Concept of Momentum

point mass $\vec{F}_{Net} = m\vec{a} = m\dfrac{d^2\vec{r}}{dt^2} = m\ddot{\vec{r}}$

when m is constant as a function of time, then ...

$$m\frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(m\frac{d\vec{r}}{dt}\right) = \frac{d}{dt}(m\vec{v})$$

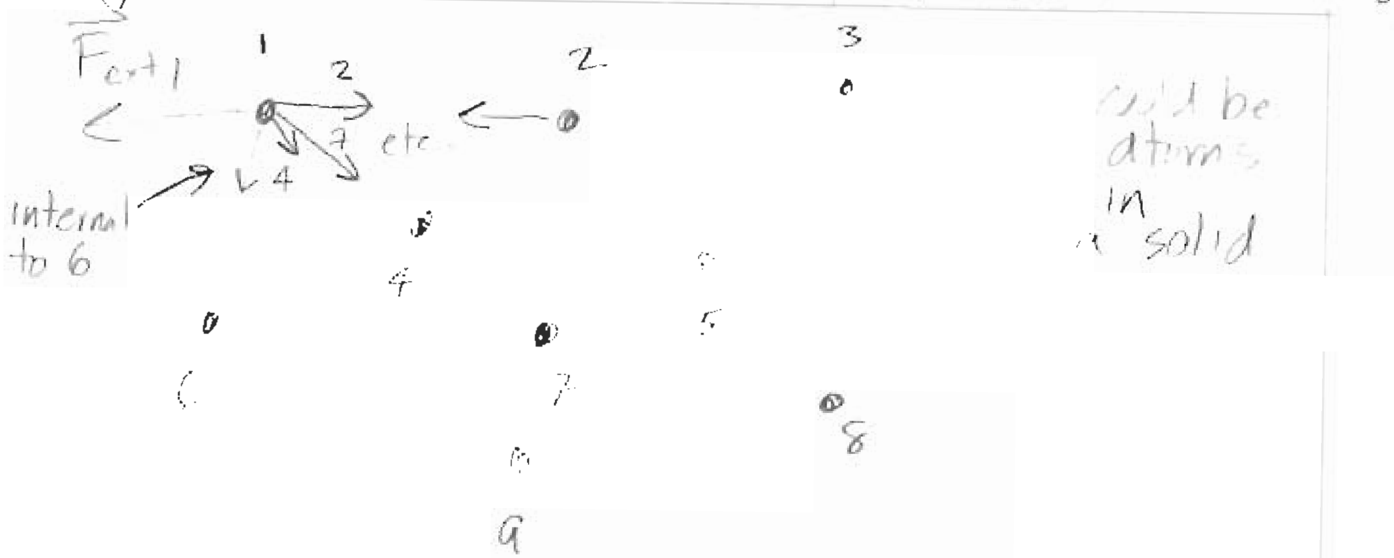
Momentum: $\vec{p} \equiv m\vec{v}$ (definition)

Why?: Some situations, $\vec{p}$ easier to work with... "deeper"

then, $\vec{F}_{Net} = \dfrac{d\vec{p}}{dt}$ ⇐ true even in relativity

Particularly useful for extended bodies... not point particles

Consider an extended body as a big throng of point particles key concept net internal force is zero ... Newton's Third Law.



Categorize forces: internal
external

when adding up, forces on #1

$$\frac{d\vec{p}_1}{dt} = \underbrace{\sum_i \vec{F}_{i1}}_{\text{include all forces, internal + external}} = \sum \vec{F}_{internal,1} + \underbrace{\vec{F}_{ext,1}}_{\text{net external force.}}$$

$$\frac{d\vec{p}_2}{dt} = \sum_i \vec{F}_{i2} = \sum \vec{F}_{internal,2} + \vec{F}_{ext,2}$$

