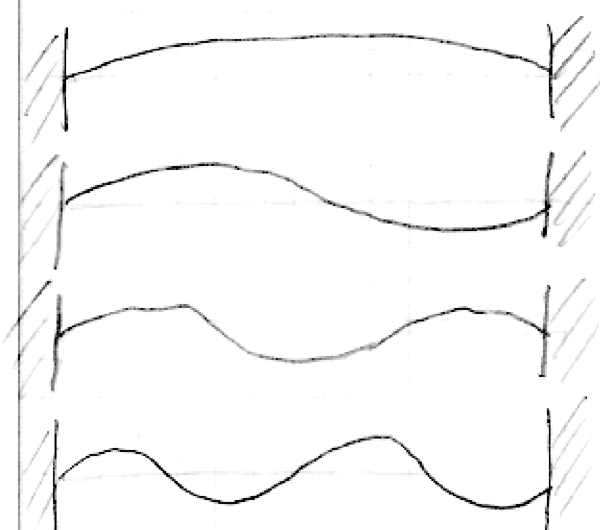
Normal Modes of String

1) endpoints – boundary conditions – determine allowed wavelengths λ_n
2) $\sqrt{\frac{F}{\mu}}$ determines speed v
3) Ear responds to frequency: $f_n = \frac{v}{\lambda_n}$
4) fixed ends: $\lambda_n = \frac{2L}{n}$, $f_n = \frac{nv}{2L}$



$n=1$	$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	
			$\frac{v}{2L}$
2	$\lambda_2 = L$	$f_2 = \frac{v}{L}$	
			$\frac{v}{2L}$
3	$\lambda_3 = \frac{2}{3}L$	$f_3 = \frac{3}{2}\frac{v}{L}$	
			$\frac{v}{2L}$
4	$\lambda_4 = \frac{1}{2}L$	$f_4 = 2\frac{v}{L}$	

equal spacing

"basis functions"

$n=1$ $\quad \xi_1(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right) \cos\left(\frac{\pi v}{L}t\right)$

$=2$ $\quad \xi_2(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi}{L}x\right) \cos\left(\frac{2\pi v}{L}t\right)$ ("faster")

$=3$ $\quad \xi_3(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right) \cos\left(\frac{3\pi v}{L}t\right)$

$=4$ $\quad \xi_4(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{4\pi}{L}x\right) \cos\left(\frac{4\pi v}{L}t\right)$

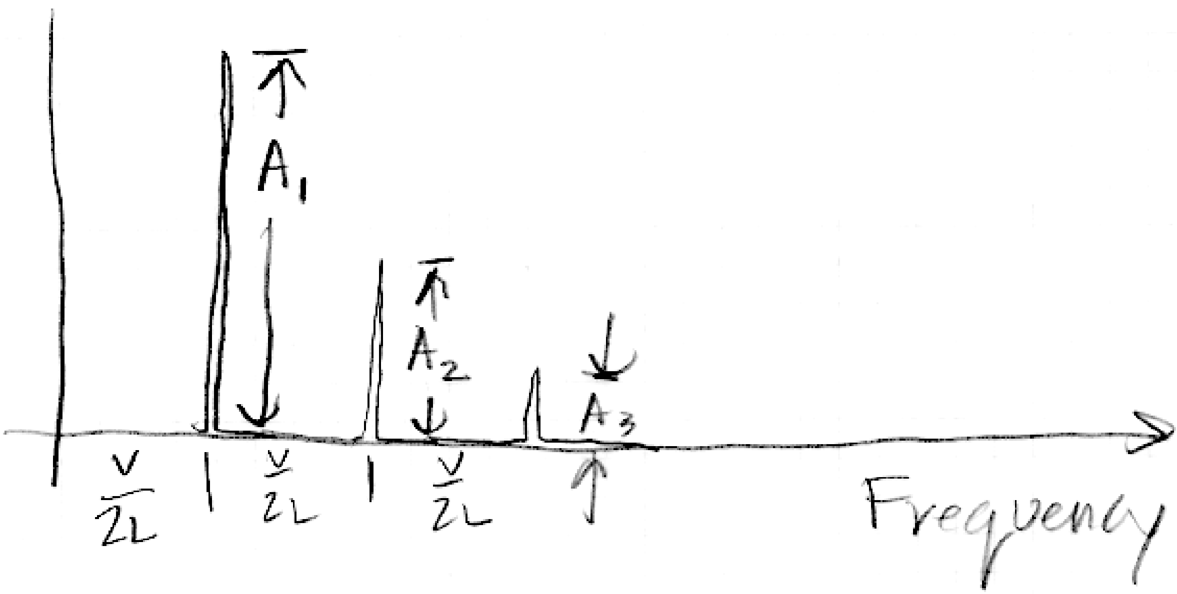
The "Fourier Analysis"... any shape you initially distort string into can be written as a series of these functions

$$\underbrace{\Delta(x,0)}_{\text{distortion, } t=0} = \sum_{n=1}^{\infty} A_n \xi_n(x,0)$$

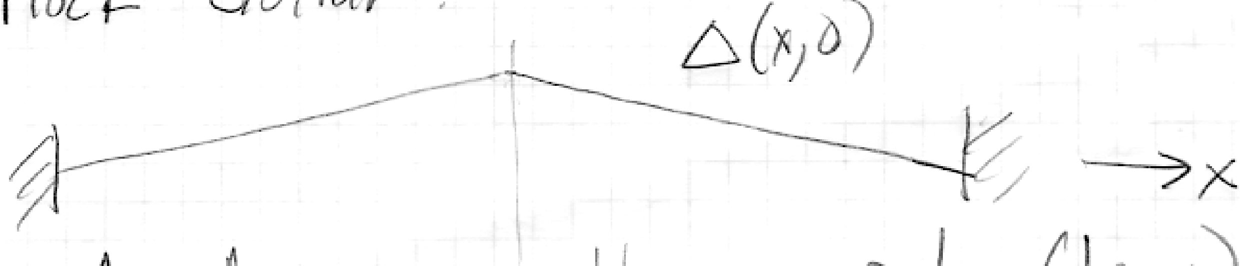
$$\int_{-L}^{0} \Delta(x,0)\,\xi_m(x,0)\,dx = \sum_{n=0}^{\infty} A_n \underbrace{\int_{-L}^{0} dx\, \xi_n(x,0)\,\xi_m(x,0)}_{\substack{=0 \text{ usually} \\ =1 \text{ when } n=m}}$$

so $$A_m = \int_{-L}^{0} dx\, \Delta(x,0)\,\xi_m(x,0)$$

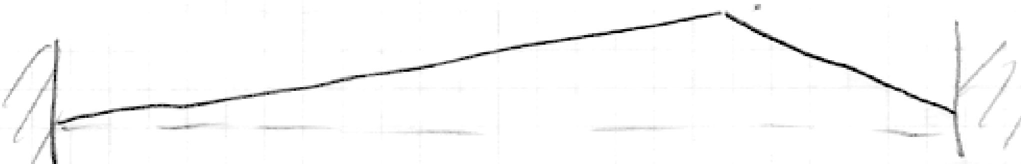
The "Fourier Analysis" window in Sigview plots spikes, whose amplitude is proportional to the coefficients:



Pluck Guitar:



$A_2, A_4, \ldots$ all are 0! (demo)
WHY?



now $A_2, A_4 \neq 0$

Musical impact of $A_2 \rightarrow$ an octave above fundamental

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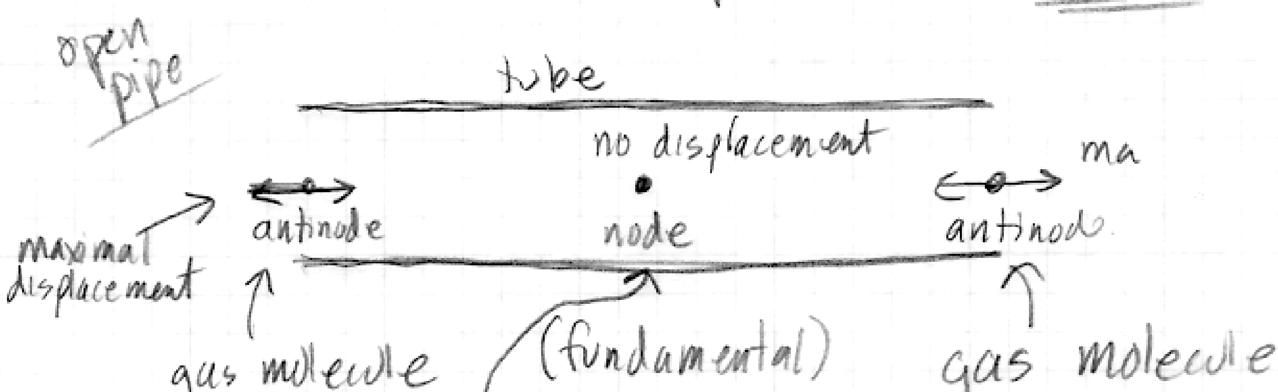
Longitudinal Waves in Cavities

→ open end a displacement <u>antinode</u>
→ close end a displacement <u>node</u>



does feel lots of <u>pressure</u> from neighbors bumping in...

displacement node = pressure antinode.

displacement antinode = $\underbrace{\text{pressure node}}$

who cares? Human ear responds to pressure: loud = pressure antinode.

Book example: 20-5

$f = 200\ Hz$

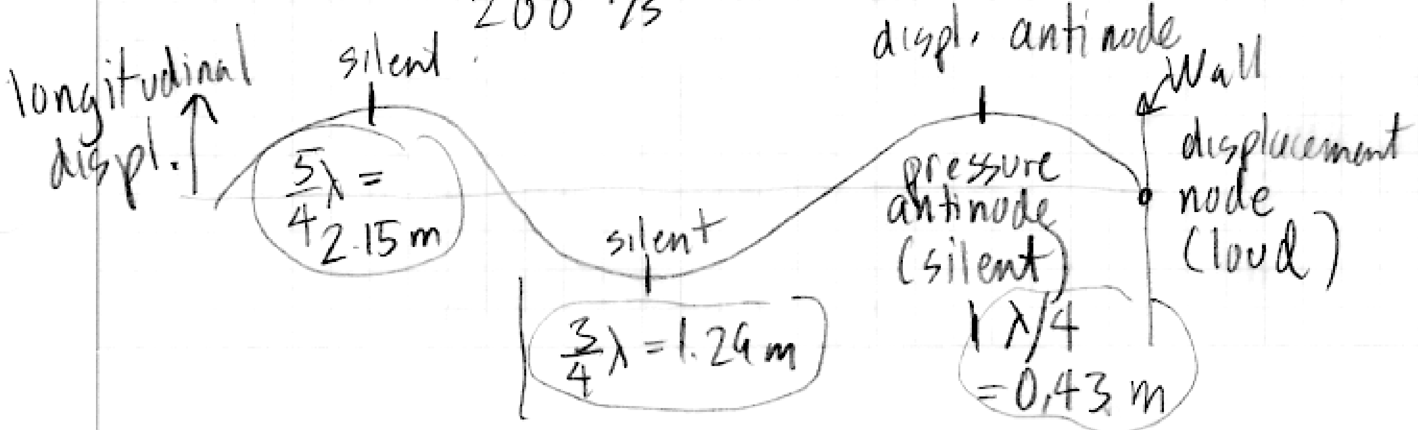
where is silence

displacement node, pressure antinode, loud

where can you here silence?

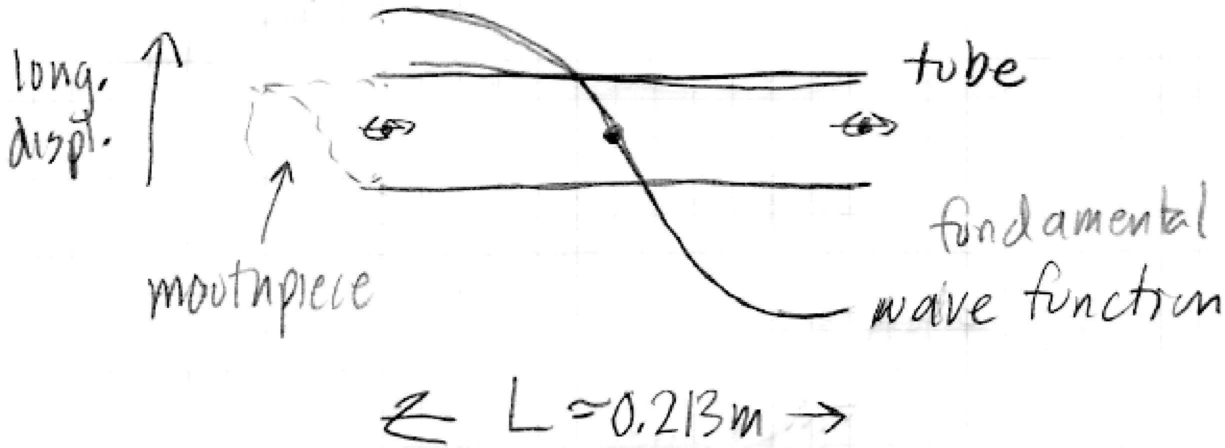
$f \cdot \lambda = v \simeq 344\ m/s$

$\lambda \simeq \frac{344\ m/s}{200\ 1/s} \simeq 1.72$ meters



Wind Instruments (both ends open)

G penny whistle... $8\frac{1}{2}$" long

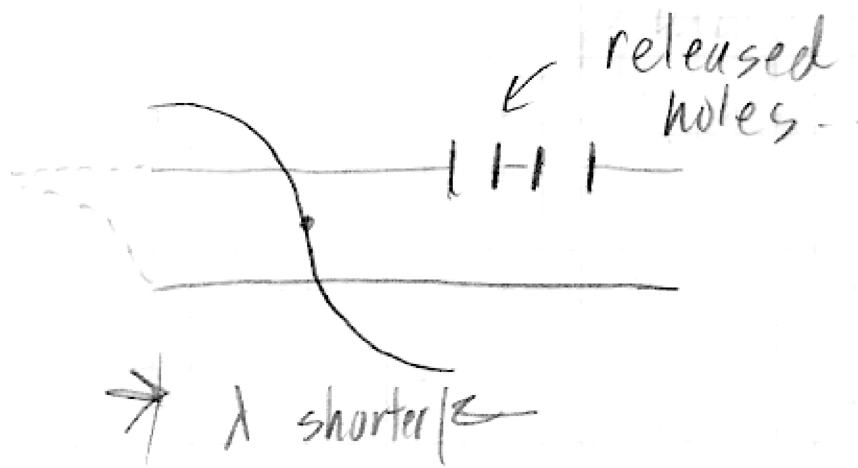


$\lambda = 2L$ (see $\frac{1}{2}$ of wave above).

$f = 780$ Hz

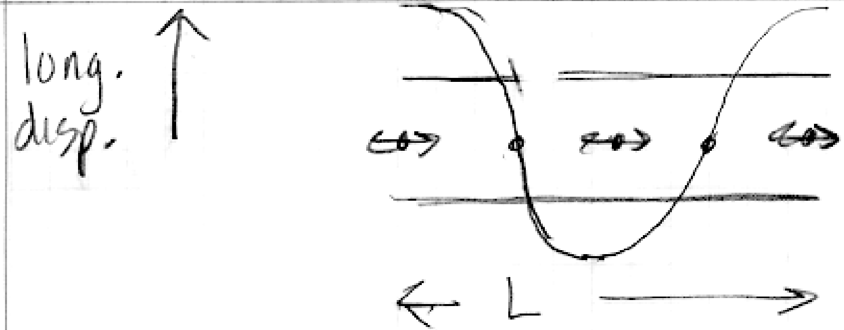
$v = \lambda \cdot f = 2L \cdot f = 2 \times 0.216\,m \cdot 780\,\frac{1}{s}$

$v \approx 337$ m/s



$f = \frac{v}{\lambda}$ higher

$\Rightarrow$ can excite overtone, octave higher

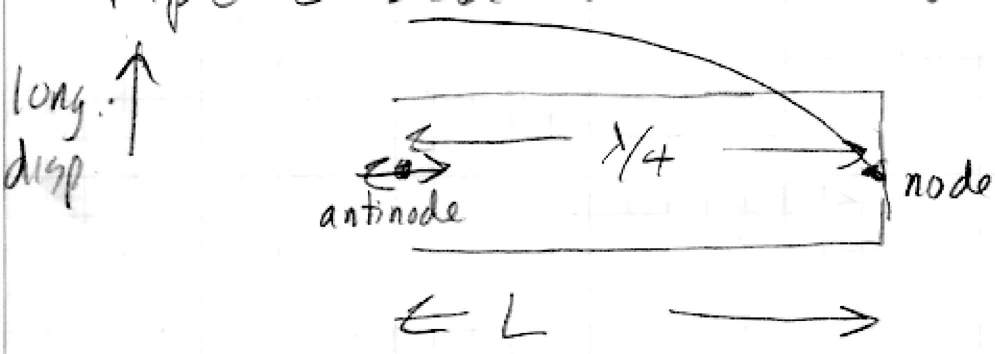


second harmonic

$\lambda_2 = L$

Open ended pipe: $\lambda_n = \frac{2L}{n}$ (like string)
$n = 1, 2, 3, \ldots$

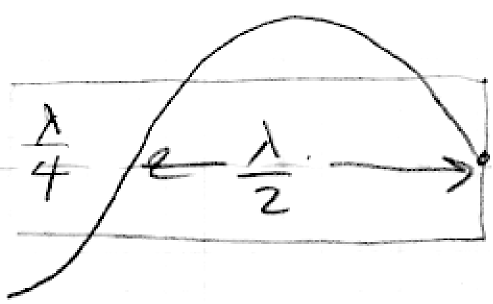
Pipe closed at one end:



Fundamental: $\frac{\lambda}{4} = L$, $\lambda = 4L$ (clarinet oboe saxophone)

Second Harmonic: $\frac{\lambda}{4} + \frac{\lambda}{2} = L$, $\frac{3}{4}\lambda = L$

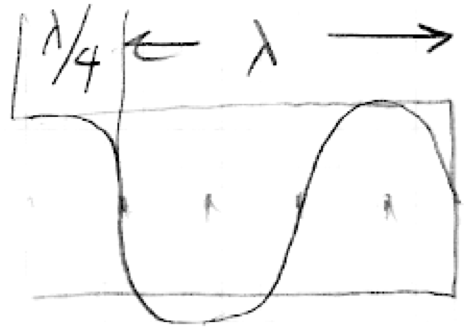
$\lambda = \frac{4}{3}L$



Third Harmonic:

$\frac{\lambda}{4} + \lambda = L$

$\lambda = \frac{4}{5}L$



$\frac{\lambda}{4} + m \cdot \frac{\lambda}{2} =$

$m = 0, 1, 2, 3$

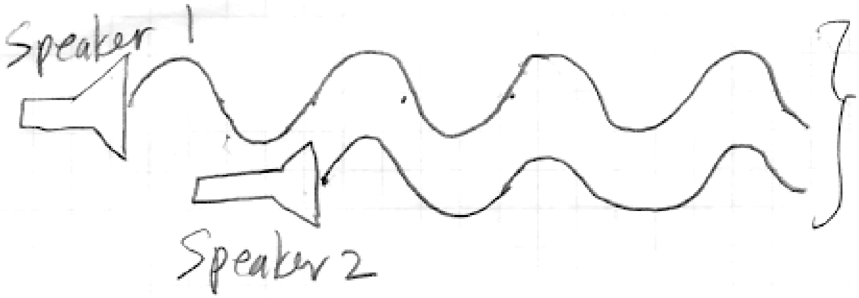
$\lambda = \frac{4L}{2m+1}$

$m = 0, 1, 2, 3,$

$$\lambda_n = \frac{4L}{n} \qquad n = 1, 3, 5, \ldots$$

(one end open, one closed)

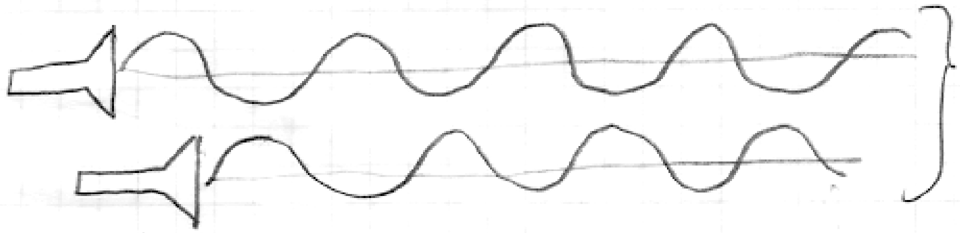
Wave Interference



constructive, louder

$\rightarrow| \ \lambda \ |\leftarrow$ when this distance $= \lambda, 2\lambda, \ldots n\lambda$

When distance is $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \ldots$

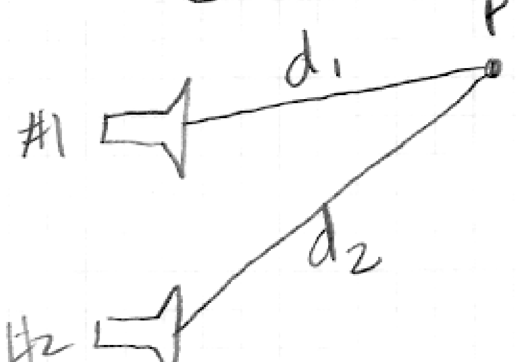


sum up to 0!

$\rightarrow| \frac{\lambda}{2} |\leftarrow$

destructive

Now go to two dimensions



look at $|d_2 - d_1|$

when $=$ integer # λ

<u>constructive</u>

$= \frac{1}{2} \times$ (odd integer) # λ

<u>destructive</u>