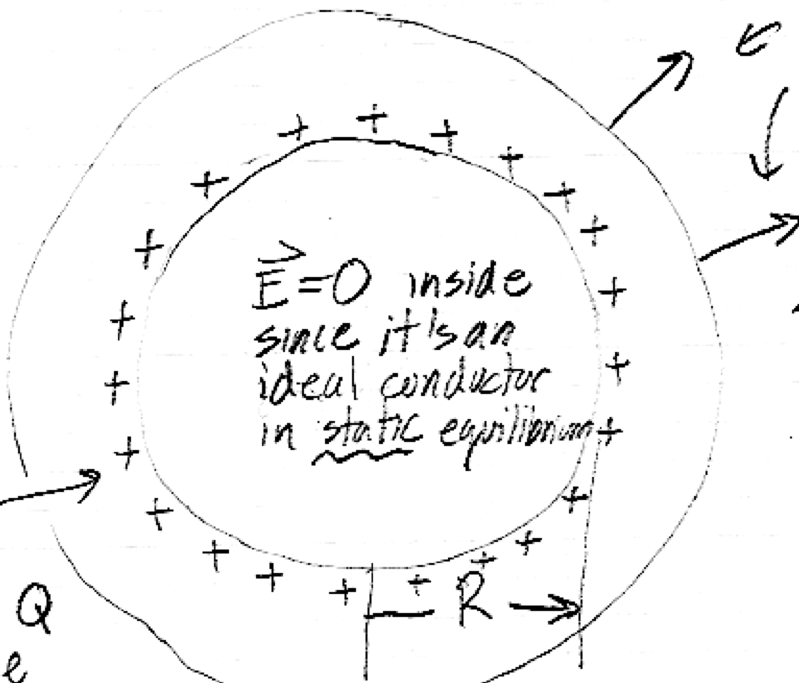
Examples With Gauss's Law

Solid Conducting Sphere

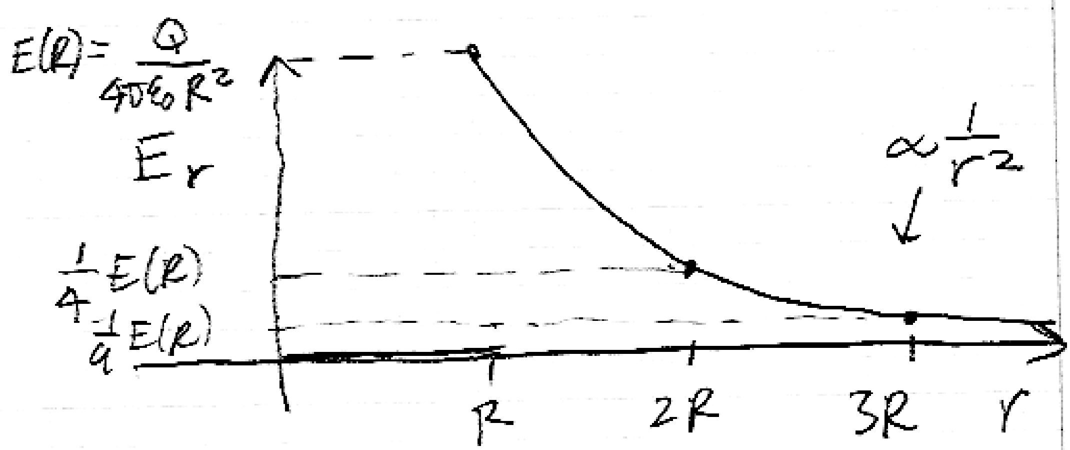


$\leftarrow \vec{E}$ has only radial components by symmetry.

$$4\pi r^2 \cdot E_r = \frac{Q}{\varepsilon_0}$$

$$E_r = \frac{Q}{4\pi\varepsilon_0 r^2}$$

charge Q can live on surface (ideally)



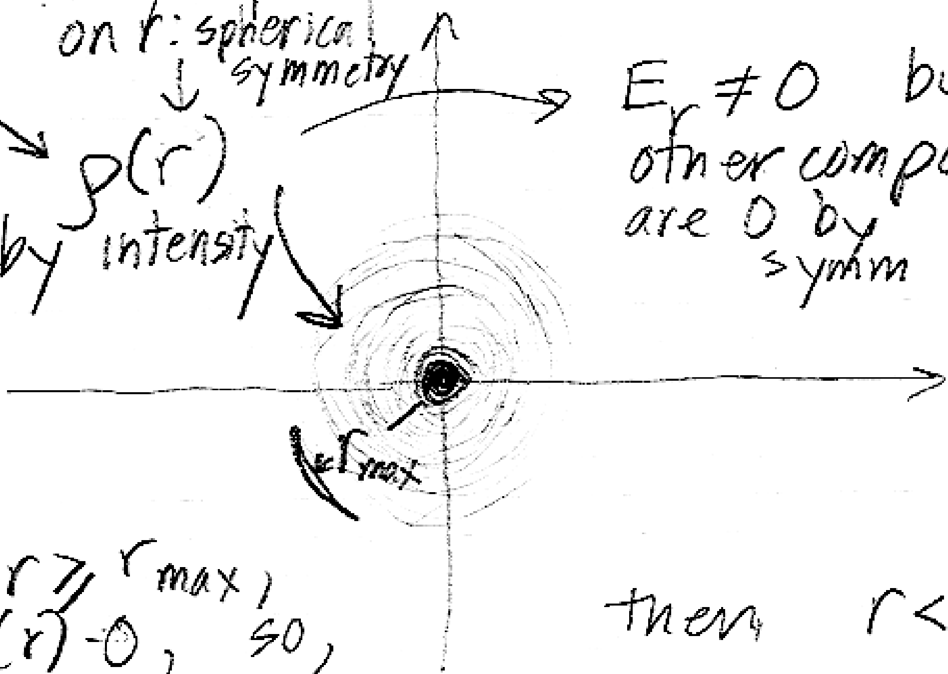
Note the $1/r^2$ dependence: just like a point charge... how general is this behavior?

