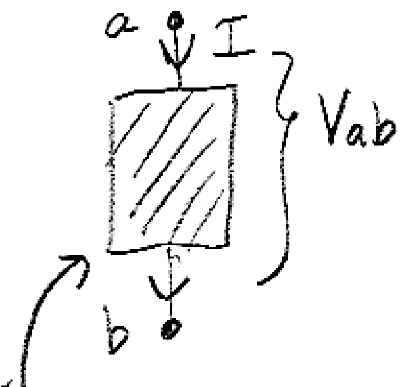
Chapter 27 – DC Circuits

1) Resistors in series + parallel
2) Kirchoff's Rules
3) Meters
4) RC Circuits
5) Power Distribution

Series + Parallel Resistors



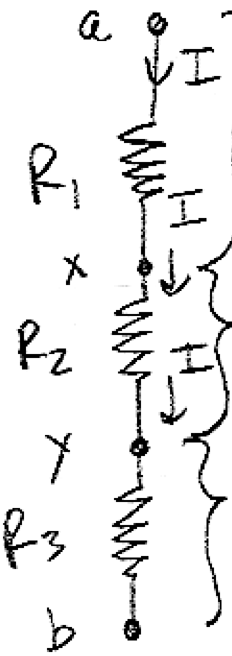
resistors in box

$V_{ab} = I R_{eq}$

defines R_{eq}

For Example

current I same!



$V_{ax} = I R_1$

$V_{xy} = I R_2$

$V_{yb} = I R_3$

$$V_{ab} = V_{ax} + V_{xy} + V_{yb}$$

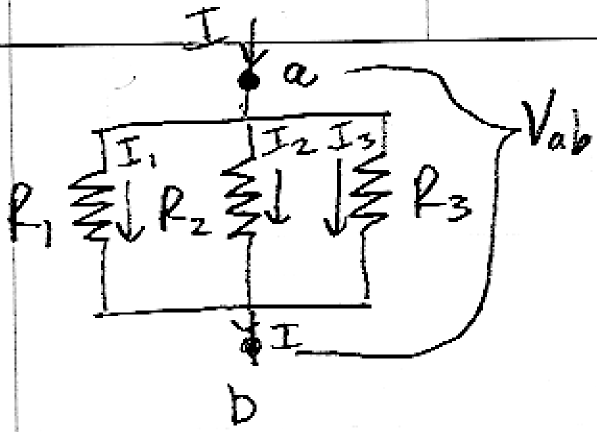
$$= IR_1 + IR_2 + IR_3$$

$$V_{ab} = I(R_1 + R_2 + R_3)$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

resistors in series

(more resistors, keep adding)



$$V_{ab} = I_1 R_1 = I_2 R_2 = I_3 R_3$$

$$I = \underbrace{I_1}_{\frac{V_{ab}}{R_1}} + \underbrace{I_2}_{\frac{V_{ab}}{R_2}} + \underbrace{I_3}_{\frac{V_{ab}}{R_3}}$$

$$I = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$R_{eq} I = V_{ab}$$

$$I = V_{ab} \cdot \frac{1}{R_{eq}}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$1/R_{eq}$ is larger than any of $1/R_1$, $1/R_2$, $1/R_3$

so, R_{eq} is smaller than any of R_1, R_2, R_3

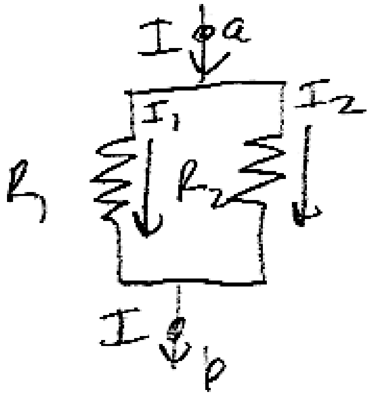
note: $V_{ab} = I_i R_i$

$$I_i = \frac{V_{ab}}{R_i} \Rightarrow I \propto \frac{1}{R}$$

parallel: most current flows through smallest resistor.

