PHYS 22 HW 9

(1) a) $L = x_B - x_A = 30\text{ cm} - 0\text{ cm} = 30\text{ cm}$

$T = t_B - t_A = 0.75\text{ ns}$

$\frac{L}{T} = \frac{30\text{ cm}}{0.75\text{ ns}} = 40 \frac{\text{cm}}{\text{ns}} > c = 30 \frac{\text{cm}}{\text{ns}} \Rightarrow$ spacelike separation

We know there exists some frame where the events appear simultaneous ($T' = 0$, $L' > 0$)

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{30}{40}\right)c = 0.75\,c}$

Check! $\boxed{\begin{array}{l} T' = \gamma\left(T - \beta\frac{L}{c}\right) = \gamma T\left(1 - \frac{\beta(L/T)}{c}\right) = \gamma T(1-1) = 0 \\ L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.511)L = 45.33\text{ cm} \end{array}}$

b) $\frac{L}{T} = \frac{-30\text{ cm}}{1\text{ ns}} = \left|-30 \frac{\text{cm}}{\text{ns}}\right| = c \Rightarrow$ on the light cone

c) $\frac{L}{T} = \frac{-60\text{ cm}}{1.5\text{ ns}} = \left|-40 \frac{\text{cm}}{\text{ns}}\right| > c \Rightarrow$ space like separation

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{c}{L/T}\right)c = \left(\frac{30}{-40}\right)c = -0.75c}$

✓ $\boxed{\begin{array}{l} T' = \gamma\left(T - \beta\frac{L}{c}\right) = \gamma T\left(1 - \frac{\beta(L/T)}{c}\right) = \gamma T\left(1 - \left(\frac{-3}{4}\right)\left(-\frac{4}{3}\right)\right) = 0 \\ L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.511)L = -45.33\text{ cm} \end{array}}$

d) $L = \sqrt{L_x^2 + L_y^2} = \sqrt{40^2 + 30^2} = 50\text{ cm}$, $T = 4 - 5 = -1\text{ ns}$

$\frac{L}{T} = \frac{50\text{ cm}}{-1\text{ ns}} = \left|-50 \frac{\text{cm}}{\text{ns}}\right| > c \Rightarrow$ spacelike separation

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{c}{L/T}\right)c = \left(\frac{30}{-50}\right)c = -0.6c}$

$\boxed{L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.25)L = 62.5\text{ cm}}$

(2) 12.2 Interval is invariant...

$$c^2t^2 - x^2 = c^2t'^2 - x'^2$$

so $$t^2 = \frac{c^2t'^2 + x^2 - x'^2}{c^2}$$

$$t^2 = t'^2 + \left(\frac{x^2 - x'^2}{c^2}\right)$$

$$t = \pm\sqrt{t'^2 + \underbrace{\left(\frac{x^2-x'^2}{c^2}\right)}_{0\,!}}$$

$$t = \pm t' \rightarrow \underline{+t', \beta = 0\,!}$$

must be $t = -t' = -4\,s$

then $$x' = \gamma(x - \beta c t)$$
$$t' = \gamma\left(t - \beta\frac{x}{c}\right)$$
$$\frac{x'}{t'} = \frac{x - \beta c t}{t - \beta\frac{x}{c}} = \frac{\frac{x}{t} - \beta c}{1 - \beta\frac{x}{ct}}$$

$$\left(1 - \beta\frac{x}{ct}\right)\frac{x'}{t'} = \frac{x}{t} - \beta c$$

$$\frac{x'}{t'} - \frac{x}{t} = \beta\left(\frac{x}{ct}\frac{x'}{t'} - c\right)$$

$$\beta = \frac{\frac{x'}{t'} - \frac{x}{t}}{\frac{x}{ct}\frac{x'}{t'} - c}$$

$$= \frac{\frac{6\cdot\cancel{10^8}}{4} - \frac{6\,\cancel{10^8}}{-4}}{\frac{2\,\,6\cdot\cancel{10^8}}{3\cdot\cancel{10^8}\,4}\frac{6\,\cancel{10^8}}{(-4)} - 3\cdot\cancel{10^8}}$$

$$= \frac{\frac{6}{2}}{-\frac{12}{16} - 3} = \frac{\frac{6}{2}}{\frac{-12-48}{16}}$$

$$= \frac{\cancel{16}^{4}\cdot\cancel{3}^{1}}{\cancel{-60}_{\cancel{20}_{5}}} = -\frac{4}{5}$$

$$\boxed{\beta = -\frac{4}{5}, \quad v = -\frac{4}{5}\cdot 3\cdot 10^8\ m/s = -\frac{12}{5}\cdot 10^8\ m/s}$$

