Physics 225b Problem Set 4

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due **Wednesday, Feb. 25 in class**

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1. The point of this problem is for you to calculate an actual cross section at the LHC.

   We discussed in class the cross-section per unit mass-squared for Drell-Yan production by $q\overline{q}$ annihilation through a virtual photon at the LHC pp collider, where the total center of mass energy $\sqrt{s} = 14\,\text{TeV}$:

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2 e_q^2}{9M^2} \times \frac{1}{s} \times \int_{-\left(1-\frac{M^2}{s}\right)\frac{\sqrt{s}}{2}}^{\left(1-\frac{M^2}{s}\right)\frac{\sqrt{s}}{2}} \frac{dp_z}{\sqrt{M^2+p_z^2}} \left[f_q(x_1)f_{\overline{q}}(x_2) + f_{\overline{q}}(x_1)f_q(x_2)\right] \quad (1)$$

   where

$$\begin{aligned}
M &= \text{the invariant mass of the final state leptons} \\
&= \sqrt{x_1 x_2 s} \\
p_z &= \text{the momentum along the beam axis of the lepton system} \\
&= (x_1 - x_2)\frac{\sqrt{s}}{2} \\
x_1 &= \text{the fraction of the rightward moving proton's momentum carried by its parton that annihilates} \\
&= \frac{\sqrt{p_z^2 + M^2} + p_z}{\sqrt{s}} \\
x_2 &= \text{the fraction of the leftward moving proton's momentum carried by its parton that annihilates} \\
&= \frac{\sqrt{p_z^2 + M^2} - p_z}{\sqrt{s}} \\
f_q &= \text{the structure function for quark species } q \text{ in the proton; this includes both valence and sea contribut} \\
f_{\overline{q}} &= \text{the structure function for quark species } \overline{q} \text{ in the proton; of course, this includes only sea contributio} \\
\alpha &= \text{the fine structure constant}, \approx 1/137 \\
e_q &= \text{relative electric charge of quark species } q;\ e_u = 2/3,\ e_d = -1/3; \text{ neglect the other quarks.}
\end{aligned}$$

   Note that I've corrected a mistake in the notes: the factor of 3 from color *suppresses* the cross section relative to $e^+e^- \to \mu^+\mu^-$, because one averages over initial states, and 2/3 of the initial quark/antiquark states due not have the correct color combination to annihilate. We've neglected the 'running' of the structure functions with $Q^2 = M^2$ of the annihilation, which, it turns out, is not too bad an approximation.

   If you look at the terms in Eq. 1, the second pair of terms depends on the beam properties, and is given a special name: the parton-parton differential luminosity function... the two partons in this case are q and $\overline{q}$, but in principle any pair of partons could be used here. It is a differential with respect to the mass-squared M^2 of the final state. The literature on this subject gives it a funny name:

$$\frac{1}{s}\int_{-\left(1-\frac{M^2}{s}\right)\frac{\sqrt{s}}{2}}^{\left(1-\frac{M^2}{s}\right)\frac{\sqrt{s}}{2}} \frac{dp_z}{\sqrt{M^2+p_z^2}}\left[f_q(x_1)f_{\overline{q}}(x_2)+f_{\overline{q}}(x_1)f_q(x_2)\right] \equiv \frac{\tau}{\hat{s}}\frac{d\mathcal{L}}{d\tau}$$

where

$$\begin{aligned} \hat{s} &= x_1 x_2 s = M^2 \\ \tau &= x_1 x_2 = M^2/s \\ \text{so, } \frac{\tau}{\hat{s}} &= \frac{1}{s} \\ \frac{d\mathcal{L}}{d\tau} &= \text{differential luminosity, with respect to a `dimensionless' } M^2,\ \tau = M^2/s \end{aligned}$$

Note that the dimensions of cross-section arise from $1/s$, which, when multiplied by $(\hbar c)^2$, becomes a cross section. The dimensions per unit mass-squared come from the first factor in Eq. 1; experimentally, working per unit M is a bit more convenient, and the transformation from M^2 to M is straightforward, so:

$$\frac{d\sigma}{dM} = \left[\frac{8\pi\alpha^2 e_q^2}{9M}\right]\frac{\tau}{\hat{s}}\frac{d\mathcal{L}}{d\tau}$$

To perform the calculation, you need to assume something for the structure functions. A parameterization from the early 1990's (Eichten, Hinchliffe, and Quigg, Phys. Rev. D **45**, 2269 (1992)) is useful:

$$f_{q/\overline{q}}(x) = a x^b (1-x^c)^d$$

where the parameters a, b, c, and d depend upon the quark species:

parton	a	b	c	d
u(valence)	2.4	-0.4	1.4	3.1
d(valence)	2.1	-0.3	1.2	4.8
$u/\overline{u}$(sea)	0.2	-1	1	11
$d/\overline{d}$(sea)	0.2	-1	1	5.8

