Physics 22 Problem Set 4

Harry Nelson

due Monday, April 30, In Class

Course Info: This week we study orbits in a gravitational (or electric) field. Read Chapter 9, sections 9.6-note 9.1, pp. 390-410.

The instructor is Harry Nelson, the TA is Joel Varley. A web page for the course is set up at http://hep.ucsb.edu/courses/ph22.

We meet MWF 1:00-1:50pm in 1640 Broida. There are **two sections**, attendance at **both** is mandatory. Joel Varley's section will take place Friday 11:00-11:50pm in 1802 Psychology, and Harry Nelson's will take place Friday 2:00-2:50pm in 2129 Girvetz. Harry Nelson's office hours will follow section until 5:00pm on Friday, either in 2129 Girvetz (if possible) or in the PSC. Joel Varley's office hours will will take place in the Physics Study Room (1019 Broida) on Tuesday from 9:00am to 10:00am, Thursday from 9:00am to 10:00am, and Friday noon-1:00pm.

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. An Earth-launched satellite is in a circular orbit of radius R_s about the center of the Earth. The satellite takes exactly $T = 24$ hours to make one complete orbit of the Earth, and the satellite is in the plane of the Earth's equator. This type of satellite is known as *geosynchronous*, because they are always above a particular Earth longitude. Find an expression for R_s/R_e, where $R_e = 6.38 \times 10^6$ m is the Earth's radius, in terms of R_e, the acceleration of gravity $g = 9.8\,\mathrm{m/s^2}$, and T, and then numerically evaluate R_s/R_e. Take a look at the following website, in particular the 'JTrack' Java Applet:

   `http://science.nasa.gov/Realtime/jtrack/3d/JTrack3D.html`

   and see if you can find the geosynchronous satellites.

2. In this problem we'll systematically work through the slingshot effect for a spaceship sitting at rest near Jupiter.

   (a) If we approximate Jupiter's orbit about the Sun as circular, its distance from the Sun is $R_J = 7.8 \times 10^{11}$ m. Find the speed of Jupiter, v_J, symbolically as a function of Newton's constant $G = 6.7 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$, the mass of the Sun $M_s = 2.0 \times 10^{30}$ kg, and R_J, and evaluate the speed numerically. *Answer:* $v_J = 1.3 \times 10^4$ m/s.

   (b) Find, symbolically and numerically, the velocity a body must achieve to escape from the Sun, assuming that the body starts from a distance R_J from the Sun. *Answer:* 1.9×10^4 m/s

   (c) Imagine Jupiter approaches a spaceship, as shown in Figure 1. Let's work through the parameterization of the orbit of the spaceship relative to Jupiter. The first step is to 'run alongside Jupiter' at the velocity v_J, and in this reference frame, which is Jupiter's rest frame, the spaceship has velocity $-v_J$. Then, evaluate the orbit of the spaceship around Jupiter in

