can continue.... $f(x) \simeq$

$$f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \frac{1}{\underbrace{3\cdot 2}_{3!}} f'''(a)(x-a)^3$$

$$+ \frac{1}{\underbrace{4\cdot 3\cdot 2}_{4!}} f''''(a)(x-a)^4 + \ldots$$

"Taylor Series"

$f(x) = \sin(x)$, $\quad\quad a = 0$

$f'(x) = \cos(x)$	$f(a) = \sin(0) = 0$
$f''(x) = -\sin(x)$	$f'(a) = \cos(0) = 1$
$f'''(x) = -\cos(x)$	$f''(a) = -\sin(0) = 0$
$f''''(x) = \sin(x)$	$f'''(a) = -\cos(0) = -1$
$f'''''(x) = \cos(x)$	$f''''(a) = \sin(0) = 0$
	$f'''''(a) = \cos(0) = 1$

$$\sin(x) \simeq 0 + \frac{1}{1!}\cdot 1\cdot x + \frac{1}{2!}\cdot 0\cdot x^2 + \frac{1}{3!}(-1)x^3$$

around $x=0$

$$+ \frac{1}{4!}\cdot 0\cdot x^4 + \frac{1}{5!}\cdot 1\cdot x^5$$

$$\sin(x) \simeq x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

around $x=0$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$$

around $x = 0$

Another famous series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$$

$$1 \stackrel{?}{=} (1-x)(1 + x + x^2 + x^3 + x^4 + \ldots)$$

$$= 1 + x - x + x^2 - x^2 - \ldots$$

$$1 = 1 \checkmark$$

as a Taylor Series

$$f(x) = \frac{1}{1-x} \qquad \text{choose } a = 0$$

$$f(0) = 1$$

$$f'(x) = \frac{-1}{(1-x)^2}(-1)$$

$$= \frac{1}{(1-x)^2} \qquad f'(0) = 1$$

$$f''(x) = -\frac{2}{(1-x)^3}(-1) = \frac{2}{(1-x)^3} \qquad f''(0) = 2$$

$$f'''(x) = \frac{-3 \cdot 2}{(1-x)^4} \cdot (-1) = \frac{3!}{(1-x)^4} \qquad f'''(0) = 3!$$

$$f''''(0) = 4!$$

$$\frac{1}{(1-x)} \simeq 1 + 1 \cdot x + \frac{2}{2!}x^2 + \frac{3!}{3!}x^3 + \frac{4!}{4!}x^4 + \ldots$$

$\underline{\tan(x):}$ around $a=0$

$$\tan(0) = 0$$

$$f'(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos(x)}{\cos(x)} - \frac{\sin(x)}{\cos^2(x)}(-\sin(x))$$

$$= 1 + \frac{\sin^2(x)}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$f'(x) = \frac{1}{\cos^2(x)} \qquad f'(0) = \frac{1}{1} = 1$$

$$f''(x) = \frac{-2}{\cos^3(x)}(-\sin(x)) = 2\frac{\sin(x)}{\cos^3(x)} \qquad f''(0) = 0$$

$$f'''(x) = 2\left(\frac{\cos(x)}{\cos^3(x)} - \frac{3\sin(x)}{\cos^4(x)}(-\sin(x))\right)$$

$$= 2\left(\frac{1}{\cos^2(x)} + \frac{3\sin^2(x)}{\cos^4(x)}\right)$$

$$f'''(0) = 2\cdot\left(\frac{1}{1} + \frac{3\cdot 0}{1}\right) = 2$$

$$\tan(x) \simeq 0 + x + 0 + \frac{2}{3!}x^3$$

$$\tan(x) \simeq x + \frac{1}{3}x^3 \left[+ \frac{2}{15}x^5 + \frac{17}{315}x^7\right]$$

Newton's Laws

First Law (N1): In absence of outside influences (Forces), the velocity of a body as viewed from an "inertial coordinate system" will stay constant.

Comments: "Inertial System"...

- no friction (outer space)
- let go of an object... if no forces on you or it, it will stay at rest.
- Then, study other bodies...
- $\frac{d\vec{V}_{body}}{dt} = 0$ as long as no messing with the body

Second Law (N2)

$$\vec{F}_{net} = m\vec{a}$$

identify all forces that act on a body. This <u>can</u> get subtle!

$$\vec{F}_{net} = \underbrace{\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_N}_{\text{tip to tail, components}}$$

$$= \sum_{i=1}^{N} \vec{F}_i$$

never forget!

m = mass (related to weight, but same on all planets...)
(measurable)

$\vec{a}$ = the one and only acceleration of the body, as observed from an inertial frame (look out when viewing from a non-inertial!)

$\vec{F} \Rightarrow$ abstraction!

$\vec{a} \Rightarrow$ measureable, most frequently $\vec{a} = 0$! (statics!)

$\vec{a} = 0$, does that mean no forces act on an object?

NOPE → no net force

Third Law (N3)

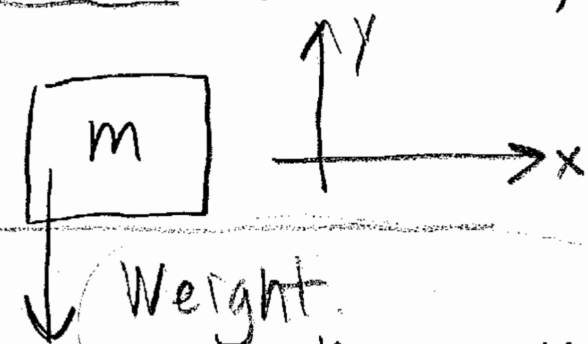
If body b pushes body a with force $\vec{F}_a$, then a pushes back on b with force $-\vec{F}_a$.

→ LOOK OUT.... 2 bodies, 2 forces, one force for each body.

Box on Table (at rest)

$\leftrightarrow$ no forces

$a_x = 0$



Weight on earth $= -m \cdot g \cdot \hat{j}$

$g =$ "acceleration of gravity"

$= 9.8 \frac{m}{s^2}$ ← not quite the same everywhere, but ≈ down toward earth's center everywhere.

moon → not g, etc.

Is that all? $\boxed{\vec{a} = 0}$, so net force 0 too!