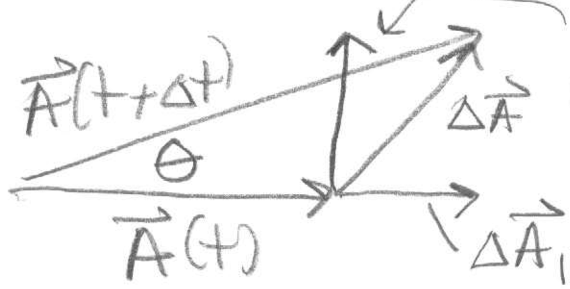
but more generally ...



$$\left|\frac{d\vec{A}_{\perp}}{dt}\right| = |\vec{A}|\frac{d\theta}{dt}$$

new

$$\underbrace{\left|\frac{d\vec{A}_{\parallel}}{dt}\right| = \frac{d|\vec{A}|}{dt}}_{\text{used to this}}$$

$$\frac{d}{dt}(f(t)g(t)) = \frac{df}{dt}g + f\frac{dg}{dt}$$

$$\left.\begin{aligned}
&\frac{d}{dt}(c\vec{A}) = \frac{dc}{dt}\vec{A} + c\frac{d\vec{A}}{dt} \\
&\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} \\
&\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}
\end{aligned}\right\}\text{generalized product rules.}$$

constant vector

$$\frac{d}{dt}(\vec{A}\cdot\vec{A}) = \frac{d\vec{A}}{dt}\cdot\vec{A} + \vec{A}\cdot\frac{d\vec{A}}{dt}$$

$$= 2\vec{A}\cdot\underbrace{\frac{d\vec{A}}{dt}}_{\perp\text{ to}}$$

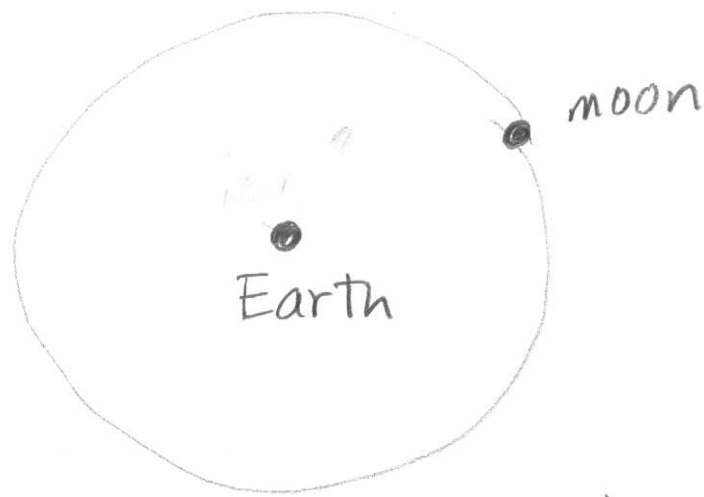
(circular motion) then

$$\frac{d}{dt}(\vec{A}\cdot\vec{A}) = 0!$$

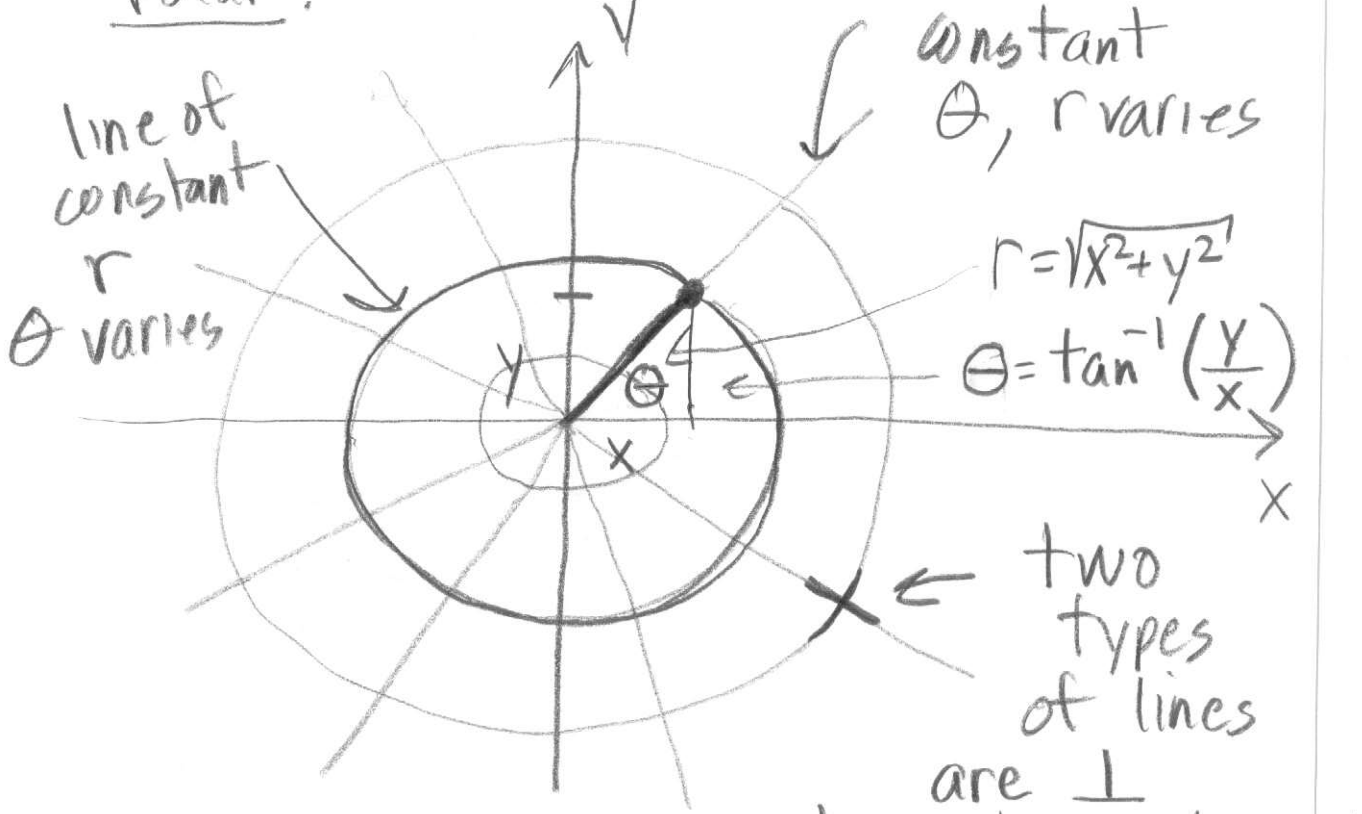
Polar Coordinates

when ... some problems have aspects that depend on distance from a central point

→ example... gravity



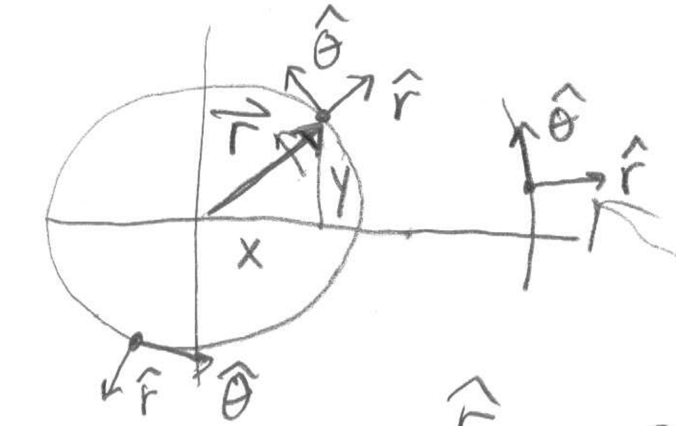
Polar:



$\hat{r}$ and $\hat{\theta}$ unit vectors

$\hat{r}$: points in direction of increasing r

$\hat{\theta}$: points in direction of increasing



$\hat{r}$ + $\hat{\theta}$ **Depend on r and θ**

$\hat{r}$ ... easier

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x}{r}\hat{\imath} + \frac{y}{r}\hat{\jmath}$$

$$\boxed{\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}}$$

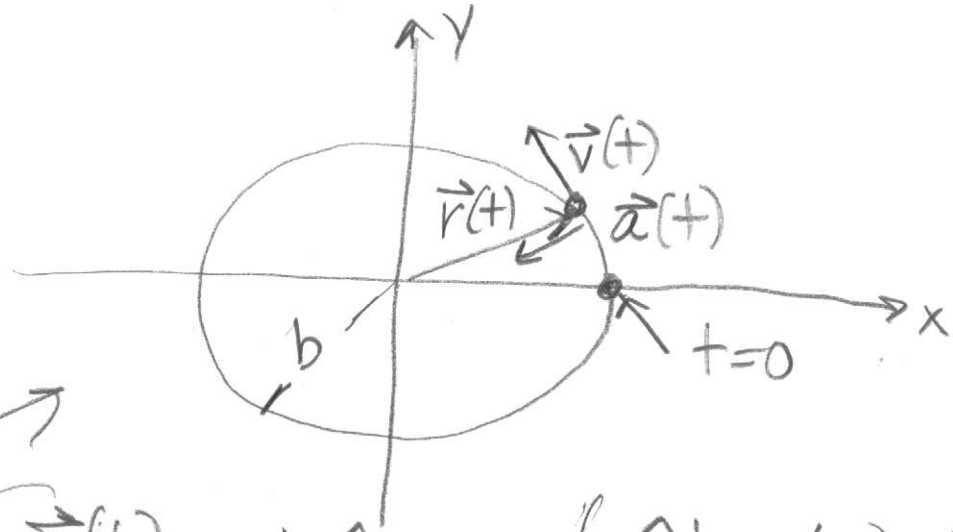
$\hat{\theta}$ ... $\perp$ to $\hat{r}$

$$\hat{\theta} = \pm\sin\theta\,\hat{\imath} \mp \cos\theta\,\hat{\jmath}$$

+ on the $\hat{\jmath}$

$$\boxed{\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}}$$

Makes Circular Motion Simpler



$$\vec{r}(t) = b\,\hat{r} \qquad \left(\hat{\imath}\, b\cos(\omega t) + \hat{\jmath}\, b\sin(\omega t)\right)$$

$$\vec{v}(t) = \omega b\,\hat{\theta} \qquad \left(b\omega\left(-\hat{\imath}\sin(\omega t) + \hat{\jmath}\cos(\omega t)\right)\right)$$

visually

$$\vec{a}(t) = -\omega^2 b\,\hat{r}$$

interpretation : $\dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(b\hat{r})$

$$\vec{v}(t) = \omega b\hat{\theta} \quad = b\left(\frac{d\hat{r}}{dt} \neq 0\,!\right)$$

<u>For circular motion</u> $\dfrac{d\hat{r}}{dt} = \omega\hat{\theta}$

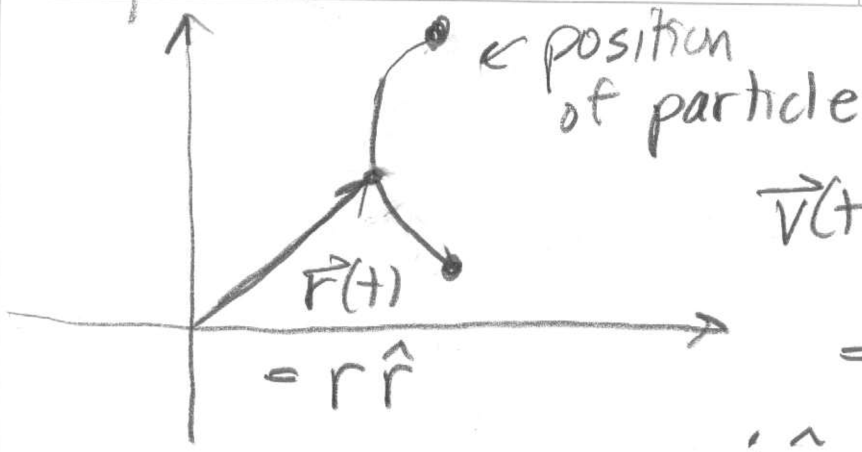
$\theta = \omega t$

in <u>general</u> $\dfrac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta} \qquad \dot{\theta} = \omega$