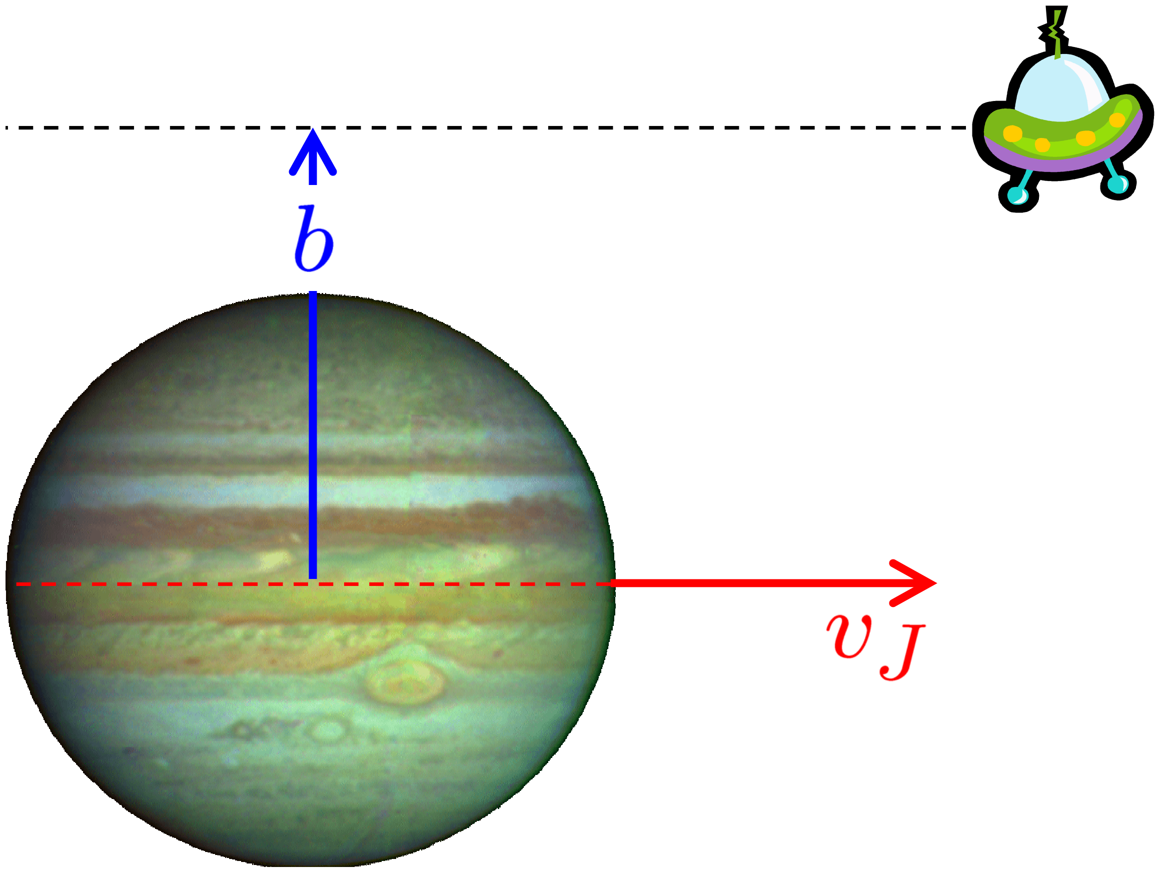

Figure 1: Spaceship being approached by Jupiter, for use in Problem 2c.

this frame. A good sequence of steps to evaluate an orbit is to find ℓ, the angular momentum of the orbit, and then find r_0, the radius of the circular orbit with the same ℓ, then find ϵ, the eccentricity, which depends on the energy relative to the energy of the circular orbit at r_0. Knowing these quantities allows the description of the orbit, as parameterized in Equation 9.21 on page 392 of your text.

i. Evaluate the angular momentum of the spaceship in the following form:

$$\ell = \alpha \times m_s \left[\frac{b}{\rho_J}\right]$$

and find the constant α both symbolically and numerically. The constant $\rho_J = 7.1 \times 10^7$ m is the radius of Jupiter itself, and m_s is the mass of the spaceship.

Answer $\alpha = 9.3 \times 10^{11}\ \mathrm{m}^2/\mathrm{s}$

ii. Find the circular orbit about Jupiter, r_0, that has the same angular momentum as the spaceship. Do this by finding the constant β, both numerically in the formula:

$$r_0 = \beta \left[\frac{b}{\rho_J}\right]^2.$$

You'll need the mass of Jupiter, $M_J = 1.9 \times 10^{27}$ kg. *Answer* $\beta = 6.8 \times 10^6$ m

iii. Find the magnitude of the total energy $|E_c|$ (the sum of potential energy and kinetic energy) of the spaceship if it were in a circular orbit of radius r_0. Put the answer in the form:

$$|E_c| = \gamma m_s \left[\frac{\rho_J}{b}\right]^2$$

and evaluate γ numerically. *Answer* $\gamma = 9.4 \times 10^9$ Joules/kg

iv. Calculate the eccentricity, which depends on the actual total energy of the spaceship, E, and the E_c computed above. Take $E = (1/2)m_s v_J^2$, which is the assumption that the spaceship starts infinitely far from Jupiter. Then, the eccentricity is:

$$\epsilon = \sqrt{1 + \frac{E}{|E_c|}}$$

Employing the approximation that:

$$\sqrt{1+x} \approx 1 + x/2$$

Evaluate the constant δ in the relationship:

$$\epsilon \approx 1 + \delta \left[\frac{b}{\rho_J}\right]^2$$

Answer $\delta = 4.6 \times 10^{-3}$

v. The main challenge in using the slingshot effect is to avoid crash landing on Jupiter. The distance of closest approach of the spaceship to Jupiter, known as the *perijove* (*perihelion* refers to the Sun, and *perigee* to the Earth; the generic term is *periapsis*), can be evaluated given r_0 and ϵ. Show that the perijove r_{min}, satisfies:

$$r_{\text{min}} \approx \frac{\beta \left[\frac{b}{\rho_J}\right]^2}{2 + \delta \left[\frac{b}{\rho_J}\right]^2}.$$

Solve for the minimum b that prevents crashing into Jupiter. *Answer* $b = 4.7\rho_J$

vi. Evaluate the angle ψ through which the spacecraft swings between its initial direction and its final direction. The angle ψ is shown on page 394 of your text.

Answer $\psi = 130°$

(d) The last step is to find the final speed of the spacecraft in the original frame. You do this by adding back the velocity of Jupiter to the proper component of the spacecraft, and then by using the Pythagorean theorem. Does the final speed exceed the escape velocity... that is, can the spaceship escape our Solar System?