PHYS 22 HW 9

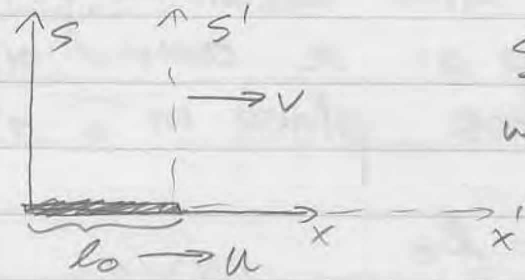
12.5

In the lab frame, we see these 2 ships flying away from one another w/ $v = 0.99c$.

So using the velocity addition formulas, to spaceship A, it is like we are moving w/ velocity $v = 0.99c$, and spaceship B is moving in our frame w/ velocity $u = 0.99c$

So $\boxed{u' = \frac{u+v}{1+uv/c^2} = \frac{2(.99c)}{1+(.99)^2c^2/c^2} = \frac{1.98}{1.9801}c = .99995c}$

12.6



So we have a rod moving w/ speed u in x in frame S. & want to know l'. Since we know for stationary objects $(u=0)$ $l' = \frac{l_0}{\gamma}$

So in S', we have $u' = \frac{u-v}{1-uv/c^2}$ as the speed.

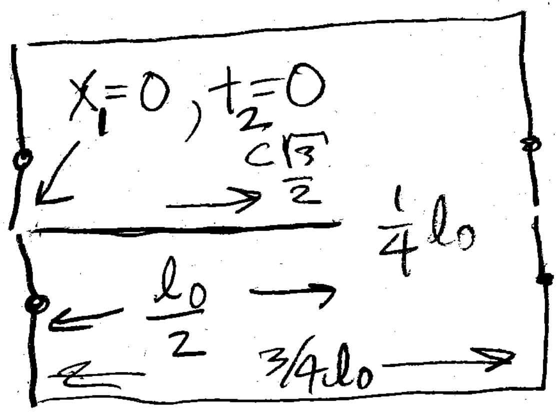
Now our $\gamma = \frac{1}{\sqrt{1-(u'/c)^2}} = \frac{1}{\sqrt{1-\frac{(u-v)^2}{\frac{c^2}{c^4}(c^2-uv)^2}}}$

$\gamma = \frac{c^2-uv}{\sqrt{(c^2-uv)^2 - c^2(u-v)^2}} = \frac{c^2-uv}{\sqrt{c^4 - c^2(u^2+v^2) + u^2v^2}}$

So back in $l' = \frac{l_0}{\gamma}$ form, $\boxed{l' = l_0 \frac{\sqrt{(c^2-u^2)(c^2-v^2)}}{c^2-uv}}$

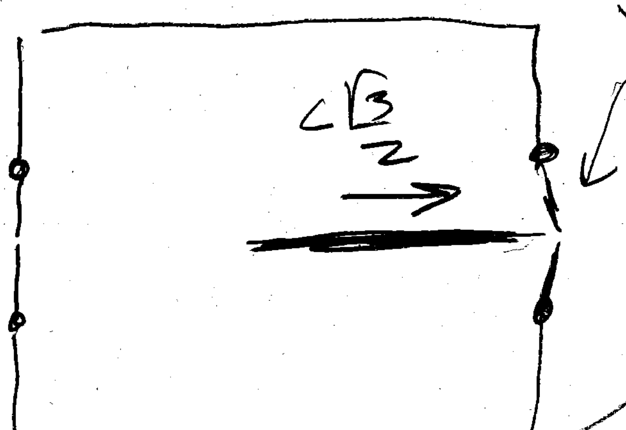
(4) 12.10 : Look at 2 events in Barn rest frame... $\gamma = \frac{1}{\sqrt{1-\frac{3}{4}}}$