(pp. 31 → 33 of text)

$$\frac{d\vec{v}}{dt} = -\omega^2 b\hat{r} = \omega b\frac{d\hat{\theta}}{dt}$$

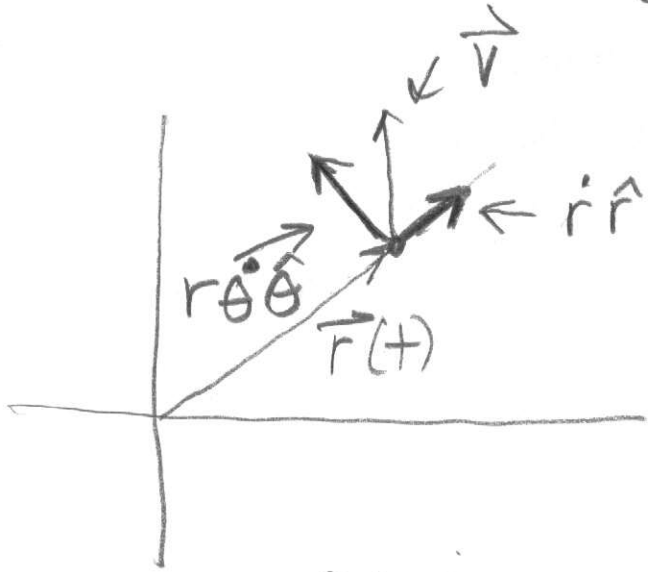
$$\frac{d\hat{\theta}}{dt} = -\omega\hat{r} \quad \Rightarrow \text{generally} \quad \frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$



$$\vec{v}(t) = \frac{d}{dt}(r\hat{r})$$

$$= \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\underline{= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$



circular

$\dot{r} = 0 \quad r = b$

$\dot{\theta} = \omega$

$\vec{v}(t) = b\omega\hat{\theta}$

again..

$$\vec{a}(t) = \ddot{r}\hat{r} + \dot{r}\underbrace{\frac{d\hat{r}}{dt}}_{\dot{\theta}\hat{\theta}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\underbrace{\frac{d\hat{\theta}}{dt}}_{-\dot{\theta}\hat{r}}$$

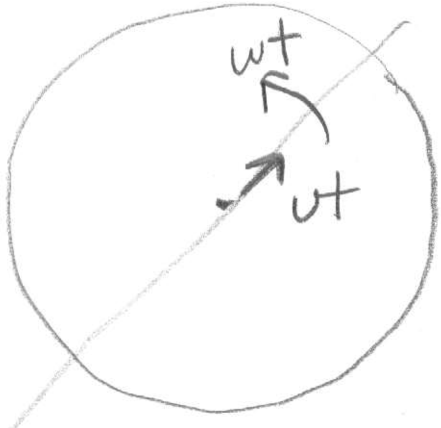
$$= (\ddot{r} - \dot{\theta}^2 r)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

circular: $r = b, \ \dot{r} = 0, \ \ddot{r} = 0$

$\theta = \omega t, \ \dot{\theta} = \omega \quad \ddot{\theta} = 0$

$$\vec{a} = -\omega^2 b\hat{r}$$

Bead on Spoke



$r(t) = vt$ (homework. $= A\theta = \frac{1}{2} A \alpha t^2$)

$\dot{r} = v$

$\ddot{r} = 0$

$\theta(t) = \omega t$ (homework, $= \frac{1}{2}\alpha t^2$)

$\dot{\theta} = \omega$

$$\vec{v}(t) = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$$

$$\vec{v}(t) = \underbrace{v}_{\text{constant}}\hat{r} + \underbrace{vt\omega}_{\text{grows}}\,\hat{\theta}$$

$$\vec{a}(t) = (0 - \omega^2 vt)\,\hat{r} + (2v\omega + r\cdot 0)\,\hat{\theta}$$

$$= \underbrace{-\omega^2 vt\,\hat{r}}_{\text{centripetal}} + \underbrace{2v\omega\,\hat{\theta}}_{\text{constant tangential}}$$