# Physics 225b Problem Set 3

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## due Monday, Feb. 9 in class

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1. As you know, when an ultrarelativistic proton(neutron) has 4-momentum $p$, the probability of finding a $u$-quark($d$-quark) with 4-momentum $xp$ (where $x$ is the momentum fraction) is $u(x)$, the structure function; the usual approximation neglects the momentum transverse to the proton momentum that arises from the uncertainty principle. Further, the structure functions for $d$, $\overline{u}$, and $\overline{d}$ quarks in the proton (or $u$, $\overline{d}$, and $\overline{u}$ quarks in the neutron) are $d(x)$, $\overline{u}(x)$, and $\overline{d}(x)$. Isospin symmetry is implicitly assumed in the labelling. The normalization of the structure functions is then set by the requirement that the *net* $u$-quark and $d$-quark probabilities are 2 and 1, respectively:

$$\int_0^1 dx[u(x) - \overline{u}(x)] = 2, \qquad \int_0^1 dx[d(x) - \overline{d}(x)] = 1$$

   (a) Explain (in a simple manner) why the essential information in electron-proton and electron-neutron inelastic scattering may be quantified (neglecting the strange-quark and charm-quark contributions) as:

$$\frac{d\sigma_{ep}}{dxdy} \propto f_p(x) = \frac{4}{9}[u(x) + \overline{u}(x)] + \frac{1}{9}[d(x) + \overline{d}(x)] \quad (1)$$

$$\frac{d\sigma_{en}}{dxdy} \propto f_n(x) = \frac{4}{9}[d(x) + \overline{d}(x)] + \frac{1}{9}[u(x) + \overline{u}(x)] \quad (2)$$

   (b) Derive the Gottfried Sum Rule:

$$\int_0^1 dx[f_p(x) - f_n(x)] = \frac{1}{3} + \frac{2}{3}\int_0^1 dx[\overline{u}(x) - \overline{d}(x)]dx$$

   (c) Experimentally, the quantity on the left-hand side of the Gottfried Sum Rule is 1/4 (approximately), not 1/3. What do you conclude? Feynman actually predicted, qualitatively, this effect. Physically, it need not imply isospin (that is, that $u$ and $d$ quarks are identical in their strong interactions) violation. Suggest a physical origin.

   (d) The best data concerning the physics in this question came from the E866 Experiment at Fermilab, shown in Fig. 1. The process they used was Drell-Yan production of $\mu^+\mu^-$ in the collision of 800 GeV protons with hydrogen, and deuterium (a source of neutrons). The momentum fraction carried by a parton in the projectile proton is called $x_1$ and that carried by a target parton is $x_2$; the E866 Experiment was most sensitive when $x_1 \approx 0.5 \gg x_2$.

      i. Draw the Feynman diagrams for the partons that cause $pp \to \mu^+\mu^- X$; no need to draw the spectator partons. Repeat for $pn \to \mu^+\mu^- X$; neglect $Z^0$ contributions.

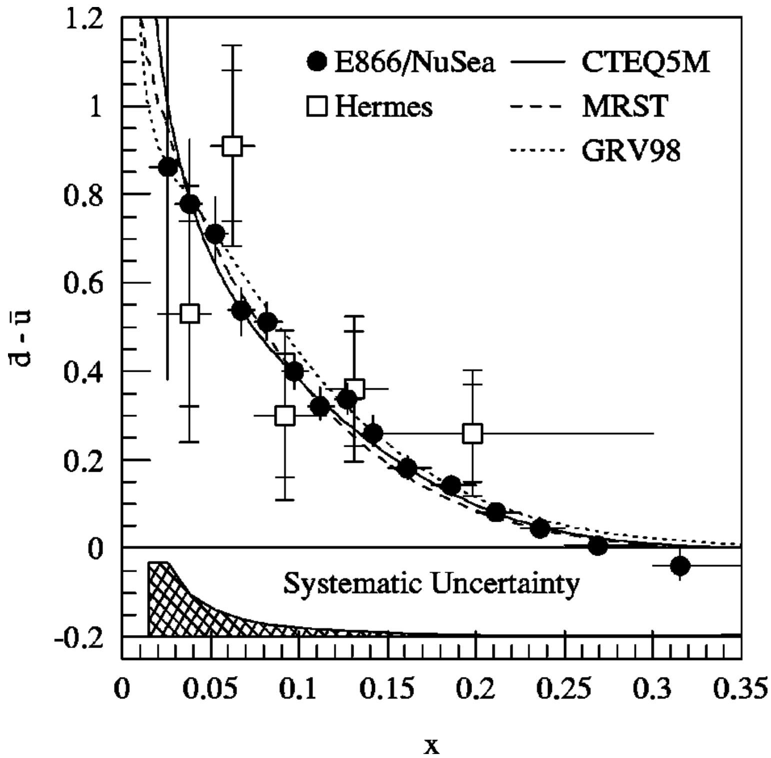


Figure 1: Reduced data from E866 on $\overline{d}(x) - \overline{u}(x)$. Isospin invariance (that $u$ and $d$ have the same strong interactions) implies that the solid curve through the filled black circles should go to 0 as $x$ goes to 0.