(1) Close back door .. $\gamma = 2$



$x_1' = t_1' = 0$ in pole frame too

(2) Open front door



$x_2 = \frac{3}{4}l_0 \quad t_2 = \frac{1/4\, l_0}{\sqrt{3}/2\, c}$

$= \frac{1}{\sqrt{3}\cdot 2}\frac{l_0}{c}$

$$x_2' = 2\left(\frac{3}{4}l_0 - \frac{1}{4}l_0\right) = l_0$$

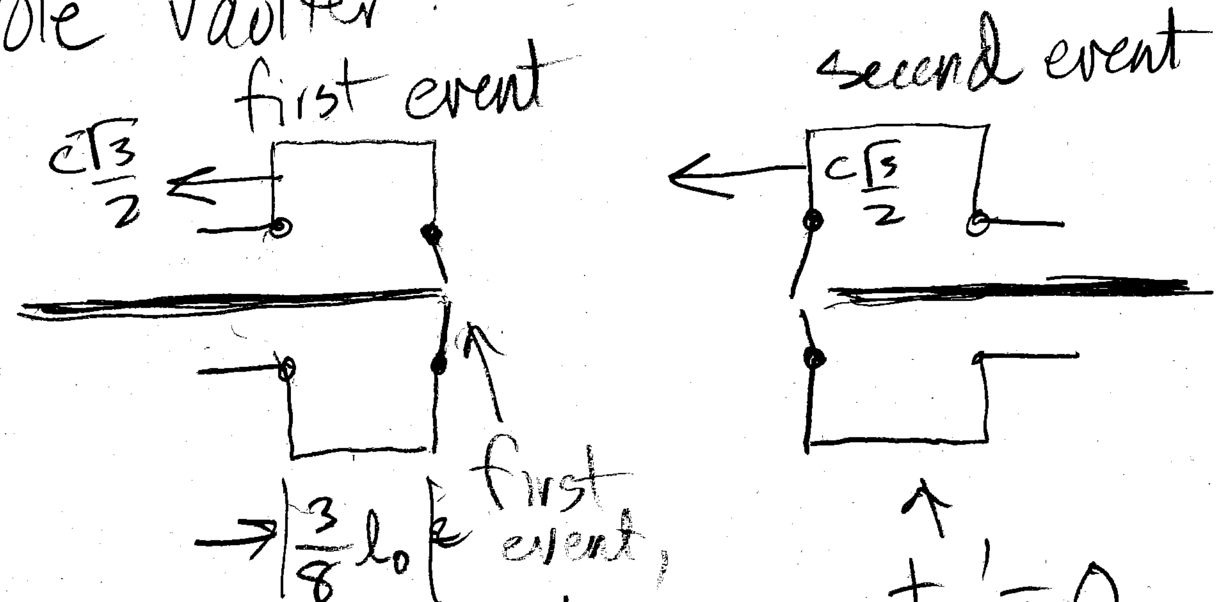
$$t_2' = \cancel{2}\left(\frac{1}{\sqrt{3}\,\cancel{2}}\frac{l_0}{c} - \frac{\sqrt{3}}{\cancel{2}}\frac{3}{4}\frac{l_0}{c}\right)$$

$$t_2' = \frac{1}{\sqrt{3}}\left(1 - \frac{9}{4}\right)\frac{l_0}{c} = -\frac{5}{4}\frac{1}{\sqrt{3}}\frac{l_0}{c} < 0$$

In the barn frame...
pole vaulter trapped in barn for a shot time.

In the pole frame... rear door opens first... barn is foreshortened...

Pole Vaulter:



$$t_2' = -\frac{5}{4}\frac{1}{\sqrt{3}}\frac{l_0}{c} \qquad t_2' = 0$$

Everything real, just sequence differs.

