Physics 22 Practice Final - 3 hours
2 Pages - turn over!!

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Monday, June 11

Write your answers in a blue book. Calculators and two page of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if necessary) after a symbolic answer is available. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

Remember the real final will take place on Wednesday, June 13 from 4:00pm to 7:00pm in 1640 Broida.

Take the acceleration of gravity near the earth's surface as $g = 10\,\mathrm{m/s^2}$.

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1. A stick of length L and mass M sits on a frictionless horizontal table, and rotates with angular velocity ω_1 about one end, which is pinned by a frictionless nail. A bug with mass $(1/3)M$ is initially on the nail, but then walks slowly out to the other end of the stick. Derive a symbolic expression for the final angular velocity ω_2 of the rotating stick with the bug at its end, in terms of ω_1.

2. Four sticks, each of mass M and length L, are fastened together to make a square frame. The frame is suspended from one of its corners, on earth, and is free to swing from that corner about the axis perpendicular to the plane of the frame. Find the period of its oscillations.

3. A mass of $m = 0.1\,\mathrm{kg}$ is on a frictionless surface and is attached to a horizontal spring of spring constant $k = 0.4\,\mathrm{N/m}$. There is a damping coefficient $b = 4 \times 10^{-3}\,\mathrm{Ns/m}$.

   (a) If a constant force $F_0 = 10^{-2}\,\mathrm{N}$ is applied to the mass, by how much does it displace from equilibrium?

   (b) What is the Q of the system?

   (c) A (co)sinusoidal force is applied to the mass,

   $$F(t) = F_0 \cos(\omega t)$$

   where F_0 is the same as that in part (a), and $\omega = 1.98\,\mathrm{s^{-1}}$. The transients in the system are allowed to die out.

      i. What is the amplitude of the oscillation of the mass?

      ii. What is the minimum time that passes between the force $F(t)$ reaching its maximum value and the amplitude of the mass reaching its maximum value?

4. Event A takes place at $x_A = 0\,\mathrm{cm}$, $y_A = 0\,\mathrm{cm}$, and $t_A = 0\,\mathrm{ns}$, and Event B takes place at $x_B = 120\,\mathrm{cm}$, $y_B = 90\,\mathrm{cm}$, and $t_B = 4\,\mathrm{ns}$ in the reference frame S.

   (a) Is the interval between the two events timelike, on the light cone, or spacelike?

   (b) If the event is timelike (spacelike) find the minimum time (distance) between the events A and B when those events are viewed from a different moving reference frame.

(c) A frame S' moves along the x axis of frame S with a speed, relative to the speed of light, $\beta = v/c = 4/5$. At $t_A = t'_A = 0$, the origins of S and S' coincide. Find the time and space coordinates of both events A and B in the S' frame.

5. When a large mass M is confined to a uniform sphere of smaller and smaller radius r, the minimum velocity for a tiny mass m to escape from r to infinitely far away becomes bigger and bigger.

   (a) Find an expression for that radius (denoted r_s) where the escape velocity becomes the speed of light, c. *Use the non-relativistic formula for the kinetic energy of the escaping particle.*

   (b) Numerically evaluate r_s when $M = 2.0 \times 10^{30}$ kg, the mass of the Sun.

   Take $c = 3.0 \times 10^8$ m/s, and $G = (2/3) \times 10^{-10}\,\mathrm{m^3kg^{-1}s^{-2}}$. The radius r_s is known as the 'Schwarzschild' radius of the mass M, and when the mass M is crushed to that radius, it becomes a black hole.

6. In this problem, design a time machine that will take a person forward in time by simply keeping them going at a speed near that of the speed of light. The total rest mass of the machine and its inhabitants is m_0.

   (a) We'd like 10 of our years to go by when the person in the time machine perceives only one year. Find the Lorentz factor γ and the speed relative to the speed of light β (to 3 digits to the right of the decimal point) of the person in the time machine.

   (b) Symbolically evaluate the momentum of the machine and its inhabitants in terms of m_0, γ, β, and c.

   (c) We'd like to implement the time machine by putting it into a gravitationally-bound circular orbit or radius R around a uniform spherical body of mass M. We'll first analyze the problem in the rest frame of that spherical body.

      i. Assume that the time machine is in a circular orbit where the magnitude of its momentum is p. Evaluate dp/dt in the rest frame of the body its orbits, in terms of m_0, γ, β, c, and R. Do not use the knowledge of the gravitational force to arrive at dp/dt.

      ii. Use your knowledge of the gravitational force to evaluate R in terms of other quantities.

      iii. Numerically evaluate R, and compare it to the radius of the Sun, $R_{\mathrm{sun}} = 7.0 \times 10^8$ m.

      iv. Numerically evaluate the acceleration of the machine as viewed in the rest frame of the Sun.

      v. Repeat the last part for the inhabitants of the time machine.
