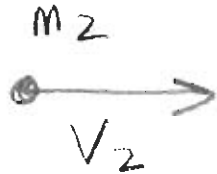
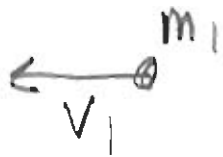
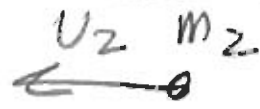
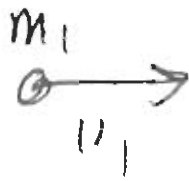


Inelastic

momentum still conserved



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Labels under the equation: "given" under  $m_1 u_1 + m_2 u_2$  and "unknowns" under  $m_1 v_1 + m_2 v_2$ .

→ why? center of mass unmoved by

now inelastic

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + Q$$

Label: "lost energy" with an arrow pointing to  $Q$ .

How big can  $Q$  be?

Can all <sup>initial</sup> energy of  $m_1, m_2$  disappear?

NO → center of mass motion cannot stop.

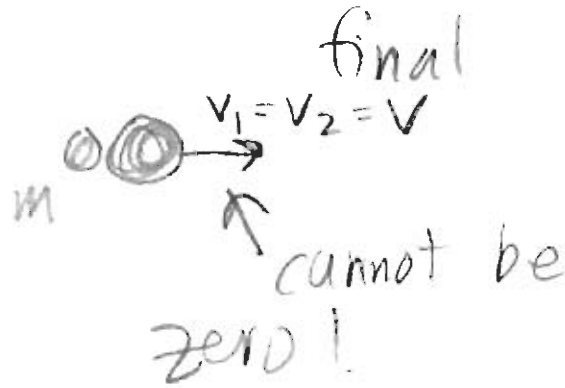
$$V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

example

Inelastic Collisions



initial



final

Why?

center of mass motion, cannot be stopped by internal forces.

$$\frac{mU + M \cdot 0}{m + M} = \frac{m}{m + M} U$$

initial velocity of COM

$$= v \quad (\text{Total velocity})$$

final

$$\frac{1}{2} (m + M) v^2 = \frac{1}{2} (m + M) \left( \frac{m}{m + M} \right)^2 U^2$$

$$= \frac{1}{2} \frac{m^2}{m + M} U^2$$

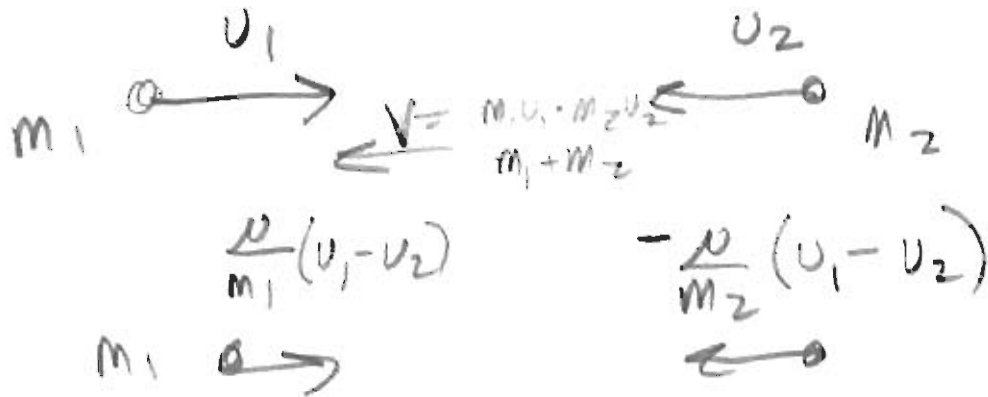
lost energy:

$$\frac{1}{2} m U^2 - \frac{1}{2} \frac{m^2}{m + M} U^2$$

$$= \frac{1}{2} m \left( \frac{m + M - m}{m + M} \right) U^2 = \frac{1}{2} m \left( \frac{M}{m + M} \right) U^2$$

$$\frac{\text{lost}}{\text{initial}} = \frac{\frac{1}{2} m \left( \frac{M}{m+M} \right) v^2}{\frac{1}{2} m v^2} = \frac{M}{m+M}$$

2-d do problem  $\theta^*$  is varia



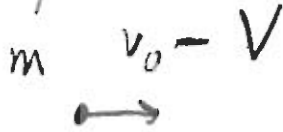
add  $v$   
back



$$\tan \theta_1 = \frac{\frac{v}{m_1}(v_1 - v_2) \sin \theta^*}{V + \frac{v}{m_1}(v_1 - v_2) \cos \theta^*}$$

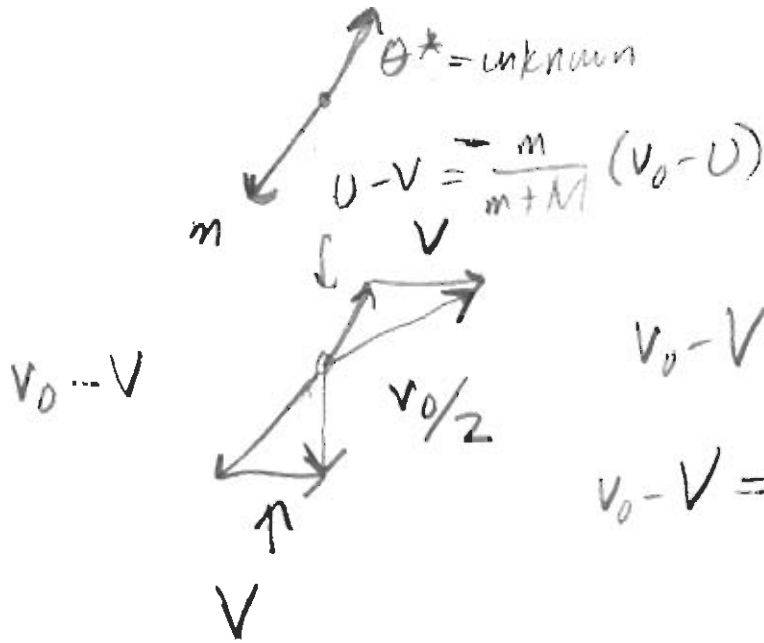
what is largest  $\theta_1$ ?





$M$

$$V = \frac{m v_0 + M U}{m + M}$$



$$v_0 - V = \frac{(m + M)v_0 - m v_0 - M U}{m + M}$$

$$v_0 - V = \frac{M}{m + M} (v_0 - U)$$

$$\tan \theta_1 = \frac{\sin \theta^*}{\frac{v}{\frac{m_1}{m_1}(u_1 - u_2)} + \cos \theta^*}$$

when  $u_2 = 0$

$$\frac{v}{\frac{m_1}{m_1} u_1} = \frac{\frac{m_1 u_1}{m_1 + m_2}}{\frac{m_1 m_2}{m_1 (m_1 + m_2)} u_1} = \frac{m_1}{m_2}$$

$$\tan \theta_1 = \frac{\sin \theta^*}{\frac{m_1}{m_2} + \cos \theta^*}$$

note:

$$\theta_1 \leq \frac{\pi}{2} \quad \text{when} \quad \frac{m_1}{m_2} > 0$$