5 12.13

We see...

$$\text{round trip} = 2 \times \underbrace{4.3}_{\text{light years}} \times \underset{\uparrow \; c/5}{5}$$

$$\boxed{t_1 = 43 \text{ years} = \text{age increment of stay at home brother}}$$

$$t_2 = \frac{43\,y}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{1}{5}\right)^2}} = 1.0206$$

$$\boxed{t_2 = 42.13\,y = \text{age increment of traveling twin}}$$

$$\boxed{\#2 \text{ younger by} \ldots \quad t_1 - t_2 = 0.87\,y}$$

⑥ 12.14

$$\left.\begin{aligned} t' &= \gamma\left(t - \beta\frac{x}{c}\right) \\ x' &= \gamma(x - \beta c t) \\ y' &= y \\ z' &= z \end{aligned}\right\}$$

do for
t_1, t_2 $\Delta t = t_2 - t_1$
x_1, x_2 $\Delta x = x_2 - x_1$
y_1, y_2 $\Delta y = y_2 - y_1$
z_1, z_2 $\Delta z = z_2 - z_1$
same: $\Delta t'$ etc.

$$(c\Delta t')^2 - (\Delta x')^2 - \cancel{(\Delta y')^2} - \cancel{(\Delta z')^2}$$

$$= \left(c\gamma\left(\Delta t - \beta\frac{\Delta x}{c}\right)\right)^2 - \left(\gamma(\Delta x - \beta c \Delta t)\right)^2 - \cancel{(\Delta y)^2} - \cancel{(\Delta z)^2}$$

$$= c^2\gamma^2(\Delta t)^2 - \cancel{2c^2\gamma^2\Delta t\beta\frac{\Delta x}{c}} + c^2\gamma^2\beta^2\frac{\Delta x^2}{c^2}$$
$$- \gamma^2\Delta x^2 + \cancel{2\gamma^2\Delta x\beta c\Delta t} - \gamma^2\beta^2c^2\Delta t^2$$

$$= c^2\cancel{\gamma^2(1-\beta^2)}(\Delta t)^2 - \cancel{\gamma^2(1-\beta^2)}(\Delta x)^2$$
$$= (c\Delta t)^2 - (\Delta x)^2 \quad ✓$$

7. $\underbrace{1\,\text{kg}\ \text{m/s}}_{\text{momentum}}$

$$1\,\text{kg} = 1\cdot(3\cdot10^8)^2\ \frac{\text{Joules}}{c^2}$$

$$1\ \text{Joule} = 6.24\cdot10^{18}\ \text{eV} = 6.24\cdot10^{12}\ \text{MeV}$$

$$1\,\text{kg} = 1\cdot(3\cdot10^8)^2\cdot6.24\,10^{12}\ \frac{\text{MeV}}{c^2}$$

$$1\ \text{m/s} = \frac{1}{3\cdot10^8}\ c$$

$$1\,\text{kg m/s} = \frac{1\cdot(3\cdot10^8)^2\cdot6.24\cdot10^{12}}{3\cdot10^8}\ \frac{\text{MeV}}{c}$$

$$= 3\cdot10^8\cdot6.24\cdot10^{12}\ \text{MeV}/c$$

$$= 18.72\cdot10^{20}\ \text{MeV}/c$$

$$\boxed{1\,\text{kg m/s} = 1.87\cdot10^{21}\ \text{MeV}/c}$$

8. $p = \gamma \beta m c$

$= \gamma \beta m c^2 \left(\frac{1}{c}\right)$

(a) Election $mc^2 = 0.511$ MeV

$12.5 \frac{\text{MeV}}{c} = \gamma\beta\, 0.511 \frac{\text{MeV}}{c}$

$\gamma\beta = 24.46 = C$

$\frac{\beta}{\sqrt{1-\beta^2}} = C$

$\beta^2 = C^2(1-\beta^2)$

$(1+C^2)\beta^2 = C^2$

$\beta^2 = \frac{C^2}{1+C^2}$

$\beta = \frac{C}{\sqrt{1+C^2}} = \frac{24.46}{\sqrt{1+(24.46)^2}}$

$\boxed{\beta = 0.99917}$

(b) $12.5 \frac{\text{MeV}}{c} = \gamma\beta \cdot \underset{\substack{\uparrow \\ mc^2 \text{ of} \\ \text{proton}}}{(938)} \frac{\text{MeV}}{c}$

$$\gamma\beta = \frac{12.5}{938} = 0.0133 = C$$

$$\beta = \frac{C}{\sqrt{1+C^2}} = \frac{0.0133}{\sqrt{1+0.0133^2}}$$

$$\boxed{\beta = 0.0133}$$