series: most voltage across largest resistor

Two parallel resistors



$$V_{ab} = R_{eq} I$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

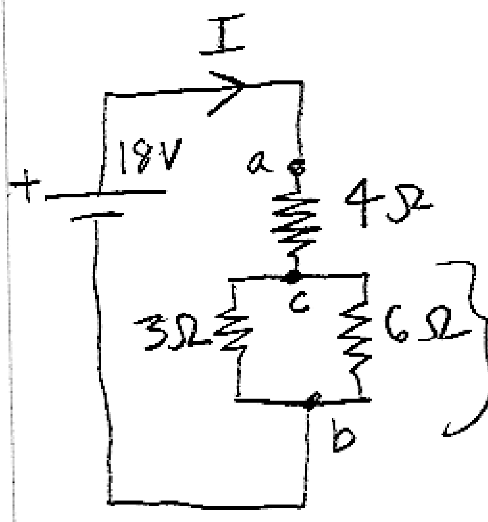
$$\frac{1}{R_{eq}} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

when $R_1 \ll R_2 \Rightarrow \frac{R_1 R_2}{R_2} = R_1$

$R_2 \ll R_1 \Rightarrow \frac{R_1 R_2}{R_1} = R_2$

"small one dominates"

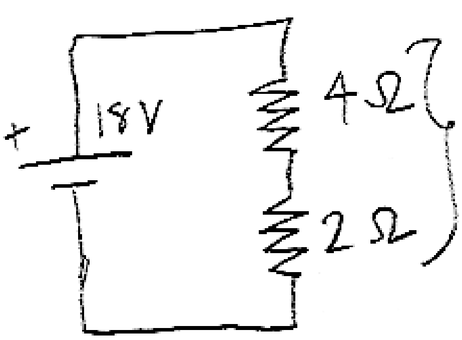


- Find I ... everywhere
- simplify resistors

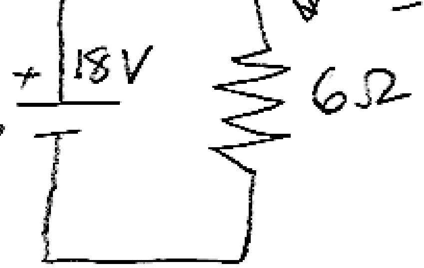
$$\frac{1}{R} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{3}{6}\frac{1}{\Omega}$$

$$R = 2\,\Omega$$

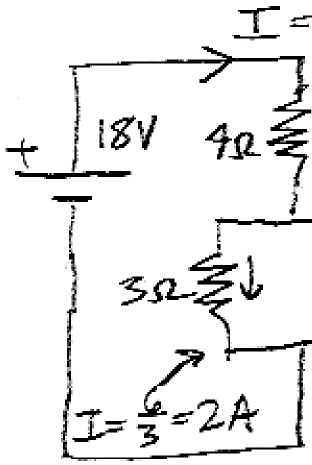
$$I = \frac{18\,V}{6\,\Omega} = 3\,A$$



$R' = 4 + 2 = 6\,\Omega \Rightarrow$



Now "unwind"



$\}$ 3×4 = 12V drop.

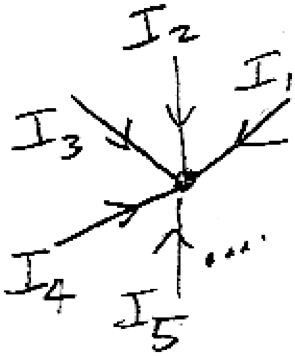
$\}$ 18 − 12 = 6V

$I = \frac{6}{6} = 1A$ note: 2+1 = 3 A.

Kirchoff's Rules

... tell you what to do for complex circuits.

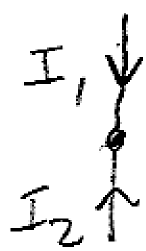
(1) at a junction of wires, all incoming current must be carried out -- mathematically



$I_1 + I_2 + I_3 + I_4 + I_5 + \ldots = 0$

or $\sum_i I_i = 0$

simple case:

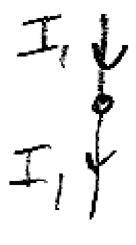


$I_1 + I_2 = 0$

$I_2 = -I_1$

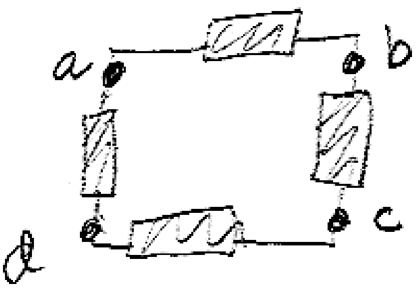
− sign means, reverse direction

means:



(charge never accumulates or disappears)