1) spherical symmetry
2) charge density $\Rightarrow 0$

charge density → $\rho(r)$ ← only depends on r: spherical symmetry

by intensity

$E_r \neq 0$ but other components are 0 by symm



for $r \geq r_{max}$, $\rho(r) = 0$, so,

$$E_r = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$Q = 4\pi \int_0^{r_{max}} dr\, r^2 \rho(r)$$

then $r < r_{max}$:

$$4\pi r^2 E_r = \frac{4\pi}{\varepsilon_0} \int_0^{r} dr'\, r'^2 \rho(r')$$

$$E_r = \frac{1}{\varepsilon_0 r^2} \int_0^{r} dr'\, r'^2 \rho(r')$$

example: $\rho(r)$ a <u>uniform</u> density

in this case:

$$Q = 4\pi\rho \int_0^{r_{max}} dr\, r^2 = \frac{4\pi}{3} r_{max}^3 \rho$$

$$\rho = \frac{3}{4\pi} \frac{Q}{r_{max}^3}$$

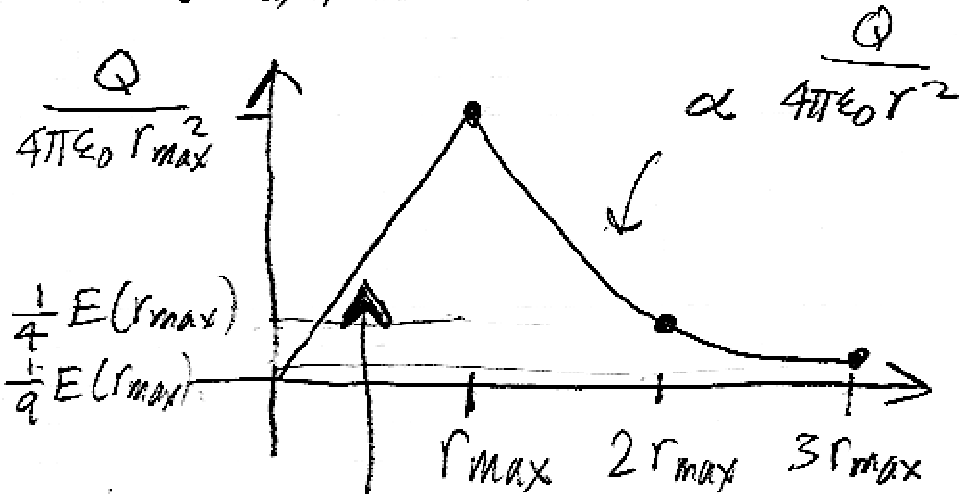
<u>Inside</u> then; $r < r_{max}$

$$E_r = \frac{1}{\varepsilon_0 r^2} \times \frac{3}{4\pi} \frac{Q}{r_{max}^3} \int_0^{r} dr'\, r'^2$$

$$= \frac{1}{4\pi\varepsilon_0 r^2} \frac{Q}{r_{max}^3} \cdot 3 \cdot \frac{1}{3} r^3$$

$$E_r = \underbrace{\frac{Q}{4\pi\varepsilon_0 r_{max}^2}}_{E(r_{max})} \cdot \left(\frac{r}{r_{max}}\right) = \frac{r}{r_{max}} E(r_{max})$$

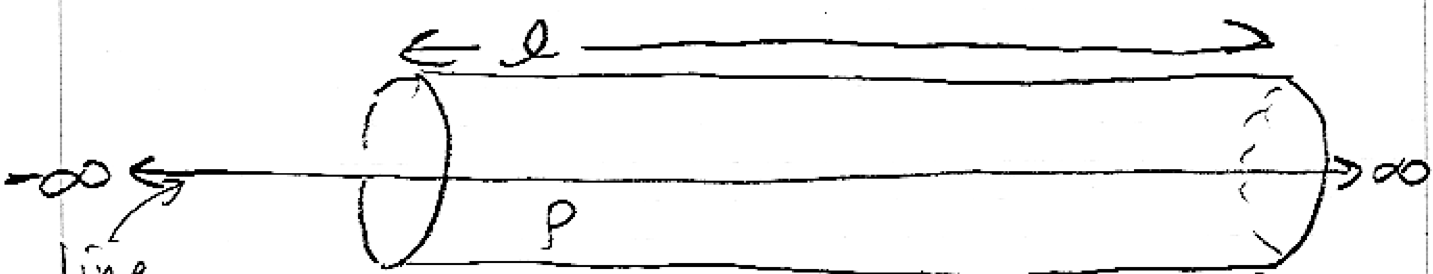
$$E(r_{max}) = \frac{Q}{4\pi\varepsilon_0 r_{max}^2}$$



<u>linear</u> when $\rho(r)$ is constant; different $\rho(r)$ will give different shape for $r < r_{max}$

Other Geometries

Cylindrical



line, linear charge density λ

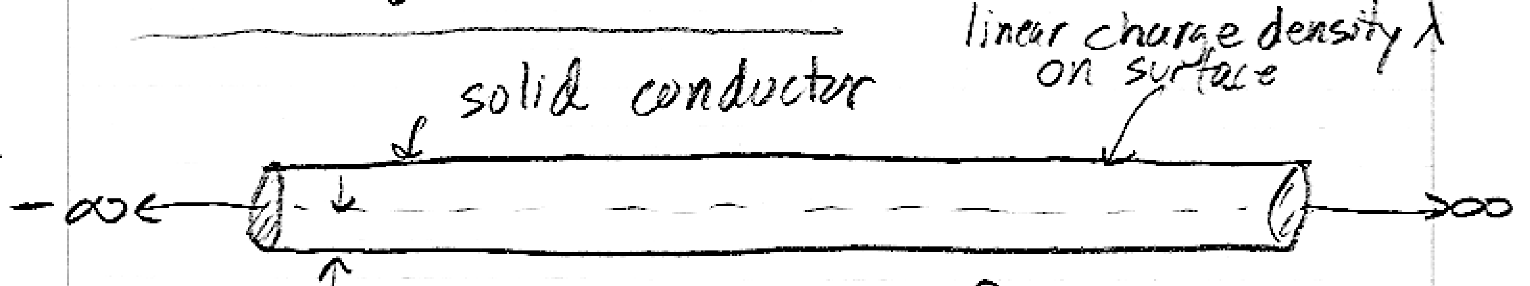
$E_\rho \leftarrow$ only ρ

Gaussian Surface, cylinder, centered on line charge

$$\underbrace{\underbrace{(2\pi\rho)\times l}_{\text{Area}} \times \underbrace{E_\rho}_{\text{E-field}}}_{\text{Flux}} = \frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E_\rho = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\rho}}$$

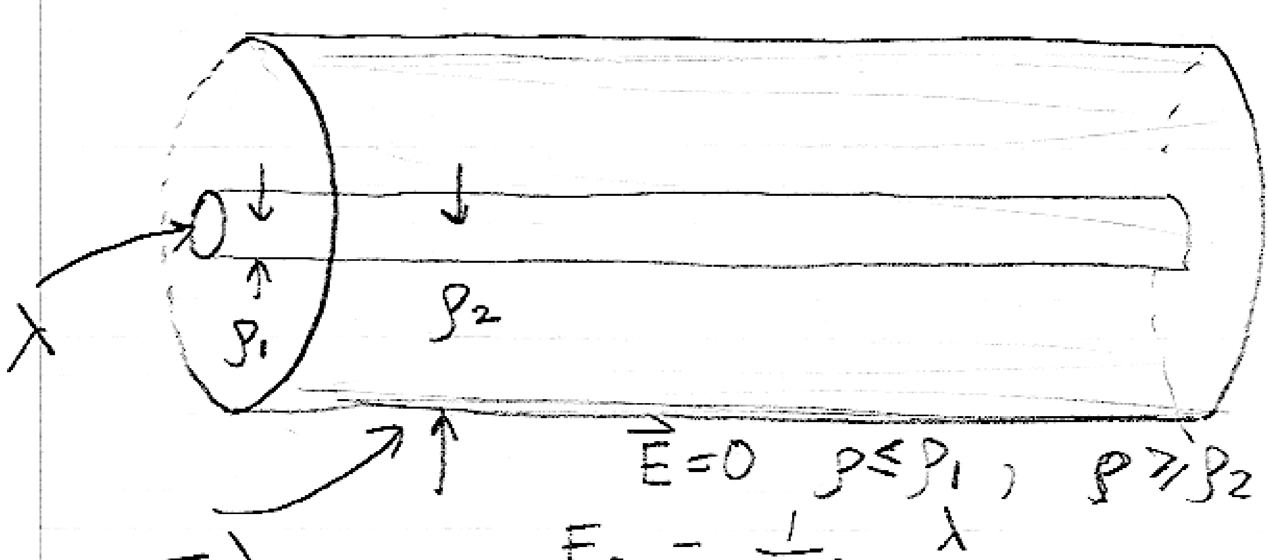
Related Problem



ρ_{max} ... $E(\rho) = 0$ for $\rho \leq \rho_{max}$

will be $E(\rho) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\rho}$ $\rho > \rho_{max}$

Coaxial Cable:



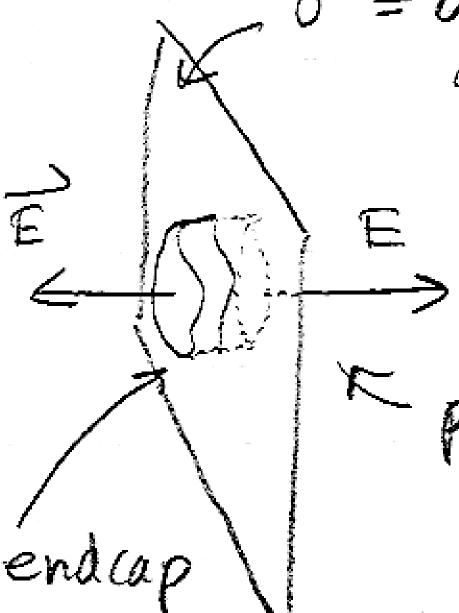
$\vec{E} = 0$ $\rho \leq \rho_1$, $\rho \geq \rho_2$

$E_\rho = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\rho}$

This is the first step in computing the capacitance of a coaxial cable...

