Physics 21 Practice Final - 3 hours
2 Pages - turn over!!

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Friday, March 16

Write your answers in a blue book. Calculators and two page of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if necessary) after a symbolic answer is available. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

Remember the real final will take place on Tuesday, March 20 from 8:00am to 11:00am in 1640 Broida.

Take the acceleration of gravity near the earth's surface as $g = 10\,\text{m/s}^2$.

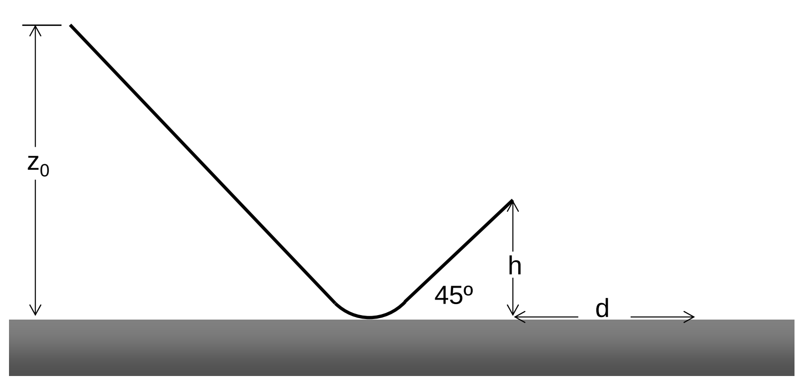

Figure 1: For use in Problem 1.

1. You build the ramp shown in Fig. 1. A small mass is released from rest at the top left. Ignore friction.

   (a) Find the horizontal distance d that the mass flies from the end of the ramp before hitting the floor in terms of z_0, h, and g.

   (b) You imagine leaving the angle of the right end of the ramp with the horizontal at 45°, but varying the height h. What choice of height h results in the largest distance d?

2. A mass of 1 kg sits on a horizontal surface on earth. There is a lot of static friction between the mass and the floor: $\mu = 0.5$. You want to move the mass horizontally, and the maximum magnitude of the force you can apply to the mass is $F = 4.8\,\mathrm{N}$.

   (a) You pull the mass horizontally. Can you move it? Explain quantitatively.

   (b) You decide to pull the mass at an angle of $\theta = 20^\circ$ above the horizontal. Can you move it now? Explain quantitatively.

3. You are driving at a speed of $v = 20\,\mathrm{m/s}$, and you want to turn in a circle of radius $R = 20\,\mathrm{m}$ on a level street. What is the minimum coefficient of friction that is needed between your tires and the ground to keep your car from skidding?

4. A mass accidently explodes at the top of its trajectory. The horizontal distance between the launch point and the point of explosion is L. The mass breaks into two pieces which fly apart horizontally. The larger piece has twice the mass of the smaller piece. The smaller piece lands right under the point of explosion. How far from the launch point does the larger piece land? Neglect air resistance and effects due to the earth's curvature.

5. A mass of 4 kg sits on a frictionless horizontal surface, and is attached to one end of a spring with equilibrium length $\ell_0 = 1\,\mathrm{m}$ and spring constant $k = 100\,\mathrm{N/m}$. The other end of the spring is attached to a wall. The mass is initially at rest. The mass is hit with a force $F = 10^3\,\mathrm{N}$ in the direction of the wall for a very brief time interval Δt.

   (a) Just after being hit, the velocity of the mass is $2\,\mathrm{m/s}$. Find the time interval Δt.

   (b) Find the distance Δx that the mass travels before coming to rest for the first time.

   (c) Find the time from the hit that it takes for the mass to return for the first time to its initial position.

6. A 10 kg mass is dropped from rest, at a distance of $h = 3200\,\mathrm{km}$ above the earth's surface. What velocity does it achieve when it hits the earth? Take the mass of the Earth as $M_e = 6\times10^{24}\,\mathrm{kg}$, the radius of the Earth as $R_e = 6400\,\mathrm{km}$, and the gravitational constant $G = (2/3)\times10^{-10}\mathrm{Nm^2/kg^2}$, and ignore air resistance.

7. A particle of mass $m = 1/5\,\mathrm{kg}$ moves in one dimension from the origin to ∞ and ia subject to the potential energy:

$$U(x) = \frac{A}{x^2} - \frac{B}{x}$$

   where $A = 5$ Joule-meter^2 and $B = 10$ Joule-meter.

   (a) Is there a stable equilibrium point for the particle, and if so, at what value of x does it occur (both symbolically and numerically)?

   (b) Determine the circular frequency ω of small oscillations about any stable equilibrium point (both symbolically and numerically).

8. A mass measuring device exploits elastic scattering. The mass to be measured (called m_2) is placed at rest, and another mass (called m_1) is thrown at the stationary mass with velocity u_1. The masses collide and recoil, all in one dimension. Then, the velocity of m_1 is measured after its elastic collision with m_2. Call m_1's final velocity $f \times u_1$, where f is a variable that has no dimensions, and might be negative. One measures f, in order to deduce m_2.

   (a) Solve symbolically for m_2 as a function of f.

   (b) Suppose $m_1 = 1\,\mathrm{kg}$, and f is measured to be $-1/2$. What is m_2?