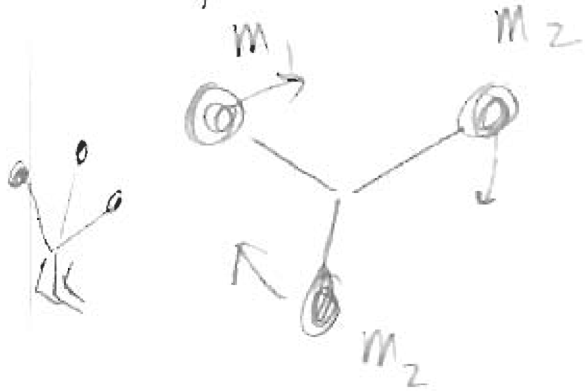
add up all

$$\sum \frac{d\vec{p}_j}{dt} = \sum_{i,j} \vec{F}_{ij} = \underbrace{0}_{\text{internal forces cancel in pairs by Newton 3}} + \sum_j \vec{F}_{ext,j}$$

$$\sum_i \frac{d\vec{p}_i}{dt} = \sum_j \vec{F}_{ext,j}$$

can neglect internal forces

Flying "bola"



$$\frac{d}{dt}(\vec{p}_1+\vec{p}_2+\vec{p}_3) = m_1\vec{g} + m_2\vec{g} + m_3\vec{g}$$

neglect string!

$\vec{P} = \vec{p}_1+\vec{p}_2+\vec{p}_3$ $\quad \frac{d\vec{P}}{dt} = \underbrace{(m_1+m_2+m_3)}_{M}\vec{g} = M\vec{a}$

motion ($\vec{p}_i(t)$) of individual is complicated

but

motion of $\vec{P}_{sum}$ is that of a single particle, mass M

Center of mass

Trajectory of precisely what follows that of something with mass M?

Find an $\vec{R}$ such that

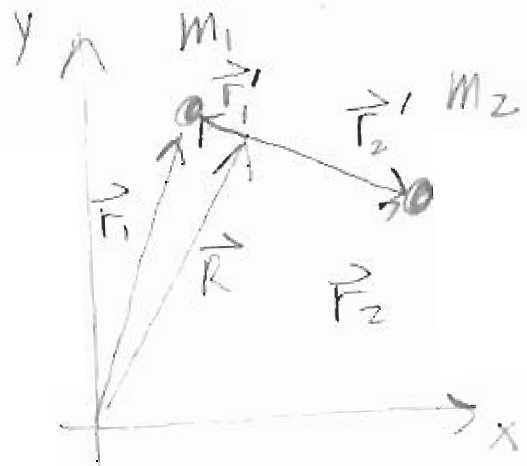
$$\vec{F} = M\ddot{\vec{R}} = \frac{d}{dt}\sum \vec{p}_i = \underbrace{\frac{d}{dt}\sum m_i\vec{v}_i}_{\text{assume } \frac{dm_i}{dt}=0} = \sum m_i\ddot{\vec{r}}_i$$

$$M\ddot{\vec{R}} = \sum m_i \ddot{\vec{r}}_i$$

$$\vec{R} \equiv \frac{1}{M}\sum m_i \vec{r}_i = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

"center of mass" "m-weighted mean position"

<u>Example</u>: baton: two masses joined by thin rod, infinitesimal mass, length ℓ



center of mass is:
1) on a line between
2) closer to the <u>heavier</u> mass

Formula: $\vec{R} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

- $\vec{r}$ to bigger one has bigger coefficient
- $\lim_{m_1\to\infty} \vec{R} = \vec{r}_1$, $\lim_{m_2\to\infty} \vec{R} = \vec{r}_2$

$$\vec{r}_1{}' = \vec{r}_1 - \vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_1 - m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 + m_2} = \frac{m_2}{m_1+m_2}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{r}_2{}' = \vec{r}_2 - \vec{R} = \frac{m_1\vec{r}_2 + m_2\vec{r}_2 - m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 + m_2} = \frac{-m_1}{m_1+m_2}(\vec{r}_1 - \vec{r}_2)$$

both $\propto \vec{r} - \vec{r}$ ...