Planar Geometry

σ = area charge density, sheet goes to ∞



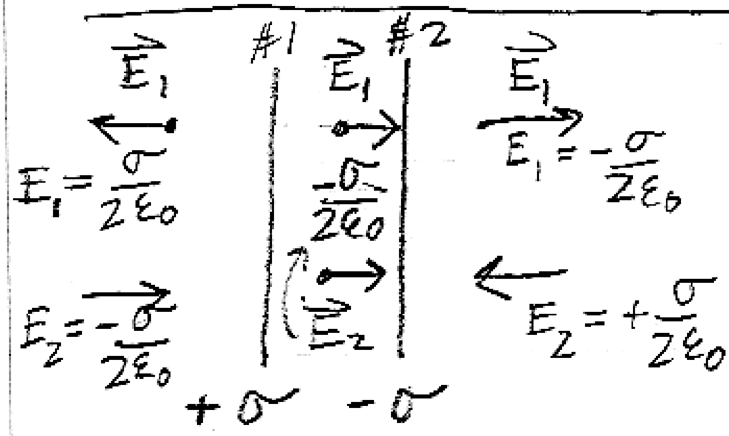
prism is split by surface

$\perp$ prism, endcap area A. E $\perp$ to surface by symmetry

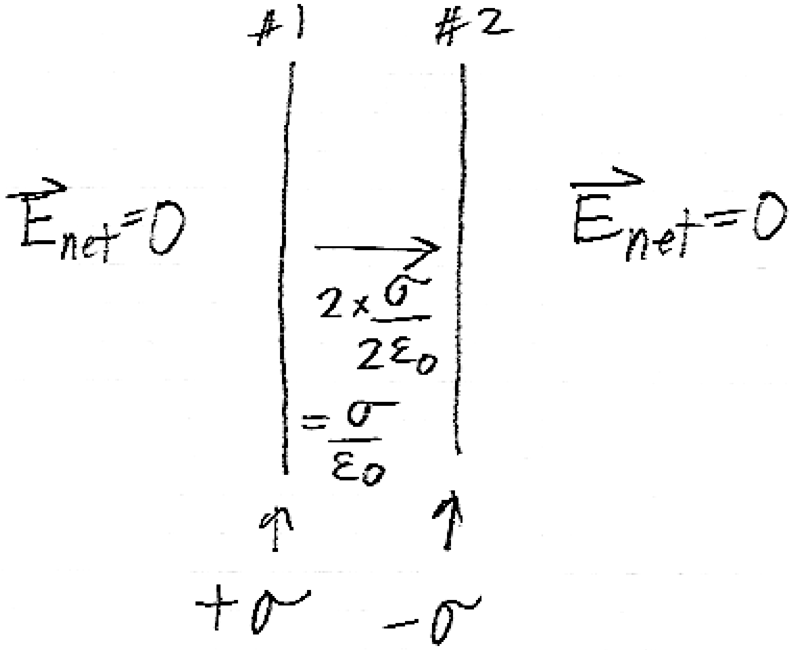
$$\underbrace{2}_{\text{two sides}}\ \underbrace{\underbrace{A}_{\text{area}} \cdot \underbrace{E}_{\text{field}}}_{\text{flux}} = \frac{Q}{\varepsilon_0} = \frac{\sigma \cdot A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

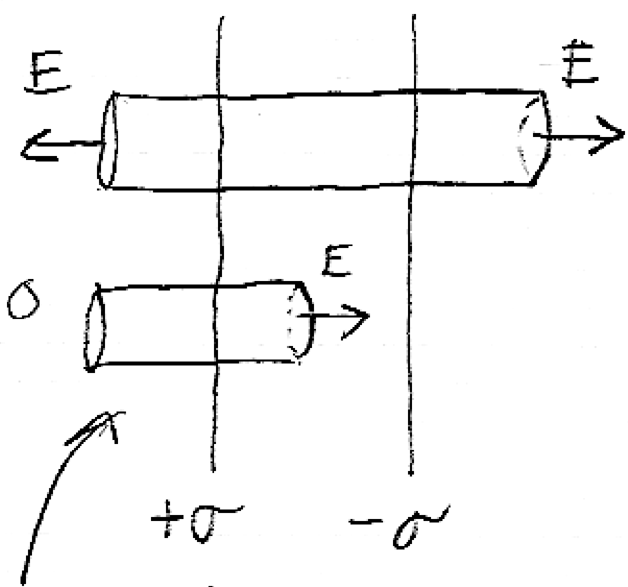
Related Problem:



now look at net $\vec{E}$ field



Another way to arrive at this:



symmetry

$2EA = 0$

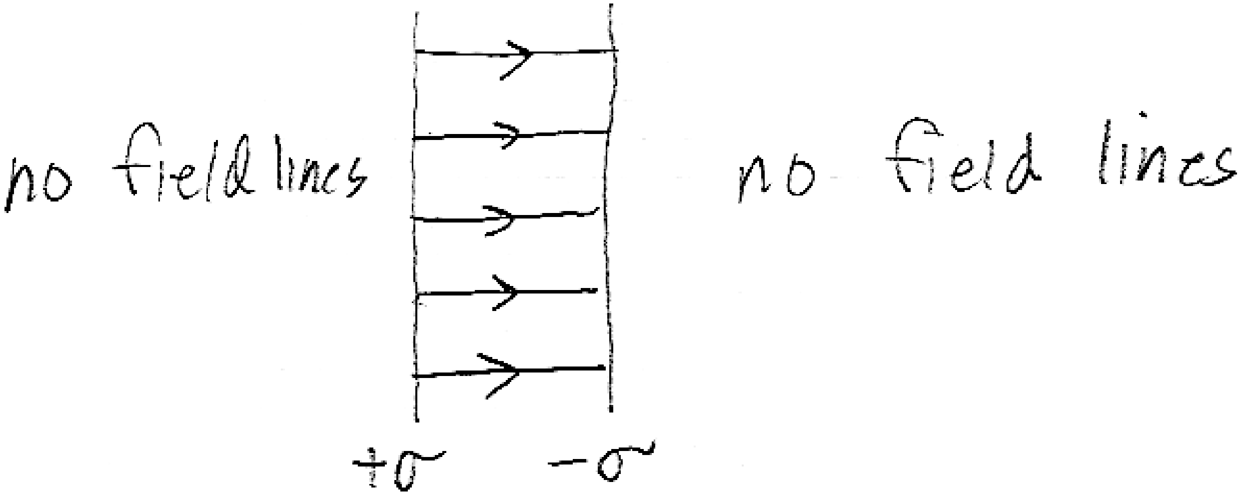
$E = 0$

outside

$$E \cdot A = \frac{\sigma}{\varepsilon_0} \cdot A$$

$$E = \frac{\sigma}{\varepsilon_0}$$ inside

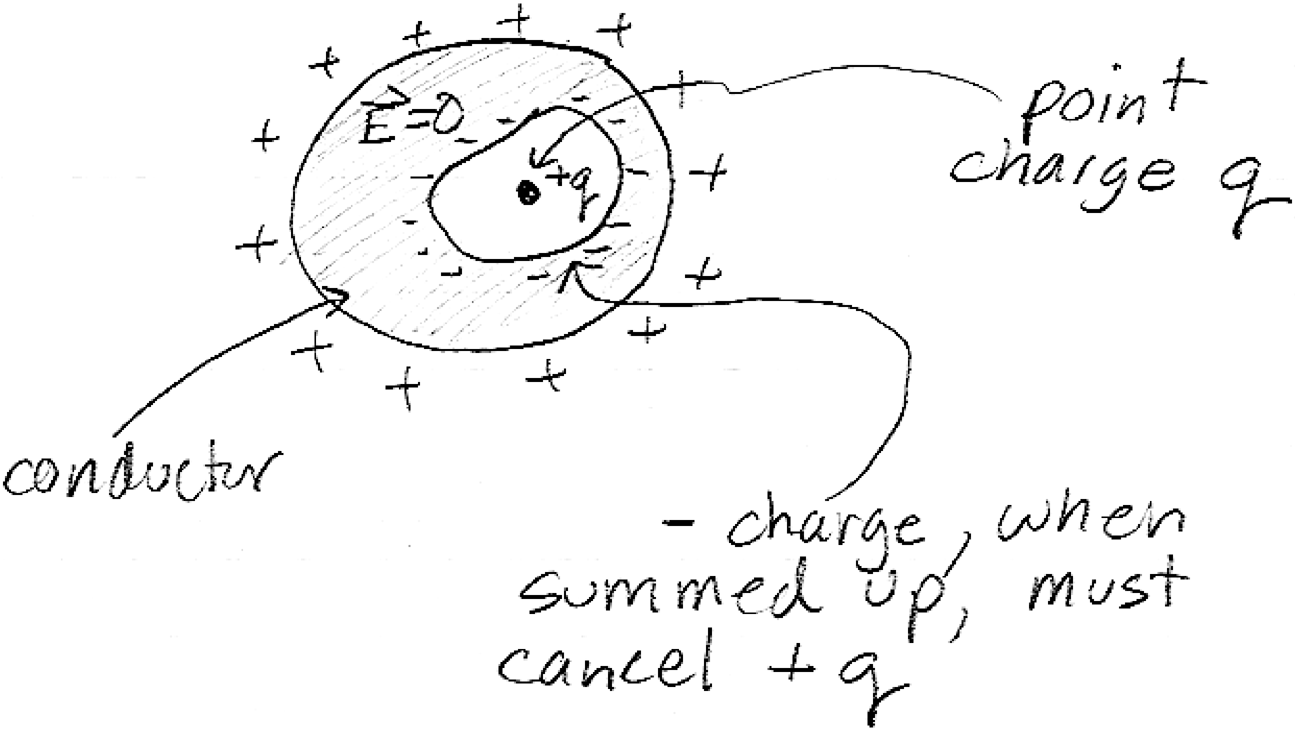
Picture: field lines equidistant



(segment of ∞ plates)

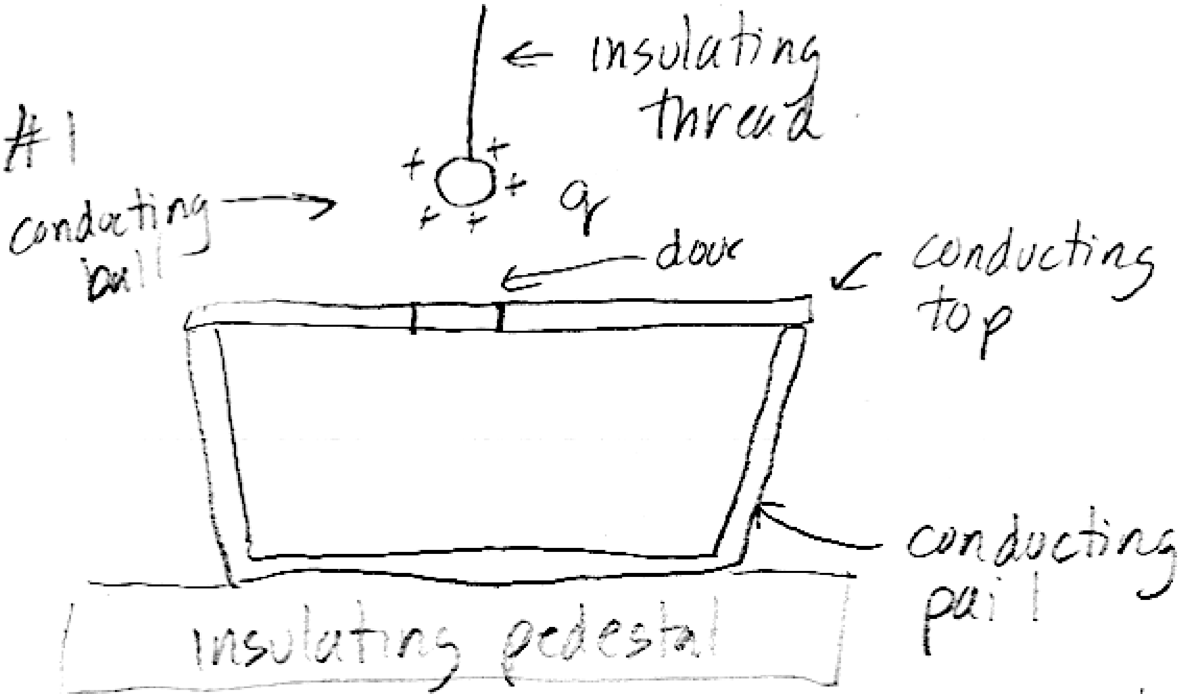
$\Rightarrow$ First step toward describing most capacitors

