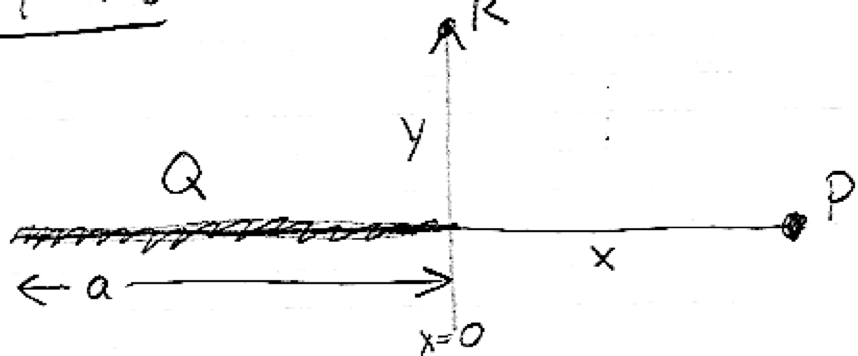
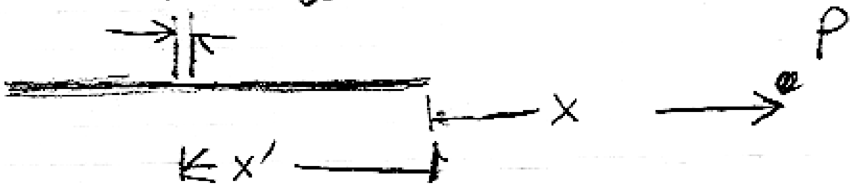
24-73:



Find potensials at points R an P, taking potential to be 0 at ∞

(a) Point P:



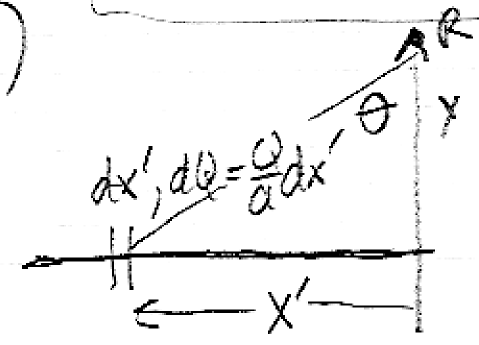
at P: $dV = \dfrac{dQ}{4\pi\varepsilon_0 (x-x')} = \dfrac{Q}{4\pi\varepsilon_0 a}\dfrac{dx'}{(x-x')}$

$$V = \int_{-a}^{0} \frac{Q}{4\pi\varepsilon_0 a}\frac{dx'}{x-x'} = \frac{Q}{4\pi\varepsilon_0 a}\left(-\ln(x-x')\right)\Big|_{-a}^{0}$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\left[-\ln x + \ln(x+a)\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{x+a}{x}\right)}$$

(b)



$dV = \dfrac{dQ}{4\pi\varepsilon_0\sqrt{x'^2+y^2}} = \dfrac{Q}{4\pi\varepsilon_0 a}\dfrac{dx'}{\sqrt{y^2+x'^2}}$

substitution:

$\cos\theta = \dfrac{y}{\sqrt{y^2+x'^2}}$ $\quad \tan\theta = \dfrac{-x'}{y}$

$$\frac{\cos\theta}{y} = \frac{1}{\sqrt{y^2+x'^2}} \qquad y\frac{d\theta}{\cos^2\theta} = -dx'$$

$$\frac{\cos\theta}{y}\cdot y\frac{d\theta}{\cos^2\theta} = \frac{-dx'}{\sqrt{y^2+x'^2}}$$

$$\frac{dx'}{\sqrt{y^2+x'^2}} = \frac{-d\theta}{\cos\theta}$$

so $$V = -\int_{\tan^{-1}(-\frac{a}{y})}^{\tan^{-1}(0)} \frac{Q}{4\pi\varepsilon_0 a}\cdot\frac{d\theta}{\cos\theta} = \frac{Q}{4\pi\varepsilon_0 a}\ln(\tan\theta+\sec\theta)\Big|_{\tan^{-1}(0)}^{\tan^{-1}(\frac{a}{y})}$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\left(\ln\left[\left(\frac{a}{y}\right)+\frac{\sqrt{y^2+a^2}}{y^2}\right] - \underbrace{\ln\left(0+\frac{\sqrt{y^2}}{y}\right)}_{0}\right)$$

$$\boxed{V(y) = \frac{Q}{4\pi\varepsilon_0 a}\ln\left[\left(\frac{a}{y}\right)+\sqrt{1+\left(\frac{a}{y}\right)^2}\right]}$$

