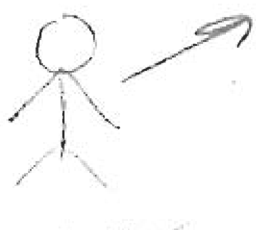
seen by the observer ?

Note true in 2-d, 3-d

$$\vec{X} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{V} = \dot{\vec{X}} = \underbrace{\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}}_{\substack{\text{before collision} \\ \uparrow \\ \text{GIVEN}}} = \underbrace{\frac{m_1 \vec{V}_1 + m_2 \vec{V}_2}{m_1 + m_2}}_{\text{after collision}}$$



runs along with velocity

observer

$$\vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

what do masses look like to the observer.

note:
$$m_1(\vec{v}_1 - \vec{V}) + m_2(\vec{v}_2 - \vec{V})$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 - (m_1+m_2)\vec{V}$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 - (m_1+m_2)\left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)$$
$$= 0$$

no net momentum! according to running observer

In detail...

$$\vec{v}_1 - \vec{V} = \vec{v}_1 - \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)$$
$$= \frac{(m_1+m_2)\vec{v}_1 - m_1\vec{v}_1 - m_2\vec{v}_2}{m_1+m_2}$$
$$= \frac{m_2}{m_1+m_2}(\vec{v}_1 - \vec{v}_2) = \frac{\mu}{m_1}\Delta\vec{v}$$

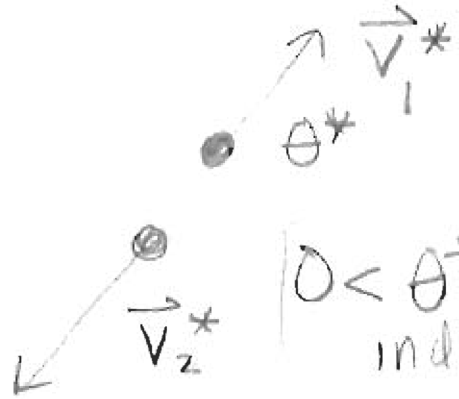
$$\vec{v}_2 - \vec{V} = \vec{v}_2 - \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)$$
$$= \frac{(m_1+m_2)\vec{v}_2 - m_1\vec{v}_1 - m_2\vec{v}_2}{m_1+m_2}$$
$$= \frac{m_1}{m_1+m_2}(\vec{v}_2 - \vec{v}_1) = -\frac{\mu}{m_2}\Delta\vec{v}$$

note:
$$m_1(\vec{v}_1 - \vec{V}) = \left(\frac{m_1 m_2}{m_1+m_2}\right)(\vec{v}_1 - \vec{v}_2) = -m_2(\vec{v}_2 - \vec{V})$$
$$\mu \qquad = -\frac{m_2 m_1}{\ldots}(\vec{v}_{\ldots} - \ldots)$$

To observer, collision looks collinear

<u>or</u>

initial



final

$0 < \theta^* < \pi$

independent variable

1-d

$\theta^* = 0$ no scatter

$\theta^* = \pi$ scatter

2/3-d

<u>still</u> $m_1\vec{V}_1^* + m_2\vec{V}_2^* = 0$

so $m_1\vec{V}_1^* = m_2\vec{V}_2^*$

$$\frac{m_1}{m_2} = \frac{|\vec{V}_2^*|}{|\vec{V}_1^*|}$$

<u>Energy</u>

$$\frac{1}{2}m_1\left(\frac{\mu\Delta\vec{u}}{m_1}\right)^2 + \frac{1}{2}m_2\left(\frac{\mu\Delta\vec{u}}{m_2}\right)^2 = \frac{1}{2}m_1|\vec{V}_1^*|^2 + \frac{1}{2}m_2|\vec{V}_2^*|^2$$

$$\frac{1}{2}\mu^2\underbrace{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}_{\frac{1}{\mu}}|\Delta\vec{u}|^2 = \frac{1}{2}\left(m_1 + m_2\cdot\frac{m_1^2}{m_2^2}\right)|\vec{V}_1^*|$$

$$\frac{1}{2}\mu|\Delta\vec{v}|^2 = \frac{1}{2}m_1^2\underbrace{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}_{\frac{1}{\mu}}|\vec{V}_1^*|^2$$