ii. Explain (in a simple manner) the following relations for the essential parts of the cross-sections for Drell-Yan production:

$$\frac{d\sigma_{pp}}{dx_1 dx_2} \propto \frac{4}{9}[u(x_1)\overline{u}(x_2) + \overline{u}(x_1)u(x_2)] + \frac{1}{9}[d(x_1)\overline{d}(x_2) + \overline{d}(x_1)d(x_2)]$$
$$\frac{d\sigma_{pn}}{dx_1 dx_2} \propto \frac{4}{9}[u(x_1)\overline{d}(x_2) + \overline{u}(x_1)d(x_2)] + \frac{1}{9}[d(x_1)\overline{u}(x_2) + \overline{d}(x_1)u(x_2)]$$

iii. Explain why the terms involving $\overline{u}(x_1)$ and $\overline{d}(x_1)$ in the previous equations can be neglected.

iv. Assume for the $p$-deuteron ($pd$) cross section:

$$\frac{d\sigma_{pd}}{dx_1 dx_2} = \frac{d\sigma_{pp}}{dx_1 dx_2} + \frac{d\sigma_{pn}}{dx_1 dx_2}$$

and assume $d(x) \ll 4u(x)$, then show:

$$\left.\frac{\frac{d\sigma_{pd}}{dx_1 dx_2}}{2\frac{d\sigma_{pp}}{dx_1 dx_2}}\right|_{x_1 \gg x_2} \approx \frac{1}{2}\left[1 + \frac{\overline{d}(x_2)}{\overline{u}(x_2)}\right]$$

2. In this problem, transform the differential cross section for the elastic electromagnetic scattering of an effectively massless electron of energy $E$ off of a quark in a proton, both at rest, with quark

charge $e_q e$ (so that $e_q = -1/3$ or $+2/3$) and mass $m$, to the center-of-momentum frame. That is, transform:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 e_q^2 E'^2}{Q^4}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m}\right)$$

where

$$\begin{aligned}
\alpha &= \frac{e^2}{\hbar c} \\
E' &= \text{electron's final energy in the frame where the quark is initially at rest} \\
Q^2 &= -q^2\text{, where } q^2 \text{ is the square of the four momentum transferred by the electron} = 4EE'\sin^2\frac{\theta}{2} \\
\theta &= \text{scattering angle in the frame where the quark is initially at rest} \\
\nu &= E - E'
\end{aligned}$$

into a cross section expressed in terms of $x = Q^2/2M_p$, $s \approx 2M_pE$ (assume $E \gg M_p$; note the use of the mass of the *proton*; this is the convention because we don't have free quarks), $y = pq/pk = \nu/E$ (in the quark or proton rest frame) where $p$ is the initial proton 4-momentum, $k$ is the initial electron 4-momentum, $q$ is the 4-momentum transfer from the electron, and $\theta^*$ is the scattering angle in the center-of-momentum frame:

$$\begin{aligned}
\frac{d\sigma}{dxdy} &= \frac{2\pi\alpha^2 e_q^2 s}{Q^4}\left[1 + (1-y)^2\right] x\delta\left(x - \frac{m}{M_p}\right) \\
&= \frac{2\pi\alpha^2 e_q^2 s}{Q^4}\left[2\cos^2\frac{\theta^*}{2} + \sin^4\frac{\theta^*}{2}\right] x\delta\left(x - \frac{m}{M_p}\right)
\end{aligned}$$

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This scattering cross section in the quark rest frame allows one to trace the similarity to Rutherford and Mott scattering; the $\sin^2(\theta/2)$ term arises from spin-flip in the quark rest frame, and is suppressed for small momentum transfers, but, that term becomes $\sin^4(\theta^*/2)$ in the center-of-mass frame. The cross section in the center-of-mass frame shows the similarity $((1 + (1-y)^2)$ to neutrino-quark scattering, as well as the $x\delta(x - m/M_p)$ term you would expect; when one imagines a distribution of quark properties in $x$, this term becomes $xf_q(x)$.

You can start from the cross section in the quark rest frame and get to the center-of-momentum frames any way you want, or, you can follow the suggestions below:

(a) Concerning $\delta\left(\nu - \frac{Q^2}{2m}\right)$:

i. Derive the following equality, which determines the scattered electron energy in the quark's initial rest frame:

$$\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{1}{1 + 2\frac{E}{m}\sin^2\frac{\theta}{2}}\delta\left(E' - \frac{E}{1 + 2\frac{E}{m}\sin^2\frac{\theta}{2}}\right)$$

ii. Derive another perspective on this delta function, useful in visualizing why $x$ is the fraction of $M_p$'s mass carried by the struck quark:

$$\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{m}{M_pEy}\delta\left(x - \frac{m}{M_p}\right).$$

(b) Concerning the change of variables:

$$\frac{d\sigma}{dxdy} = \left| \frac{\partial(E', \Omega)}{\partial(x, y)} \right| \frac{d\sigma}{dE' d\Omega},$$

work out the Jacobian by writing $E'$ and $\Omega = 2\pi(1 - \cos\theta)$ in terms of $x$ and $y$.

(c) Derive these relationships between center-of-momentum and lab quantities, assuming $m \ll E$; recall that the collision occurs between the quark of mass $m$ and the electron, so the pertinent 'reduced' Mandelstam variable is $\hat{s} \approx 2mE$:

$$\begin{aligned} \frac{E'}{E} &= \cos^2\frac{\theta^*}{2} = (1 - y) \\ \theta &\approx \sqrt{\frac{2m}{E}} \tan\frac{\theta^*}{2} \end{aligned}$$

(d) Put these all together to transform the differential cross section in the quark rest frame to that in the center-of-mass frame. Recall that you should work in the limit $m, M_p \gg E$.

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