(c) need: $\lim_{\varepsilon\to 0} \ln(1+\varepsilon) \simeq \varepsilon$

to see this: $\left.\frac{\partial \ln(1+\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=0} = \left.\frac{1}{1+\varepsilon}\right|_{\varepsilon=0} = 1$

so: $\ln(1+\varepsilon) \simeq \ln(1) + \varepsilon\left.\frac{\partial\ln(1+\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon_0}$

$$\simeq 0+\varepsilon \simeq \varepsilon$$

(a) $\lim_{x\to\infty}\frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{x+a}{x}\right) \Rightarrow \lim_{x\to\infty}\frac{Q}{4\pi\varepsilon_0 a}\ln\left(1+\frac{a}{x}\right)$

$$\simeq \frac{Q}{4\pi\varepsilon_0 \cancel{a}}\,\frac{\cancel{a}}{x} = \frac{Q}{4\pi\varepsilon_0 x}$$

$$\lim_{y\to\infty} V(y) = \frac{Q}{4\pi\epsilon_0 a}\lim_{y\to\infty}\ln\left[\left(\frac{a}{y}\right)+\sqrt{1+\left(\frac{a}{y}\right)^2}\right]$$

note: $\lim\limits_{\varepsilon\to 0}\sqrt{1+\varepsilon^2} \simeq 1+\frac{1}{2}\varepsilon^2$

so $$\lim_{y\to\infty} V(y) = \frac{Q}{4\pi\epsilon_0 a}\lim_{y\to 0}\ln\left[\left(\frac{a}{y}\right)+1+\left(\frac{a}{y}\right)^2\right]$$

small compared to $\left(\frac{a}{y}\right)$

$$\lim_{y\to\infty} V(y) = \frac{Q}{4\pi\epsilon_0 a}\times\frac{a}{y}$$

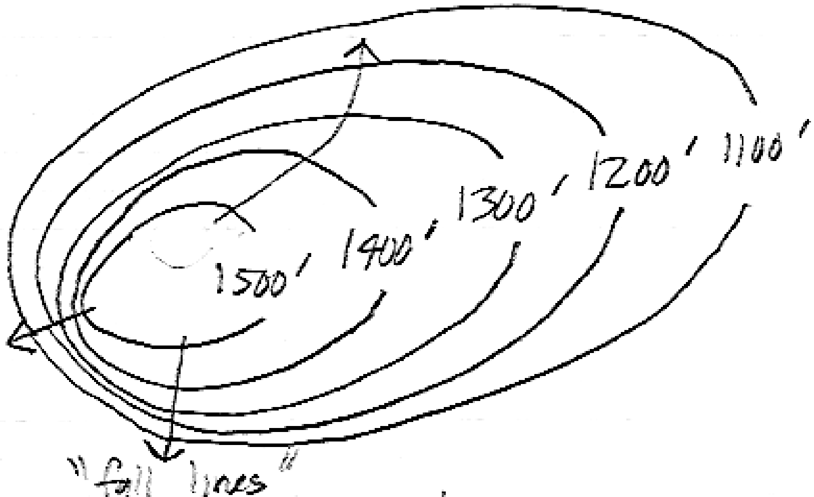
$$\boxed{\lim_{y\to\infty} V(y) = \frac{Q}{4\pi\epsilon_0 y}}$$

Equipotentials (lines, 2-d surfaces, 3-d).

Analogy with topographic maps

Mountain →

Lines indicate loci of constant elevation



(1) no work to walk always on same line (similarly, when skiing, to stay totally still, point skis along a line of constant elevation).