This parameterization was originally intended for momentum transfers $Q^2 \approx 5\,\text{GeV}^2 = M^2$; of course, at the LHC, the interesting momentum transfers are more like $M^2 = 10^6\,\text{GeV}^2$. The structure functions will 'run' due to QCD corrections and be substantially different at the LHC; surprisingly, the simple parameterization above does pretty well for $M \approx 1\,\text{TeV}$. You can actually plot and download properly QCD-corrected structure functions (up to date as of 2002) at the web site: `http://durpdg.dur.ac.uk/hepdata/pdf.html`, but try the simple parameterization first, and you need not use the more sophisticated parton distribution functions.

For the LHC, with proton-proton collisions at $\sqrt{s} = 14\,\text{TeV}$, compute $(1/s)d\mathcal{L}/d\tau$ for $u\overline{u}$ and $d\overline{d}$, and compute the differential cross section $d\sigma/dM$ for Drell-Yan lepton pair production through a virtual photon. Do this numerically... for example on a spreadsheet. Here is what I get (**revised 2/27/2009** - I had a numerical bug on my spreadsheet that associated the wrong energy in the integral).

M (TeV)	$\frac{1}{s}\frac{dL}{d\tau}(u\overline{u})$ (fb)	$\frac{1}{s}\frac{dL}{d\tau}(d\overline{d})$ (fb)	$\frac{d\sigma}{dM}$ (fb/TeV)
0.02	1.3×10^9	1.0×10^9	5.2×10^6
0.05	1.9×10^8	1.4×10^8	2.9×10^5
0.1	4.2×10^7	3.1×10^7	3.3×10^4
0.2	9.0×10^6	6.4×10^6	3.5×10^3
0.5	1.0×10^6	7.4×10^5	1.6×10^2
1	1.6×10^5	1.2×10^5	13
2	1.6×10^4	1.2×10^4	0.61
3.5	9.6×10^2	9.5×10^2	2.3×10^{-2}
5	6.4×10^1	8.7×10^1	1.1×10^{-3}
8	9.7×10^{-2}	3.7×10^{-1}	1.6×10^{-6}
10	1.9×10^{-4}	2.4×10^{-3}	5.2×10^{-9}
12	5.2×10^{-9}	5.9×10^{-7}	8.4×10^{-13}
13.9	1.2×10^{-28}	3.9×10^{-22}	4.6×10^{-28}

With luck, you'll see an integrated luminosity of 30 1/fb at the LHC. At about which M can you expect to see 1 event per TeV?

2. Probably the most important process expected at the LHC is the production (and decay) of the Higgs boson. In the minimal standard model, and in the 'narrow width' approximation, the cross-section for Higgs production from a pair of gluons is:

$$\sigma(gg \to H) = \frac{\alpha_s^2 \, G_F M_H^2}{\pi\sqrt{2}\, 2^5\, 3^2}\delta(\hat{s} - M_H^2) \tag{2}$$

where

$$\begin{aligned} \alpha_s &= \text{the strong interaction coupling constant,} \approx 0.1 \text{ at the mass scale that pertains here} \\ G_F &= \text{the Fermi constant, } 1.17 \times 10^{-5}\,\text{GeV}^{-2} \\ M_H &= \text{mass of the Higgs boson; based on LEP evidence, between 114 and 165 GeV}/c^2 \end{aligned}$$

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One important comment: Eq. 2 pertains when the mass of the Higgs is well less than twice the mass of the top quark; use this simplifying assumption in this problem. Further, Eq. 2 is based on the assumption that there are no new heavy quarks or other heavy particles that could couple the Higgs to gluons... hence, some of the interest in the whole topic. BTW, Eq. 2 was first discussed in 1977... Phys. Rev. Lett **40**, 692 (1978).

Make a numerical calculation of the total cross section for production of the Standard Higgs at the LHC... pp collisions with $\sqrt{s} = 14$ TeV. Use the following gluon structure function (from the same source as used in the previous problem):

$$G(x) = (\frac{2.6}{x} + 9.2)(1 - x)^{5.9}.$$

Actually, the production cross section for the Higgs from gluon-gluon fusion is thought to be substantially higher, due to higher-order QCD corrections, and from the 'running' of G(x) from a 5 GeV^2 scale to the LHC scale... a nice bonus.