Charges on Conductors

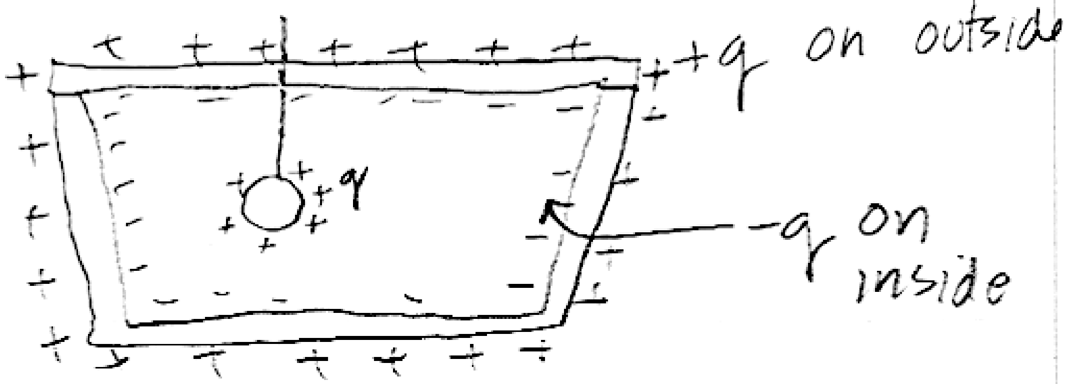


Faraday's Ice Pail:

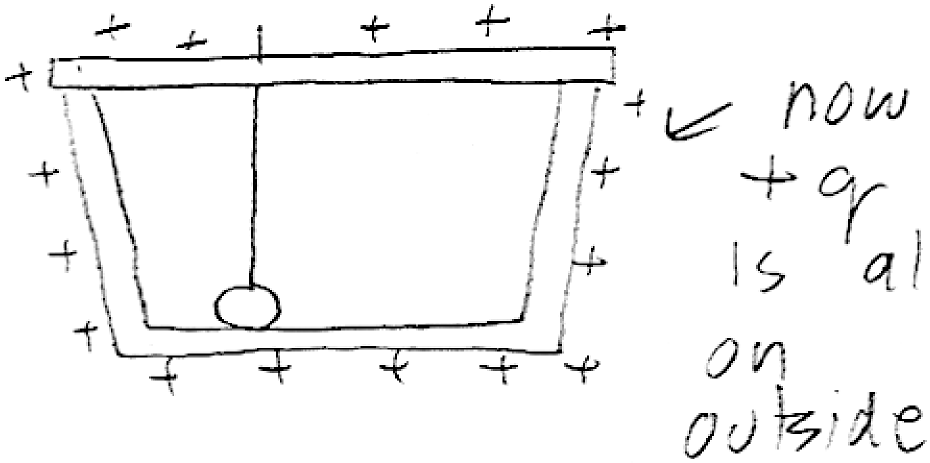
#1



#2

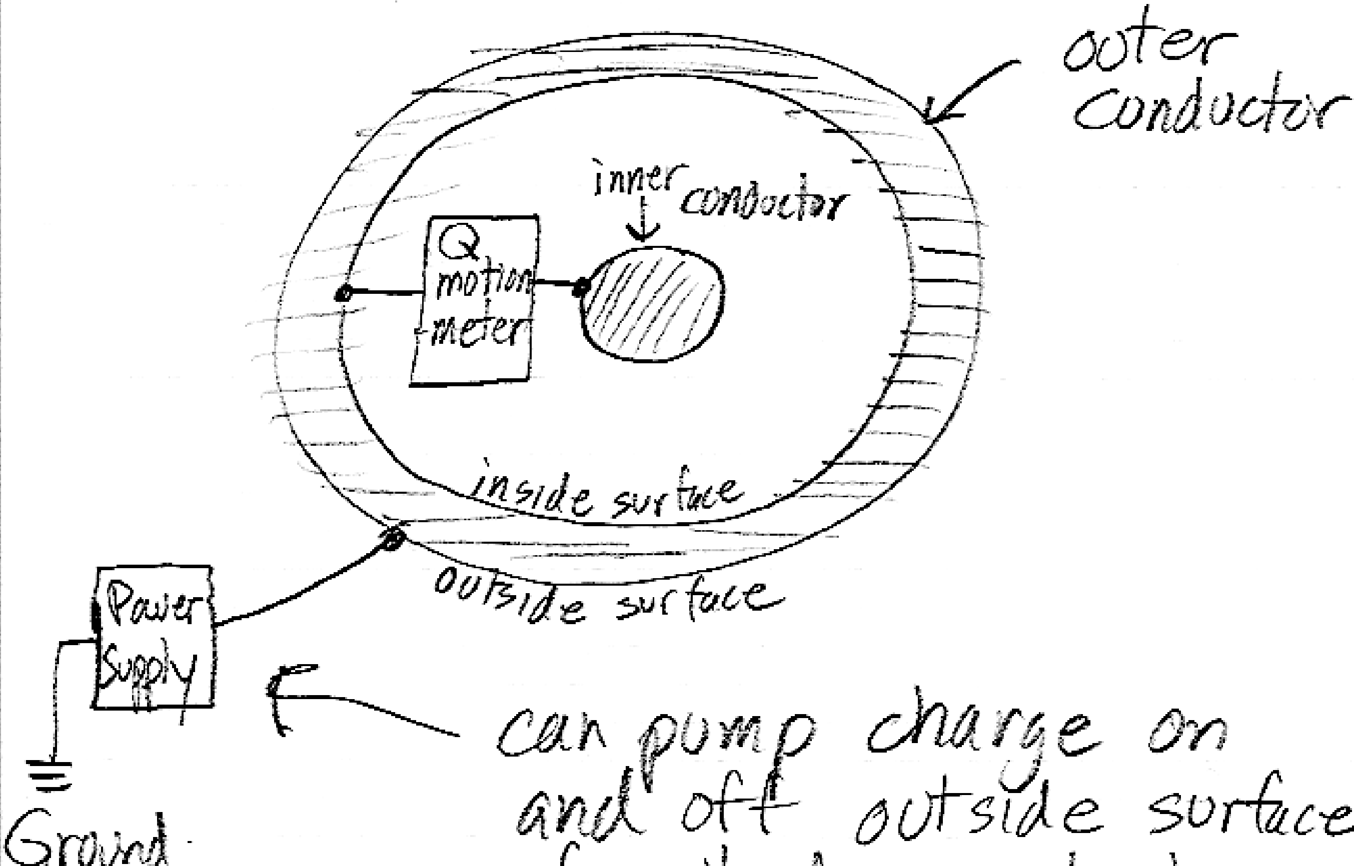


#3 drop ball on to bottom...



Charge is perfectly transfered.

Gauss's Law $\longleftrightarrow$ $1/r^2$ field



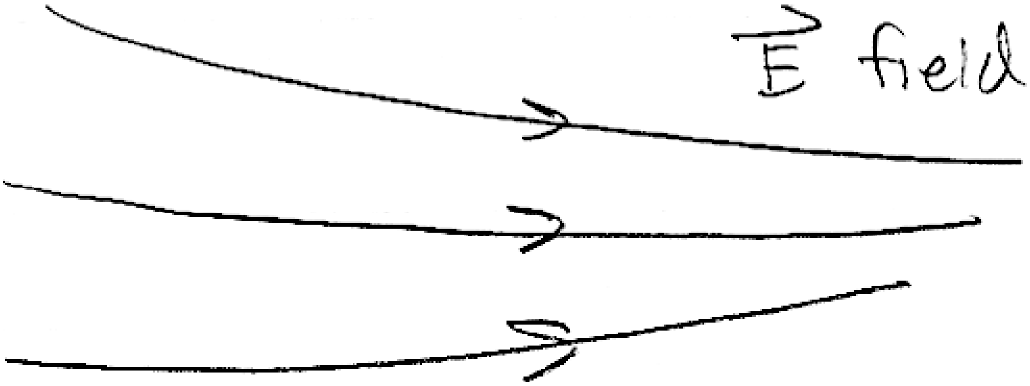
can pump charge on and off outside surface of outside conductor.

Idea is: the "Q-motion meter" will never measure anything even when power supply pumps lots of charge on and off outside ... <u>if</u> Gauss's Law is right ... or in other words, if $E \propto 1/r^2$ for point charge ...

$E \propto \frac{1}{r^{2+\varepsilon}}$ $\qquad \left|\frac{\varepsilon}{2}\right| \lesssim 2 \cdot 10^{-16}$