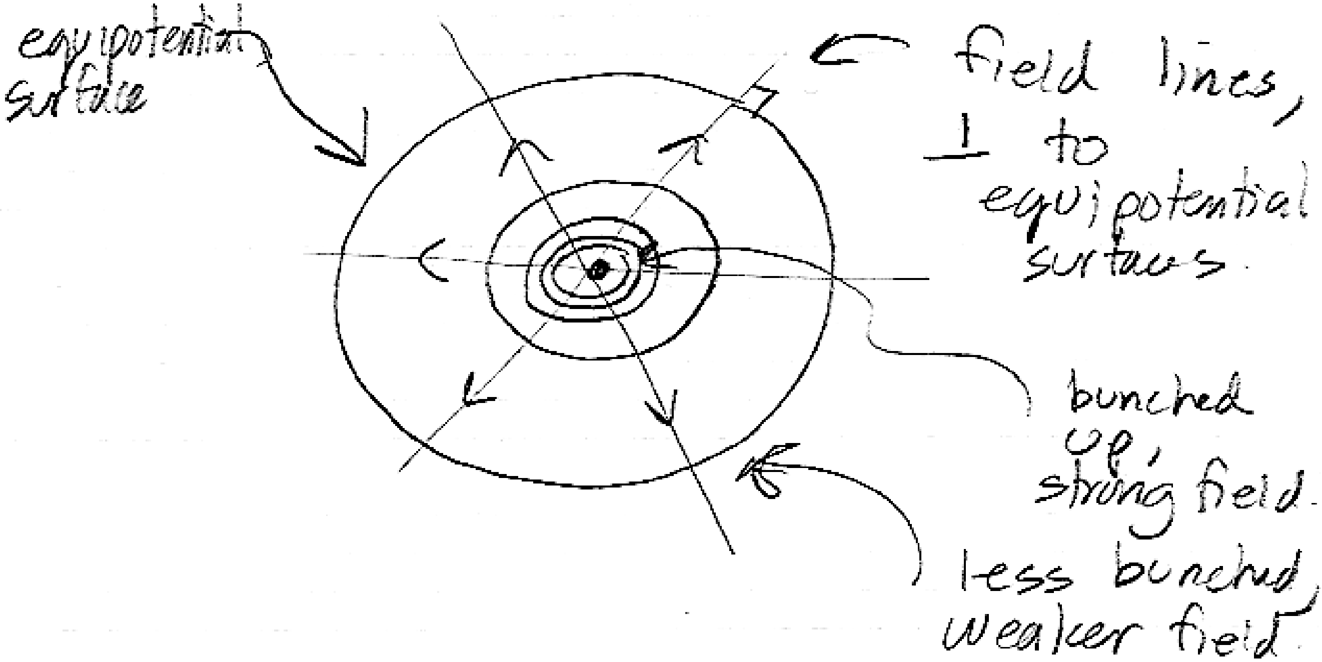
(2) where is it steep? Where lines bunch up. You will <u>fall</u> along a line perpendicular to lines of constant elevation

line of constant elevation: equipotential

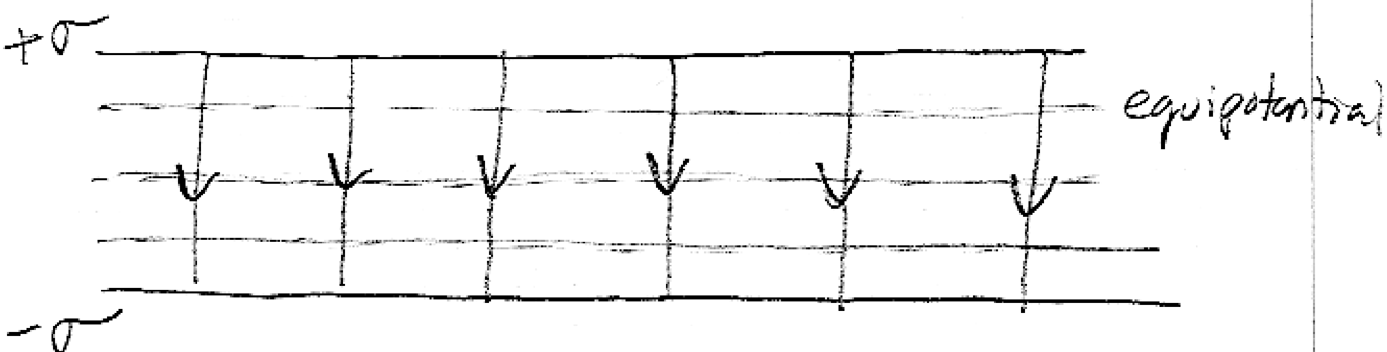
gravity force: $\vec{E}$ field

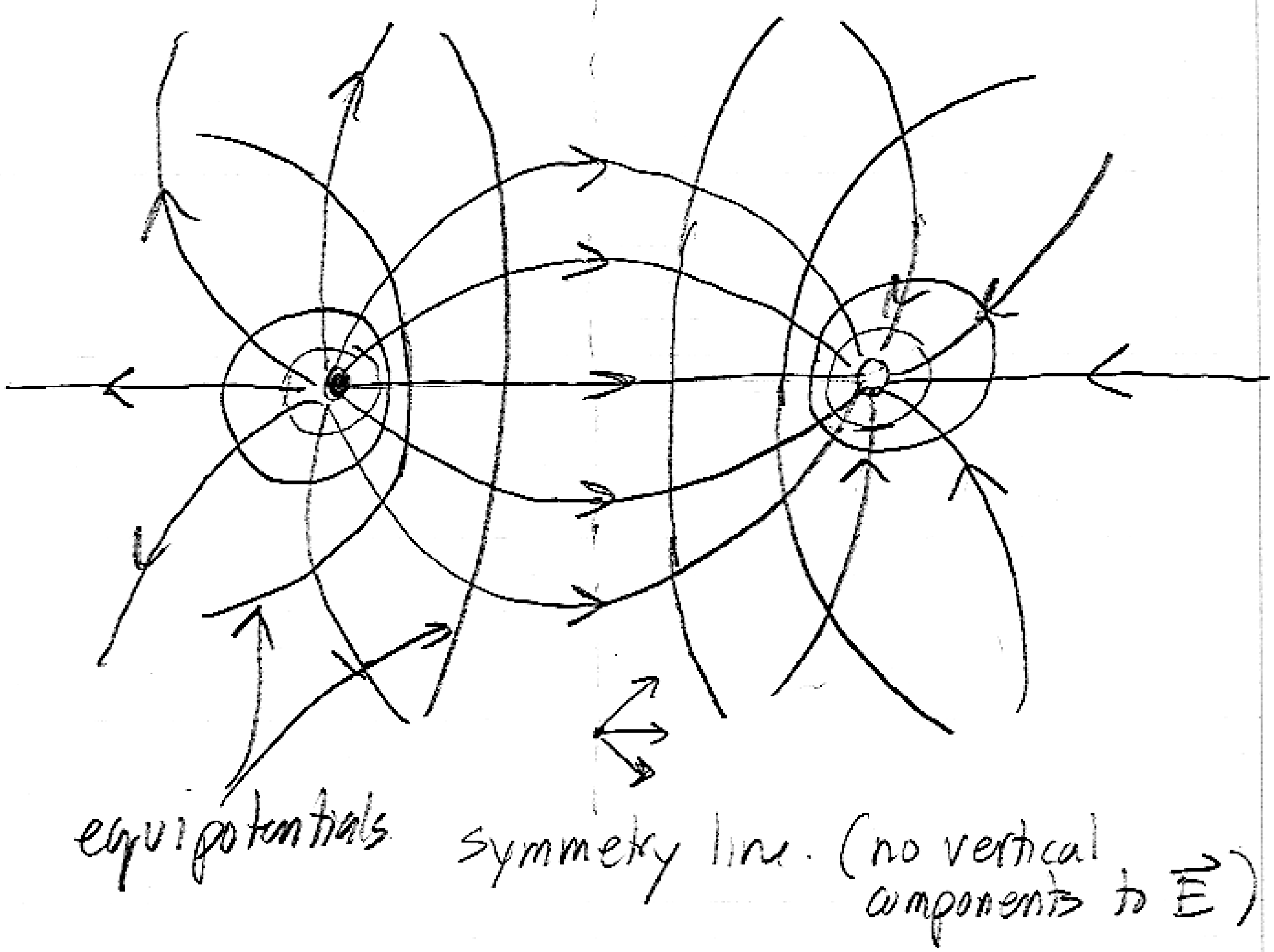
Some equipotenial surfaces:

Point Charge:



sheet charges:





Conductors are always equipotentials, since $\vec{E} = 0$ inside a conductor: never any work to move a charge around: no change in potential energy, no change in potential.

So, $\vec{E}$ always $\perp$ to surface of conductor

Physical argument: imagine the <u>contrary</u>

24-31

$$V(x,y,z) = Axy - Bx^2 + Cy$$

(a) $E_x = -\frac{\partial V}{\partial x} = -Ay - 2Bx$

$E_y = -\frac{\partial V}{\partial y} = -Ax + C$

$E_z = -\frac{\partial V}{\partial z} = 0$

(b) what (x,y,z) are $E_x = E_y = E_z = 0$

$E_x = -Ay - 2Bx = 0$

$E_y = -Ax + C = 0$, $\boxed{x = \frac{C}{A}}$

$E_x = -Ay - 2B \times \frac{C}{A} = 0$

$$\boxed{y = -2\frac{BC}{A^2}}$$

$z \Rightarrow$ any.