio were exactly 1.5, then G_4 would be exactly an octave below the *second overtone* of C_4; the second overtone has a frequency exactly 3 times higher than the fundamental.

Long ago, what people did to make a chromatic scale was devise various ratios of 3^x and 2^y to evaluate the twelve steps in the octave. This set of steps is sometimes called the 'Pythagorean Tuning'. Rather then spend a lot of time explaining it, just compute the missing entries in the following table, where (1) denotes the 'equal temperament' and (2) denotes 'Pythagorean Tuning'. For the equal temperament, use the $2^{i/12}$ factor to compute ν_i/ν_0; for Pythagorean Tuning, use the factor indicated. The interval that many people are sensitive to is related to the ratio ν_{i+7}/ν_i; to complete this column in the table, you'll have to extend a few notes beyond $i = 12$ (think how to do this...). In the last column, but the difference between frequencies for the two tunings for the specified note, (1)-(2). Your ear can actually hear $\Delta\nu$, as a so-called 'beat', which we'll discuss later.

		Equal Temperament (1)			Pythagorean Tuning (2)			
Step	Note (piano key color)	ν_i/ν_0	ν_i (Hz)	ν_{i+7}/ν_i	ν_i/ν_0	ν_i (Hz)	ν_{i+7}/ν_i	$\Delta\nu$ (Hz)
0	C_4 ○	1	261.6	1.498	1	261.6	1.500	0
1	$C\sharp_4/D\flat_4$ ●				$2^8/3^5$			
2	D_4 ○				$3^2/2^3$			
3	$D\sharp_4/E\flat_4$ ●				$2^5/3^3$			
4	E_4 ○				$3^4/2^6$			
5	F_4 ○				$2^2/3^1$			
6	$F\sharp_4/G\flat_4$ ●				$3^6/2^9$			
7	G_4 ○				$3^1/2^1$			
8	$G\sharp_4/A\flat_4$ ●				$2^7/3^4$			
9	A_4 ○				$3^3/2^4$			
10	$A\sharp_4/B\flat_4$ ●				$2^4/3^2$			
11	B_4 ○				$3^5/2^7$			
12	C_5 ○	2	523.3	1.498	2	523.3	1.500	0

4. Design a thin solid rod of length L with that has a fundamental frequency of C_4, middle C, which has frequency $\nu_{C_4} = 261.6\,\text{Hz}$; assume the rod is fixed at one end and free at the other end. Numerically evaluate L for aluminum, which has a density of $2.7\,\text{gm/cm}^3$ and a Young's modulus of 60×10^9 Pa, and for lead, which has a density of $11.4\,\text{gm/cm}^3$ and a Young's modulus of 16×10^9 Pa. Actually, lead has a small 'Q', meaning it dissipates energy, so it does not 'ring' very well.

5. Repeat the last problem, but do so for a column of air which is open at both ends.