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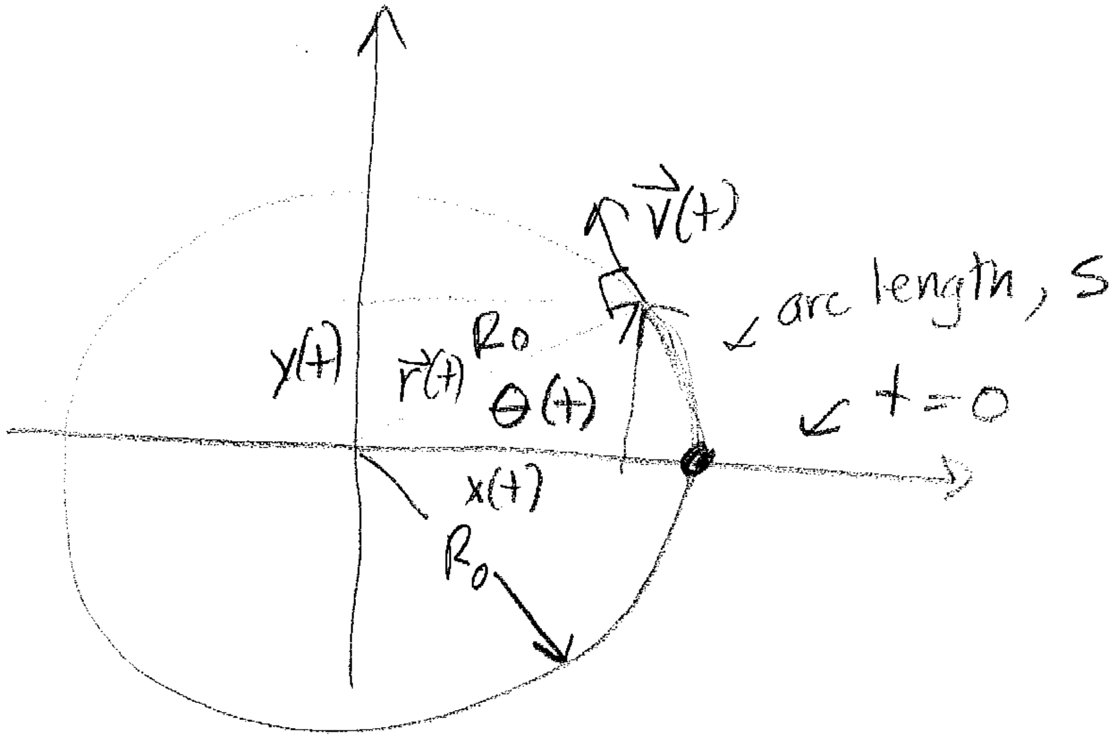
visualize...

Classic is <u>Circular Motion</u>




$\theta(t) = \omega t$ $\quad$ $\omega$ = "angular velocity"

radians

$= \frac{s(t)}{r}$

one complete revolution is

$s(T) = 2\pi r$

period

$s(T) = \frac{2\pi r}{r} = 2\pi$

$\omega T = 2\pi, \; T = \frac{2\pi}{\omega}$

$x(t) = R_0 \cos \omega t$

$y(t) = R_0 \sin \omega t$

$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$

$|\vec{r}(t)| = \sqrt{x(t)^2 + y(t)^2}$

$= \sqrt{R_0^2 \cos^2 \omega t + R_0^2 \sin^2 \omega t}$

$= \sqrt{R_0^2 \cdot 1} = R_0$

$\vec{v}(t) = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

$$\frac{dx}{dt} = \frac{d}{dt}\left(R_0 \cos(\omega t)\right) = R_0(-\sin(\omega t)) \times \omega$$

chain rule

$$= -\omega R_0 \sin(\omega t) = v_x$$

important.

$$\frac{dy}{dt} = \omega R_0 \cos(\omega t) = v_y$$

note: $\vec{v} \cdot \vec{r} = \frac{dx}{dt} \cdot x + \frac{dy}{dt} \cdot y$

$$= \left(-\omega R_0 \sin(\omega t)\right) \cdot \left(R_0 \cos(\omega t)\right) + \left(\omega R_0 \cos(\omega t)\right) \cdot \left(R_0 \sin(\omega t)\right) = 0!$$

makes sense.

$$|\vec{v}| = \text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(-\omega R_0 \sin(\omega t)\right)^2 + \left(\omega R_0 \cos(\omega t)\right)^2}$$

$$|\vec{v}| = \sqrt{(\omega R_0)^2 \cdot \underline{1}} = \omega R_0 \leftarrow \text{units?}$$

$\omega \to 1/s$

$R_0 \to m$

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$= -\omega^2 R_0 \cos(\omega t)\,\hat{i} - \omega^2 R_0 \sin(\omega t)\,\hat{j}$$

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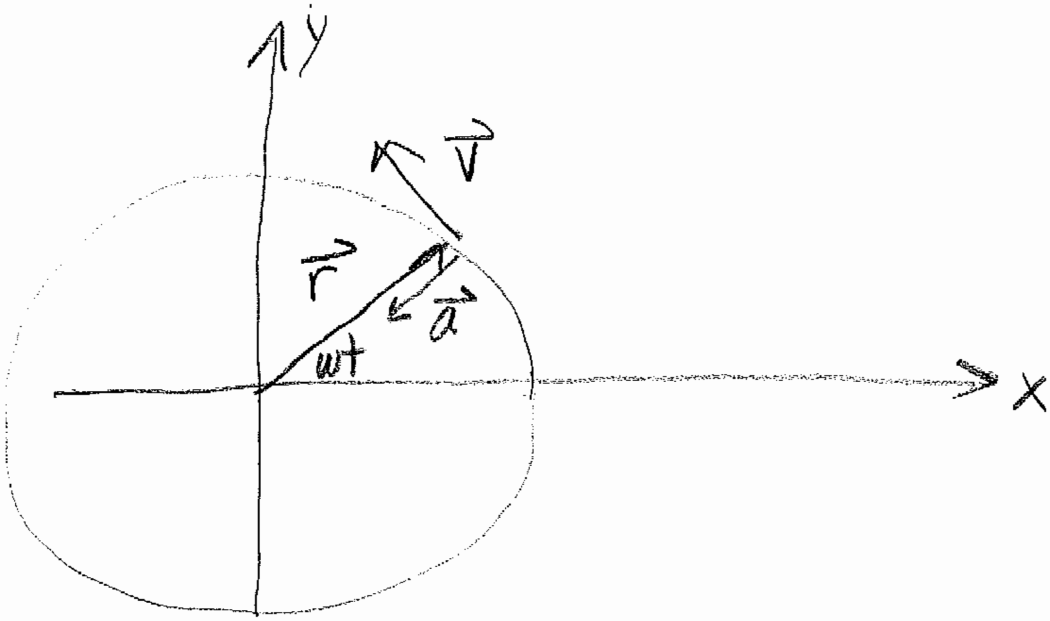
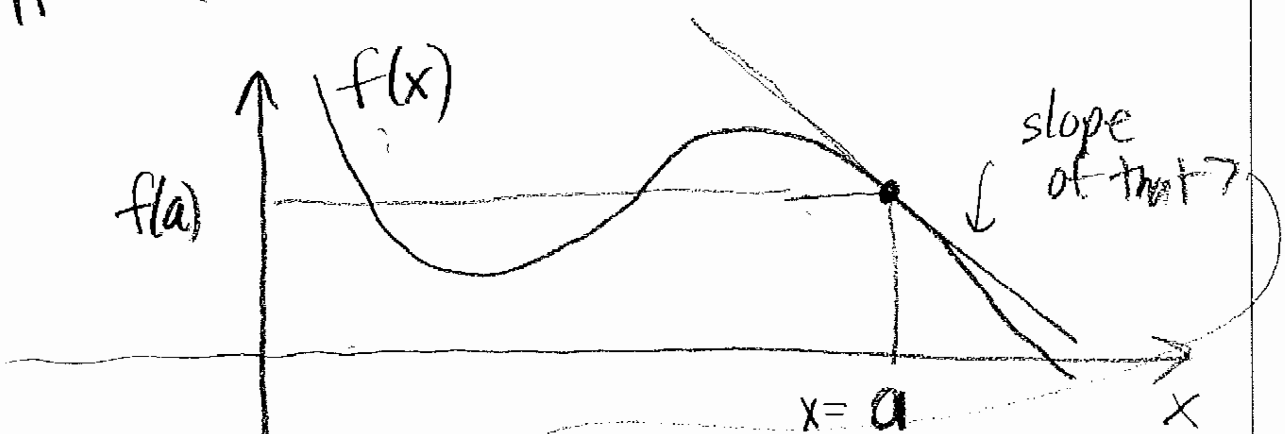
$$\vec{a} = -\omega^2 \Big[ \underbrace{R_0 \cos(\omega t)\,\hat{\imath} + R_0 \sin(\omega t)\,\hat{\jmath}}_{\vec{r}(t)\,!} \Big]$$

$$\vec{a} = -\omega^2 \vec{r}(t) \qquad [\omega^2] = \frac{1}{s^2} \qquad [\vec{r}] = m$$




## Taylor Series

Use calculus to find a way to approximate a function.




$$= f'(a) = \left.\frac{df}{dx}\right|_a$$

(here, it is negative)

Get an <u>equation</u> for the line...

$$x = a, \quad \ell(x) = f(a)$$

$$\text{slope} = f'(a)$$

$$\ell(x) = f(a) + f'(a)\underbrace{(x-a)}_{0 \text{ when } x=a}$$

<u>note</u>: $\ell'(a) = 0 + f'(a)$

$$\ell'(a) = f'(a)$$

Line has same <u>value</u> and same <u>slope</u> as function... can we take this even further, to... second derivative?

$$\text{curve}(x) = f(a) + f'(a)(x-a) + \gamma(x-a)^2$$

$c(x)$

adjust to get same second derivative as $f \to f''(a)$

$$c'(x) = f'(a) + 2\gamma(x-a)$$

$$c''(x) = 2\gamma = f''(a)$$

$$\gamma = \frac{1}{2} f''(a)$$

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