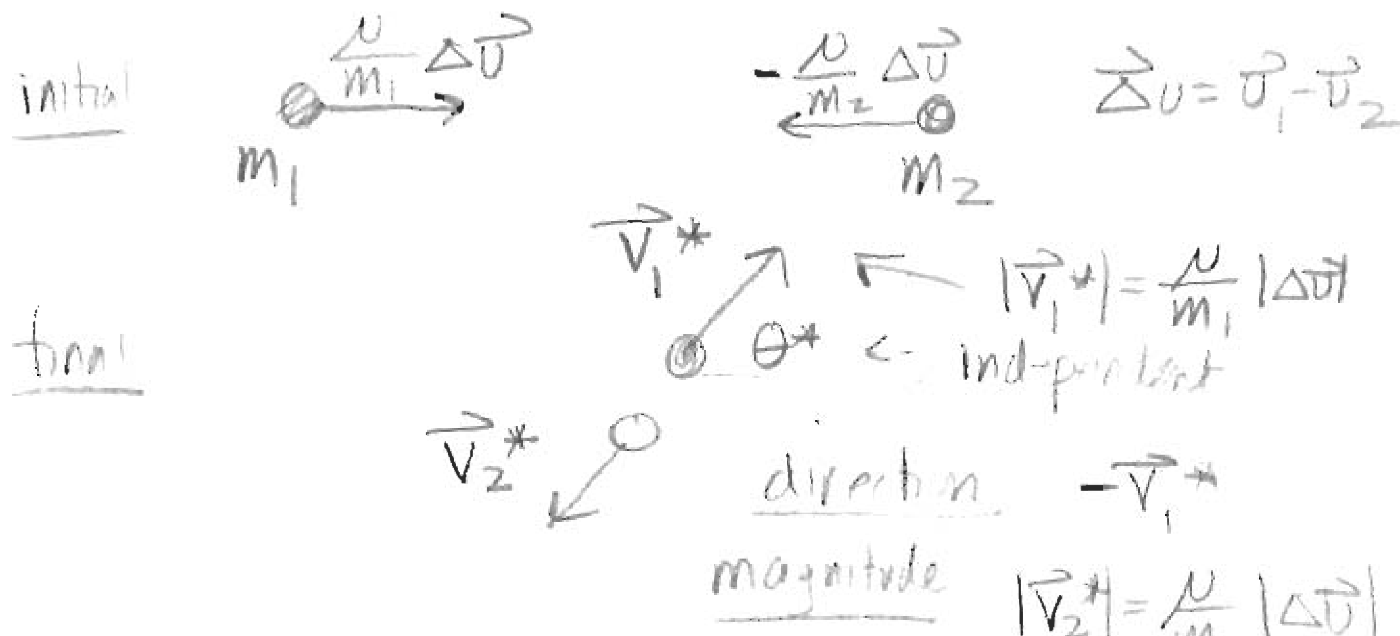
$$|\vec{V}_1^*|^2 = \frac{\mu^2}{m_1^2}|\Delta\vec{v}|^2$$

$$|\vec{V}_1^*| = \frac{\mu}{m_1}|\Delta\vec{v}| = |\vec{v}_1-\vec{V}|$$

$$|\vec{V}_2^*| = \frac{m_1}{m_2}|\vec{V}_1^*| = \frac{\mu}{m_2}|\vec{V}_2^*| = |\vec{v}_2-\vec{V}|$$

Speed does not change !!

in the "Center of Mass" frame



θ^* is totally independent

in 1-d, will be $\theta^* = \pi$ always

(If anything happens at all)