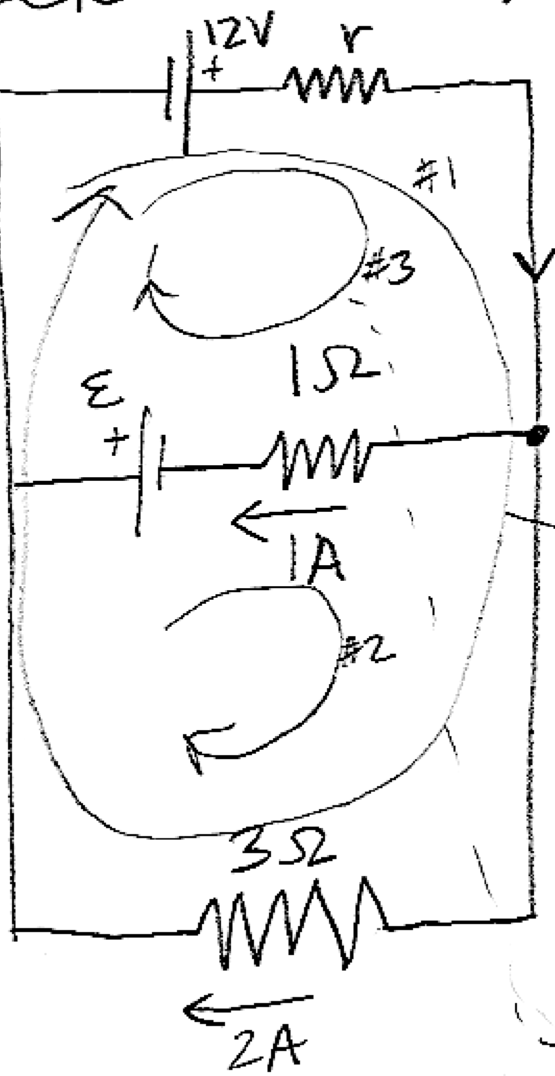
② Around a loop of circuit elements, there is no net voltage...



$$V_{ab} + V_{bc} + V_{cd} + V_{da} = 0$$

$$\sum_i V_i = 0$$

<u>Example</u>: Battery Charging



$I = 1+2$
$= 3A$

Find ε and r

→ 2 unknowns

→ 2 loops

this loop, #1

$$\underbrace{-I \cdot r}_{\text{drop}} \underbrace{- 2 \cdot 3}_{\text{drop}} + \underbrace{12}_{\text{rise}} = 0$$

$$-3 \cdot r + 6 = 0$$

$$\boxed{r = 6/3 = 2\Omega}$$

this loop, #3

$$-Ir - 1 \cdot 1 + \varepsilon + 12 = 0$$

$$-6 - 1 + \varepsilon + 12 = 0$$

$$\boxed{\varepsilon = -5V}$$

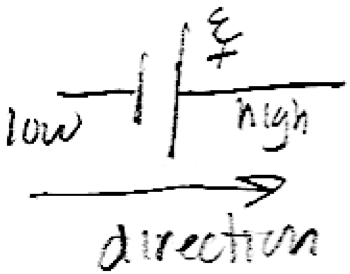
Cross check #2:

$$-\varepsilon + 1.1 - 3.2 = 0$$

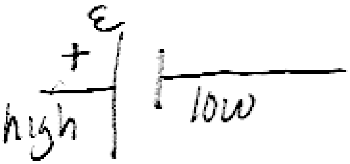
$$+5 + 6 - 6 = 0$$

$$0 = 0$$

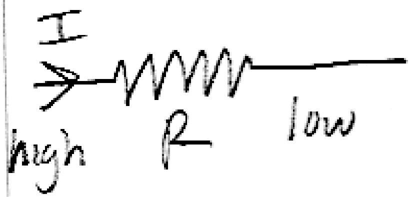
Important:



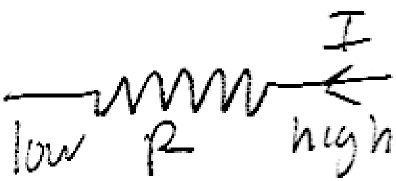
$+\varepsilon$



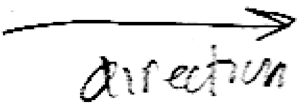
$-\varepsilon$

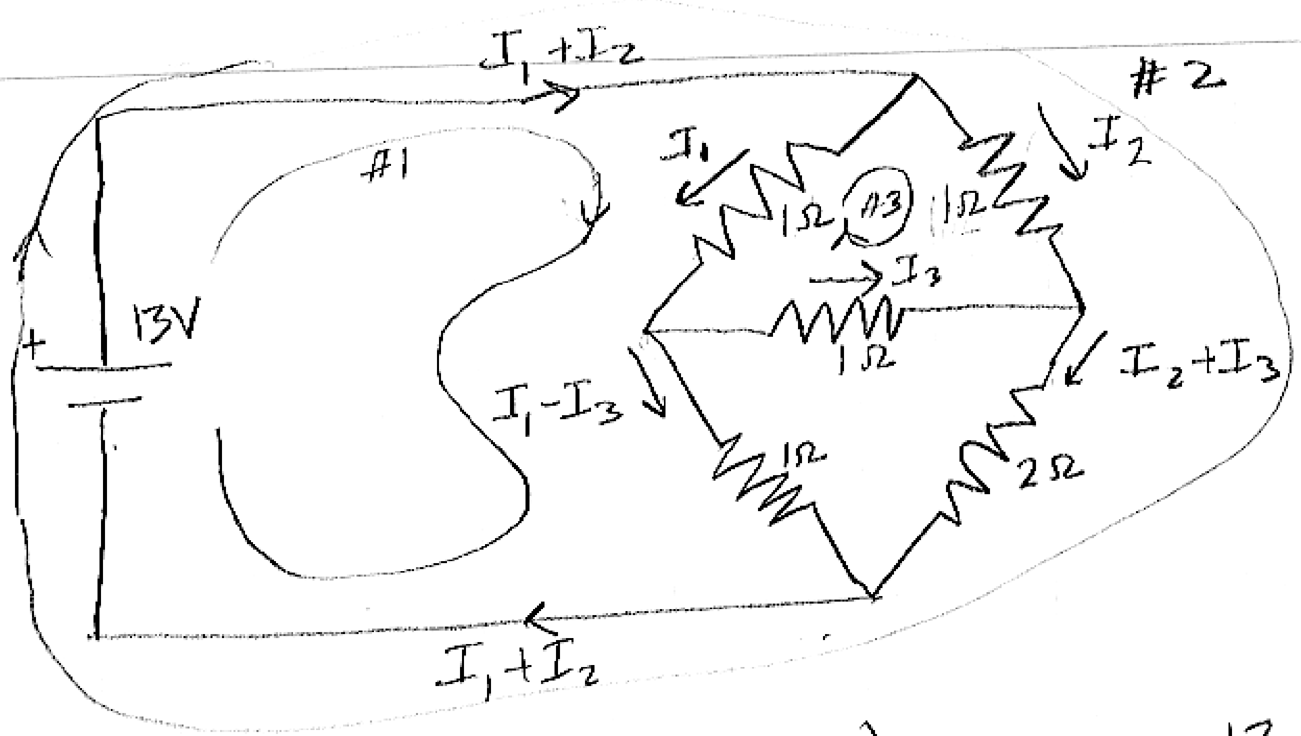


$-IR$



$+IR$





$\underline{3}$ unknowns

need 3 equations

loops: #1 : $13V - I_1 \cdot 1 - (I_1 - I_3) \cdot 1 = 0 \Rightarrow 13 - 2I_1 + I_3 = 0$

#2 : $13V - I_2 \cdot 1 - (I_2 + I_3) \cdot 2 = 0 \Rightarrow 13 - 3I_2 - 2I_3 = 0$

#3 : $I_1 \cdot 1 - I_2 \cdot 1 + I_3 \cdot 1 = 0 \Rightarrow I_1 - I_2 + I_3 = 0$

$I_2 = I_1 + I_3$

#1: $13 = 2I_1 - I_3 \qquad = 2I_1 - I_3$

#2 + #3 $\Rightarrow 13 = 3 \cdot (I_1 + I_3) + 2I_3 = 3I_1 + 5I_3$

$$\#1 \times 5: \quad 65 = 10I_1 - 5I_3$$

$$\#2': \quad 13 = 3I_1 + 5I_3$$

$$78 = 13I_1$$

$$\boxed{I_1 = 6A}$$

sub into #2:

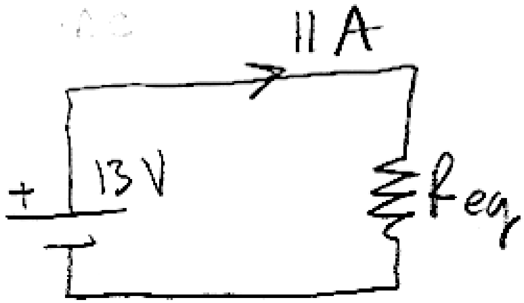
$$13 = 3 \cdot 6 + 5 \cdot I_3$$

$$13 - 18 = -5 = 5I_3$$

$$\boxed{I_3 = -1A}$$

$$\boxed{I_2 = I_1 + I_3 = 6 - 1 = 5A}$$

$$I_1 + I_2 = 6 + 5 = 11 \text{ A}$$



$$R_{eq} \cdot 11A = 13V$$

$$\boxed{R_{eq} = \frac{13}{11} = 1.2\,\Omega}$$