Then

$$U(\vec{r}_b) + K_b = U(\vec{r}_a) + K_a$$

$$U(\vec{r}) = -\int^{\vec{r}}_{\text{starting point}} \vec{F}(\vec{r}) \cdot d\vec{r}$$

or

$$U(\vec{r}_b) - U(\vec{r}_a) = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) \cdot d\vec{r}$$

Examples

Uniform

$$\vec{F}(\vec{r}) = -mg\hat{k}$$

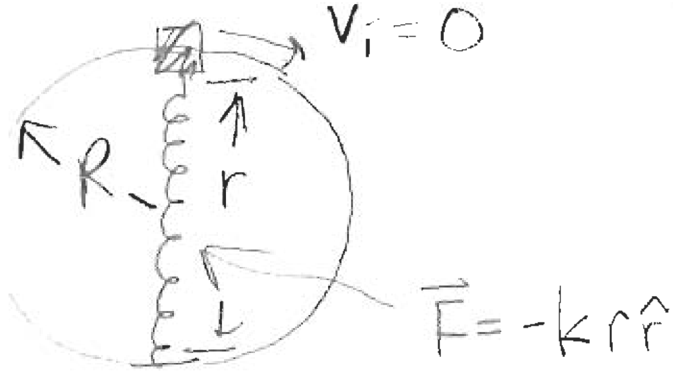
$$\vec{F}(\vec{r}) \cdot d\vec{r} = -mg\,dz$$

$$U(z_b) - U(z_a) = --\int_{z_a}^{z_b} mg\,dz$$

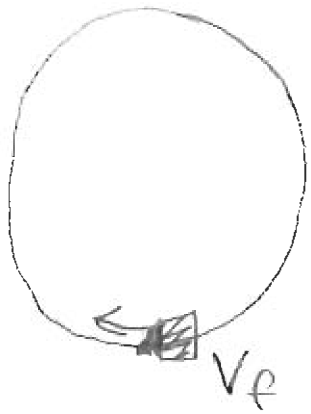
$$U(z_b) - U(z_a) = mg(z_b - z_a)$$

$$U(z) = mgz + \underbrace{\text{constant}}_{\text{arbitrary}}$$

Then $K(z_a) + U(z_a) = K(z_b) + U(z_b)$



Initial



Top: $E = K + U$

initial $= \frac{1}{2}m\cancel{v_i^2}^{\,0} + \frac{1}{2}k(2R)^2 + \underline{mg(2R)}$ relative to bottom

$$E = 2kR^2 + 2mgR$$

final: $= \frac{1}{2}mv_f^2 + \frac{1}{2}k(0)^2 + mg \cdot 0$

so $\frac{1}{2}mv_f^2 = 2kR^2 + 2mgR$

$$\boxed{v_f = 2\sqrt{\left(\frac{k}{m}\right)R^2 + gR}}$$

Given Force $\rightarrow$ $U = -\int F(x)\,dx$

Given $U(x)$ $\rightarrow$ $F(x) = -\frac{dU}{dx}$

$U(x)$ helps visualize $F(x)$

$$\frac{1}{2}mv_a^2 + mgz_a + \text{Constant} =$$

$$\frac{1}{2}mv_b^2 + mgz_b + \text{Constant}$$

Central

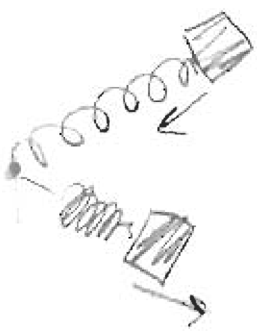
$$\vec{F}(\vec{r}) = f(r)\hat{r}$$

$$\vec{F}(\vec{r})\cdot dr \quad f(r)\,dr$$

$$U(r_b) - U(r_a) = -\int_{r_a}^{r_b} f(r)\,dr$$

Central Spring

$$\vec{F}(\vec{r}) = -k(r-r_0)\hat{r}$$



$$U(r_b) - U(r_a) = -k\int_{r_a}^{r_b}(r-r_0)\,dr$$

$$= +\frac{1}{2}k(r-r_0)^2 \Big|_{r_a}^{r_b}$$

$$U(r_b) - U(r_a) = \frac{1}{2}k\left[(r_b-r_0)^2 - (r_a-r_0)^2\right]$$

$$U(r) = \frac{1}{2}k(r-r_0)^2 + \text{Constant}$$

$$f(r) = \frac{A}{r^2}$$

$$\rightarrow U(r) = \frac{A}{r} + \text{Constant}$$