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1-d Original Frame

$\vec{v}_1$ $\qquad\qquad$ $\vec{v}_2$

$m_1 \bullet\!\longrightarrow \qquad\qquad \longleftarrow\!\bullet\ m$

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Center of Mass Fram

$\vec{v}_1 - \vec{V}$ $\qquad\qquad$ $\vec{v}_2 - \vec{V}$

$\bullet\!\longrightarrow \qquad\qquad \longleftarrow\!\bullet$

or $\qquad \frac{\mu}{m_1}\Delta\vec{v} \qquad -\frac{\mu}{m_2}\Delta\vec{v} \qquad \Delta\vec{v} = \vec{v}_1 - \vec{v}_2$

$\bullet\!\longrightarrow \qquad\qquad \longleftarrow\!\bullet$

1-d $\boxed{(\theta^* = \pi}$ in 2-d)

$-\frac{\mu}{m_1}\Delta\vec{v} \qquad\qquad \frac{\mu}{m_2}\Delta\vec{v}$

$\longleftarrow\!\bullet \qquad\qquad \bullet\!\longrightarrow$

$m_1 \qquad\qquad m_2$

go back to original frame

add $\vec{V}$ back

$$\vec{v}_1 = -\frac{\mu}{m_1}\Delta\vec{v} + \vec{V} \qquad\qquad \frac{\mu}{m_2}\Delta\vec{v} + \vec{V} = \vec{v}_2$$

$\leftarrow\!\bullet \qquad\qquad \bullet\!\longrightarrow$

$$-\frac{\mu}{m_1}\Delta\vec{v} + \vec{V} \qquad -\frac{m_2}{m_1+m_2}(\vec{v}_1 - \vec{v}_2) + \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\vec{v}_1 = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_1 + m_2}$$

$$\vec{V}_2 = \frac{(m_2 - m_1)\vec{U}_2 + 2m_1\vec{U}_1}{m_1 + m_2}$$

$\theta^* = \pi$ only

Newton's Cradle

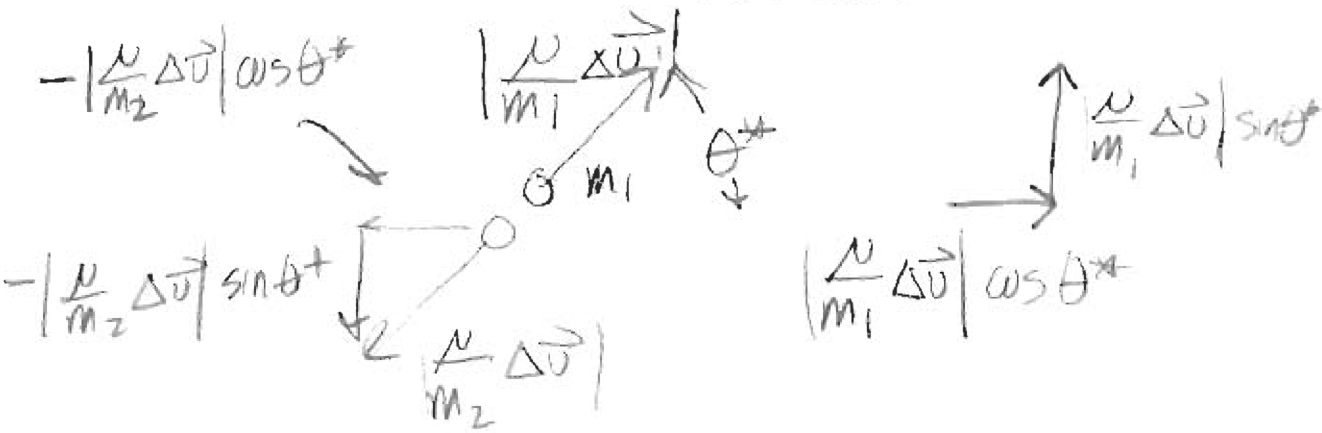
$m_1 = m_2 \qquad \vec{U}_2 = 0$

$$\vec{V}_1 = \frac{(m_1 - m_1)\vec{U}_1 + 0}{m_1 + m_2} = 0$$

$$\vec{V}_2 = \frac{(m_1 - m_1)\cdot\vec{U}_2 + 2m_1\vec{U}_1}{m_1 + m_1} = \vec{U}_1$$

What happens in 2-d?

θ^* is an independent variable



$\vec{V}_1$: