## Vectors

**Topics**

- Definition
- Manners of Description
  - → magnitude and direction
  - → components, unit & base vectors
- "position" of a vector is elusive
  - → translation usually conceptually OK
  - → occasionally vector is "nailed down"
- Addition
  - → geometric → "head to tail"
  - → components.
- Multiplication by a scalar
  - → scalar can be negative, which reverses direction of the vector
- Products of 2 Vectors
  - → Dot Product or "Scalar" Product
    - ☆ Geometric
    - ☆ Algebraic
    - ☆ "How much of one vector is along direction of another?
  - → Cross Product or "Vector" Product, "3-d"
    - ☆ Geometric
    - ☆ Algebraic
    - ☆ "How much of one vector is ⊥ to direction of another?
  - → Tensor Product — brief mention

- Velocity + Acceleration Vectors
  - → $[x(t), y(t), z(t)]$ parameterization
  - → circular motion



**Vector** ... a quantity with both magnitude and direction

"a directed line segment"

Not just a real number, because of direction

notation: books – **A** (bold-face A)

writing – $\vec{A}$ ← arrow on top indicates direction.

magnitude: $|\vec{A}|$ = positive (or 0) real #

Vectors can live in any number of dimensions.

Spatial dimensions are special because we have intuitive ideas about rotations + translations.. spatial vectors transform under rotations.

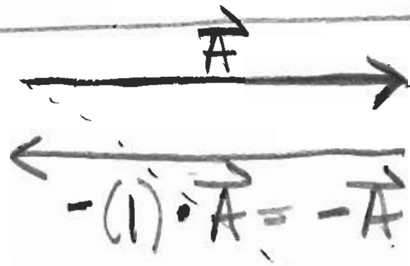
$\boxed{\text{One Dimension}}$ (horizontal)


magnitude is

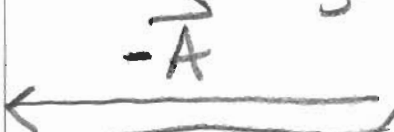
direction → left to right ≈ adequate! don't need to be fancy!

only one other direction. right to left.

**Multiplying by −1 reverses direction of a vector**



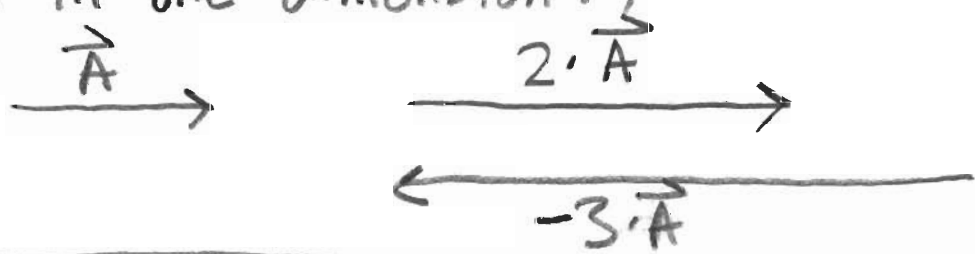

"Parallel Translation" →
(maintain <u>direction</u> + length, but <u>move</u>)


Multiplying a vector $\vec{A}$ by a real number (b) (a.k.a. a <u>scalar</u>) does the following:

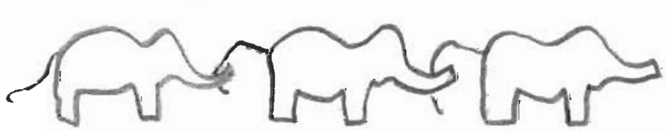
1. changes the magnitude to $|b|\cdot|\vec{A}|$
2. if $b < 0$, <u>reverses</u> direction of $\vec{A}$

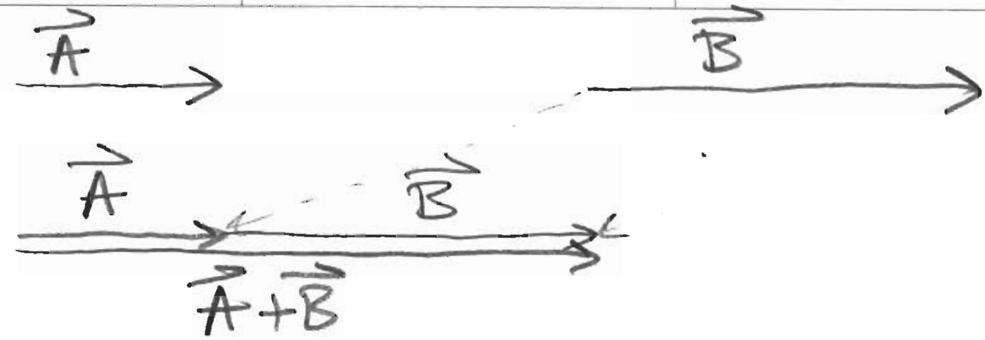
(still in one dimension!)




Adding Vectors: $\vec{A}$ and $\vec{B}$, to get $\vec{A}+\vec{B}$, do parallel translation of $\vec{B}$ so its <u>tail</u> is on top of $\vec{A}$'s <u>tip</u>

"Elephant Walk"






In <u>one dimension</u>, this is kind of trivial...

$\vec{A} \rightarrow$ like real number $A$

$\vec{B} \rightarrow$ like real number $B$

$\vec{A}+\vec{B} \rightarrow$ like $A+B$

note: $\vec{A} + (-\vec{A}) =$


$\vec{A}+(-\vec{A}) = \vec{0}$ or $0$ as expected.

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<u>Unit vector</u> $\rightarrow$ can be constructed from <u>any</u> non-zero vector

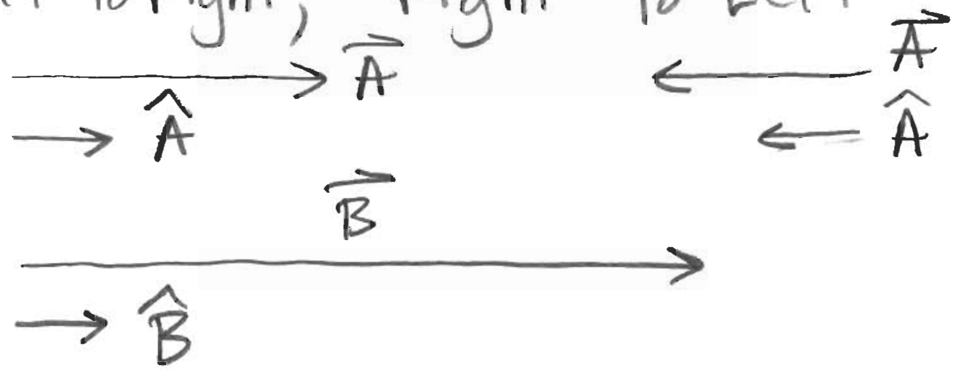
to do it: $\hat{A} \equiv \dfrac{\vec{A}}{|\vec{A}|}$ = unit vector in $\vec{A}$ direction

$\rightarrow$ has no dimensions!

$|\text{unit vector}| = 1 = \dfrac{|\vec{A}|}{|\vec{A}|} = 1$

in one dimension, only has 2 possible directions!

Left to Right, Right to Left

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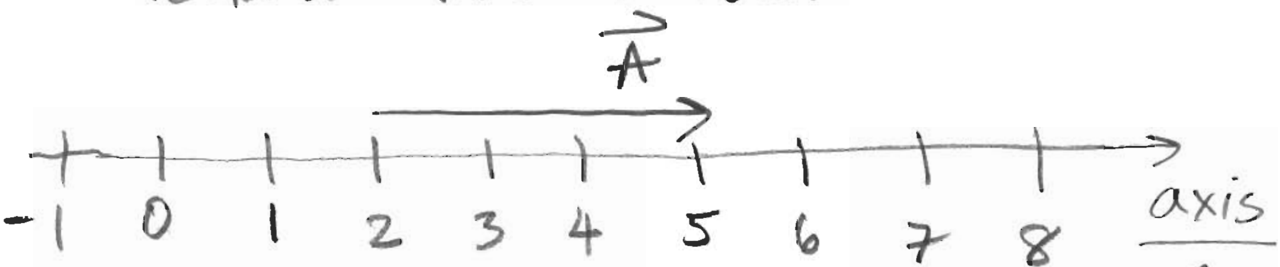
Base Vectors. (1 dimension still)

→ hold up a <u>number line</u> to your vector.. like a ruler




#'s → kind of arbitrary

usually <u>x-axis</u> in 1-d

- when $\vec{A}$ is a displacement, meters etc
- " velocity, m/s etc
- " an acceleration $m/s^2$ etc

useful to define a <u>unit vector</u> in the direction of increasing x ... call that

