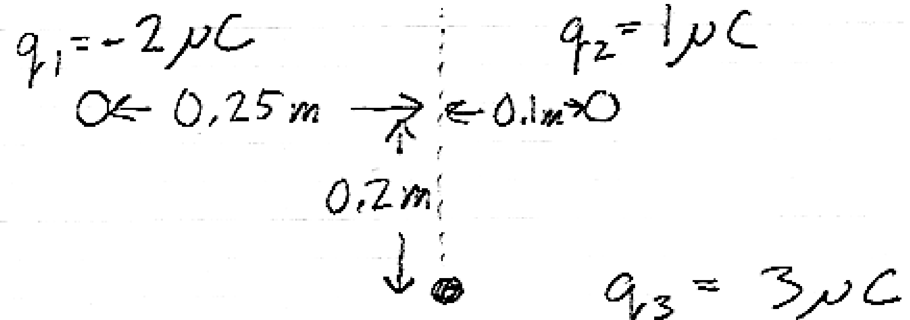
Vector Addition of Coulomb Forces



Net force on q_3?

"Magnitude" due to q_1 ; $F_{31} = k \frac{q_1 q_3}{r_{13}^2}$

$k \simeq 9 \cdot 10^9 \frac{N\,m^2}{C^2}$

$$r_{13}^2 = 0.25^2 + 0.2^2$$
$$= 0.0625 + 0.04$$
$$r_{13}^2 = 0.1025\ m^2$$

$$F_{31} = 9 \cdot 10^9 \cdot \frac{N \cancel{m^2}}{\cancel{C^2}} \cdot \frac{(-2 \cdot 10^{-6})(3 \cdot 10^{-6})}{0.1025} \frac{\cancel{C^2}}{\cancel{m^2}}$$

$$= -\frac{54}{1.03} \cdot \frac{10^9 \cdot 10^{-12}}{10^{-1}} = -53 \cdot 10^{-2}$$

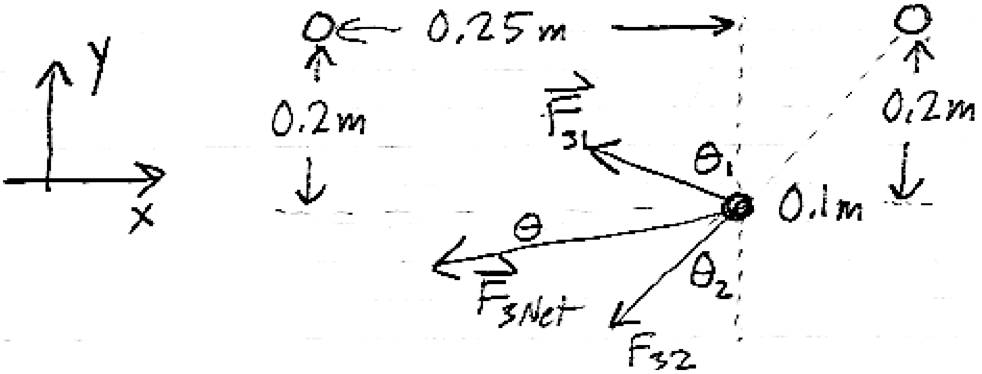
$F_{31} \simeq -0.53 N$: − sign means attractive

"Magnitude" due to q_2 : $F_{32} = k \frac{q_2 q_3}{r_{23}^2}$

$$r_{23}^2 = 0.1^2 + 0.2^2 = 0.01 + 0.04 = 0.05\ m^2$$

$$F_{32} = 9 \cdot 10^9 \cdot \frac{1 \cdot 3 \cdot 10^{-12}}{0.05}\ N$$

$F_{32} \simeq 27 \times 20 \times 10^{-3} = 0.54\ N$ + sign means repulsive



$$F_{31y} = |\vec{F}_{31}| \cos\theta_1 = 0.53 \cdot \frac{0.2}{\sqrt{0.2^2+0.25^2}} = 0.33\,N$$

$$F_{32y} = -|\vec{F}_{32}| \cos\theta_2 = 0.54 \cdot \frac{0.2}{\sqrt{0.2^2+0.1^2}} = -0.48\,N$$

$$\boxed{F_{3Net\,y} = -0.15\,N}$$

$$F_{31x} = -|\vec{F}_{31}| \sin\theta_1 = -0.53 \cdot \frac{0.25}{\sqrt{0.2^2+0.25^2}} = -0.41\,N$$

$$F_{32x} = -|\vec{F}_{32}| \sin\theta_2 = -0.54 \cdot \frac{0.1}{\sqrt{0.2^2+0.1^2}} = -0.24\,N$$

$$\boxed{F_{3Net\,x} = -0.65\,N}$$

$$\boxed{|\vec{F}_{3Net}| = \sqrt{0.15^2+0.65^2} = 0.67\,N}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{0.15}{0.65}\right) = 13.3^\circ}$$

Electric Field

Abstraction: what sort of thing is present when just one charge is present?

→ "Electric Field"

→ confusion: every charge has its field...

Idea:

$F_{12} = k\dfrac{q_1 q_2}{r_{12}^2}$ (+ means repulsive)

$F_{21} = k\dfrac{q_1 q_2}{r_{12}^2}$ (+ means repulsive)

(diagram of direction: $\vec{F}_{12} = -\vec{F}_{21}$)

$F_{12} = \underbrace{\left[k\dfrac{q_2}{r_{12}^2}\right]}_{\text{depends on \#2}} \cdot q_1$

$F_{21} = \underbrace{\left[k\dfrac{q_1}{r_{12}^2}\right]}_{\text{depends on \#1}} \cdot q_2$

$\Downarrow$ focus

$E_1 = k\dfrac{q_1}{r_{12}^2}$

$F_{21} = q_2 E_1$

E_1 caused by q_1

F_{21} caused by q_2 interacting with E_1

$E_2 = \dfrac{k q_2}{r_{12}^2}$

E_2 caused by q_2

F_{12} caused by q_1 interacting with E_2

<u>Note</u>: although $|E_1| \neq |E_2|$

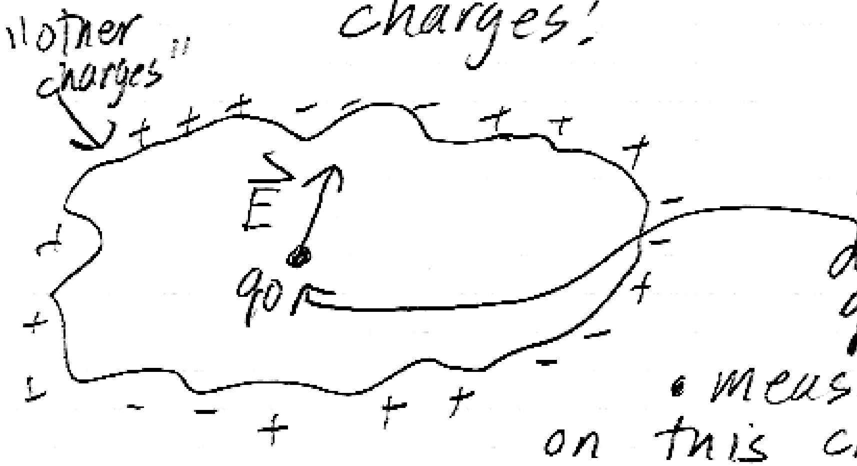
when $q_1 \neq q_2$

(as drawn, $q_1 > q_2$)

still $|\vec{F}_{12}| = |\vec{F}_{21}|$

"Test Charge" q_0

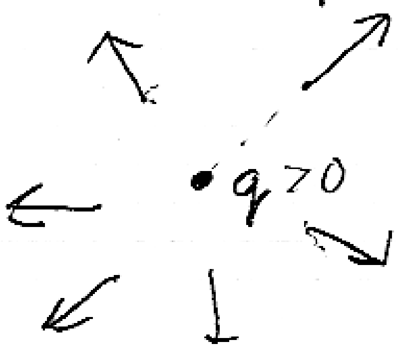
Conceptually: region of space, pre-existing electric field from other charges!



- imagine plonking down "test charge" $q_0 > 0$
- measure force, $\vec{F}_0$, on this charge.
- then $\vec{E} = \vec{F}_0 / q_0$

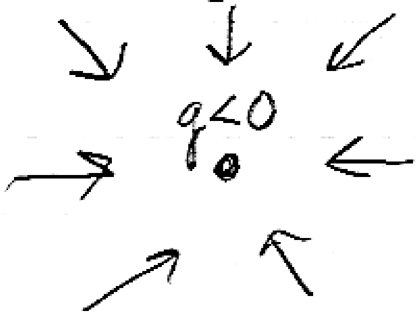
Electric Field Direction

① away from a positive charge



E.. points away at random points (q_0 would be repelled)

② toward a negative charge



E.. points toward (q_0 would be attracted)

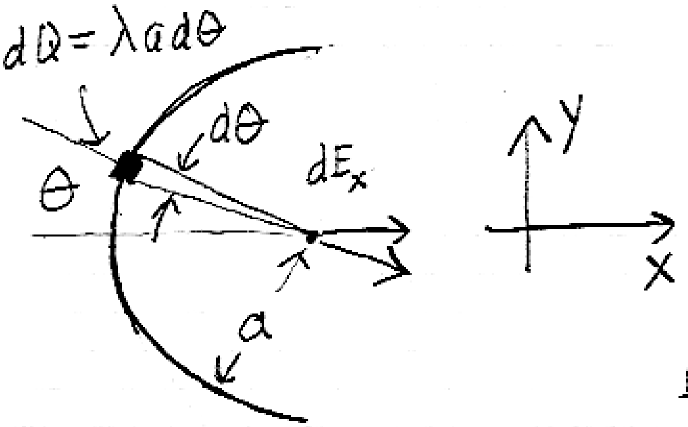
+ charge: $\vec{F} \parallel \vec{E}$: $q \bullet \longrightarrow \vec{E}$, $\longrightarrow \vec{F} = q\vec{E}$

− charge: $\vec{F} \nparallel \vec{E}$; $\longleftarrow \bullet \longrightarrow \vec{E}$, $\vec{F} = -|q|\,\vec{E}$

Electric Field Calculations

Exercises in: symmetry
vector addition
calculus

Charged Line, Semicircle radius a
Total Charge Q, linear charge density: $\lambda = \frac{Q}{\pi a}$



symmetry: $E_y = 0$ } as much λ $y>0$, $y<0$

E_x : $dE_x = \left(k \frac{\lambda a\, d\theta}{a^2}\right) \cos\theta$

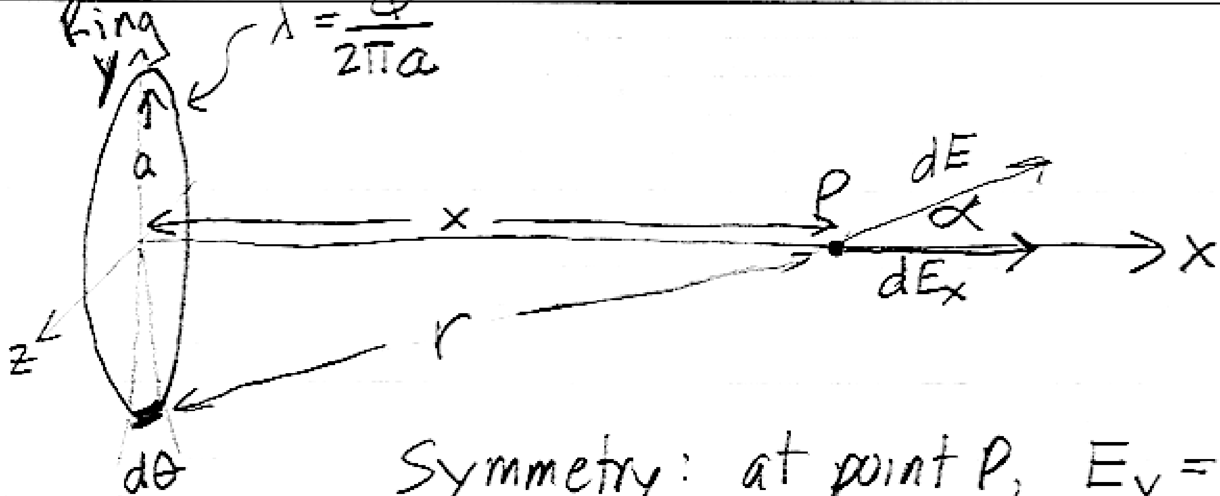
$$E_x = \int_{-\pi/2}^{\pi/2} dE_x = k \frac{\lambda}{a} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$E_x = k \frac{\lambda}{a} \sin\theta \Big|_{-\pi/2}^{\pi/2} = k\frac{\lambda}{a}(1-(-1))$$

$$= 2k\frac{\lambda}{a}$$

$$\boxed{E_x = \frac{2}{\pi} k \frac{Q}{a^2}}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$



$dQ = \lambda a \, d\theta$

Symmetry: at point P, $E_y = 0$

$E_z = 0$

$$dE_x = k \frac{dQ}{r^2} \times \cos\alpha$$

$$= k \frac{\lambda a \, d\theta}{x^2 + a^2} \cdot \frac{x}{\sqrt{x^2+a^2}}$$

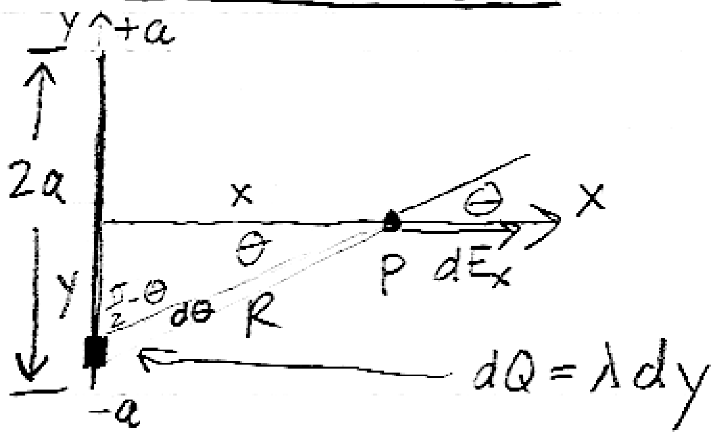
$$E_x = k \frac{\lambda a x}{(x^2+a^2)^{3/2}} \int_0^{2\pi} d\theta = k \frac{2\pi \lambda a x}{(x^2+a^2)^{3/2}}$$

$$= k \frac{2\pi \cdot \frac{Q}{2\pi a} \cdot a x}{(x^2+a^2)^{3/2}}$$

$$\boxed{E_x = k \cdot \frac{Q x}{(x^2+a^2)^{3/2}} \qquad k = \frac{1}{4\pi\varepsilon_0}}$$

Line Charge

$\lambda = \frac{Q}{2a}$



at point P' $E_y = 0$

$$dE_x = k \frac{dQ}{R^2} \cos\theta$$

$$= k \frac{\lambda \, dy}{R^2} \cos\theta$$