Shielding External Field, σ

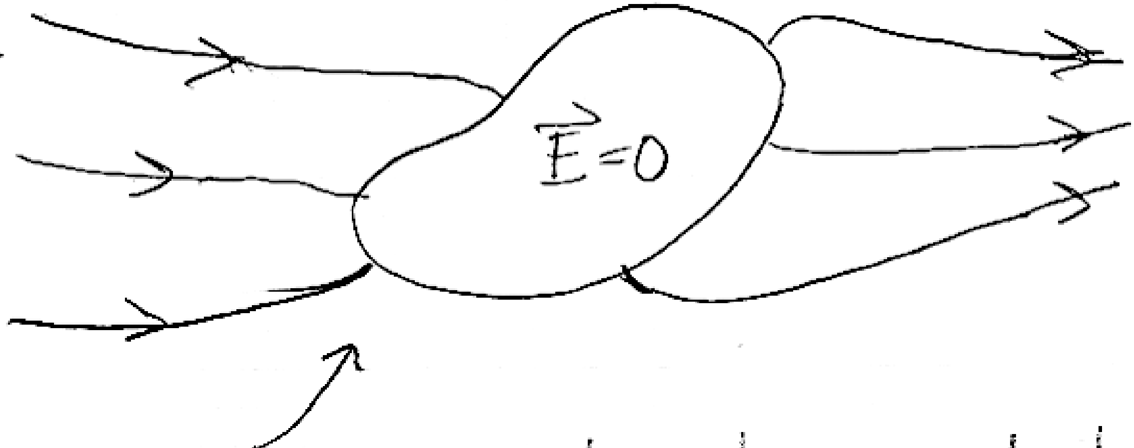
#1



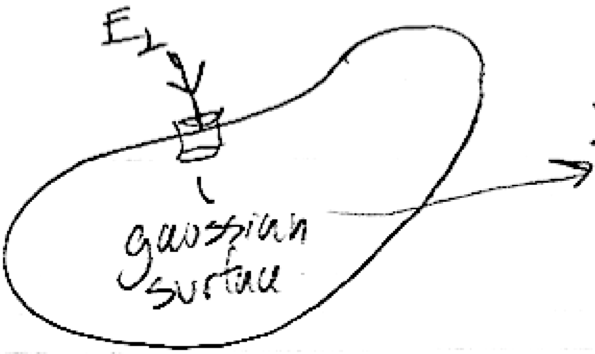
$\vec{E}$ field in space.

introduce a conductor

#2



field lines must enter conductor $\perp$ to surface.. $E_{\parallel}$, if non-zero, would drive currents until new charge distribution causes $-E_{\parallel}$.

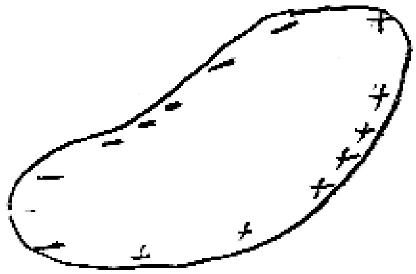


inward

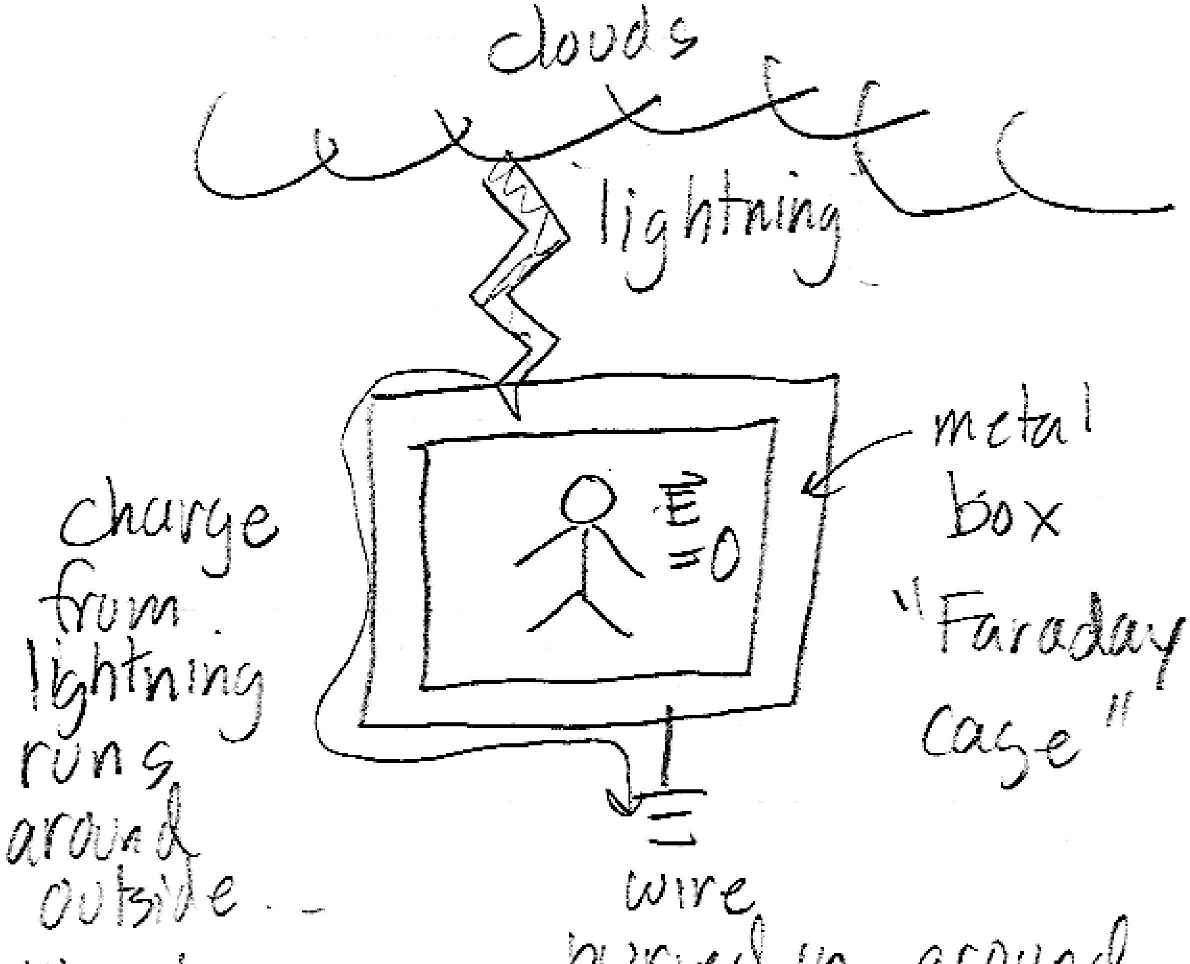
$$-A \cdot E = \frac{\sigma}{\varepsilon_0} \cdot A$$

$$E_\perp = -\frac{\sigma\,(\text{spot})}{\varepsilon_0}$$

In a sense, σ(spot) provides a kind of map of the pre-existing $\vec{E}$-field.



Also, conductor with hole shields



Thank Gauss!