$\hat{\imath}$ ← when handwritten; **i** is boldface in text

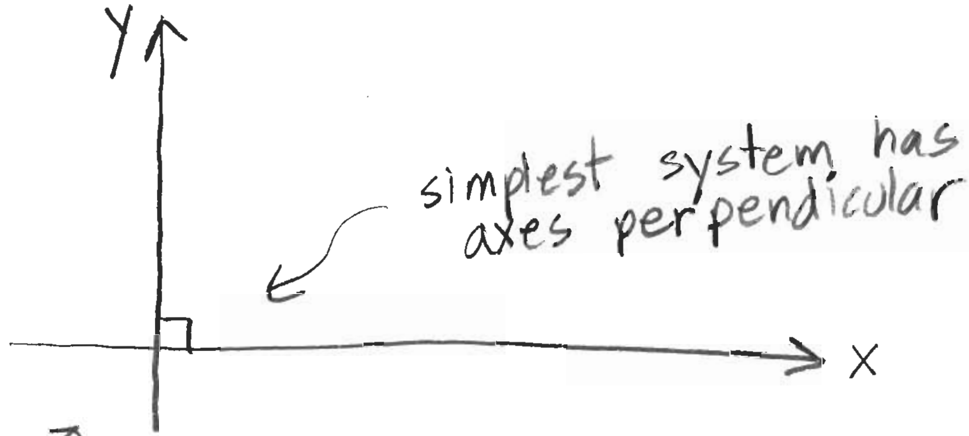
then... $\vec{A} = A_x \hat{\imath}$ → defines direction.

$A_x$ → magnitude, + or − gives direction

$A_x$ called "x-component"

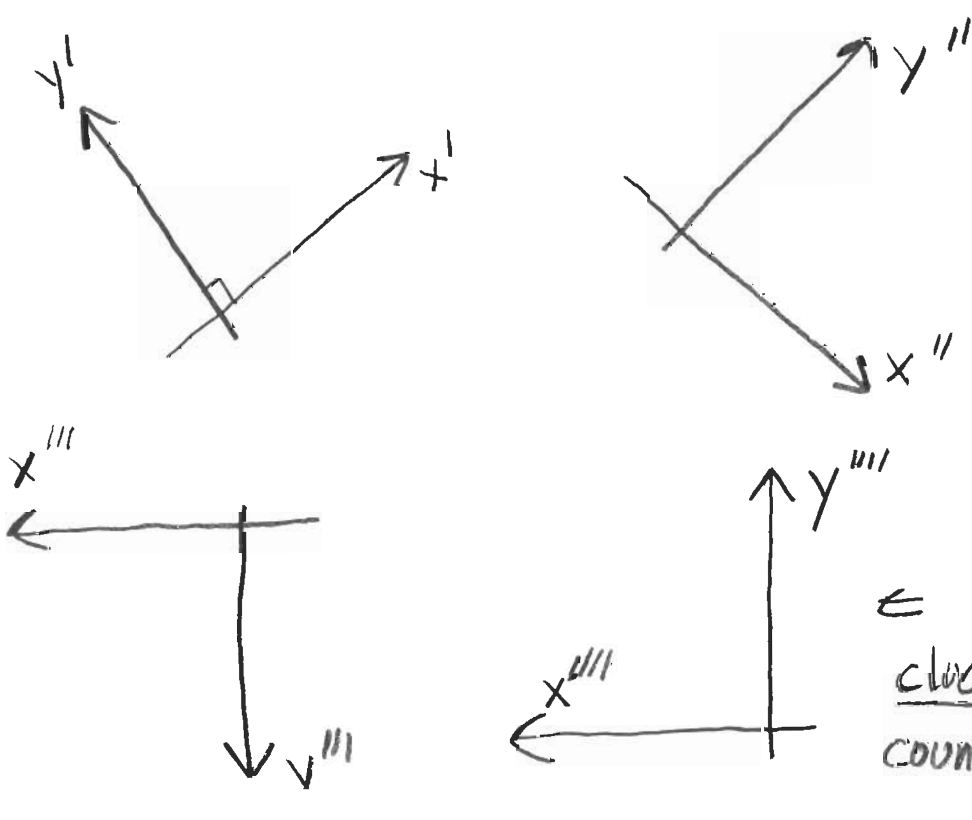
## Two Dimensions

Now imagine, say, a plane. Need two axes to describe points in a plane... and 2-d vectors




origin? For vectors, not always pertinent, because "parallel translation"

$\boxed{\text{Rotations}}$ very important in 2-d..

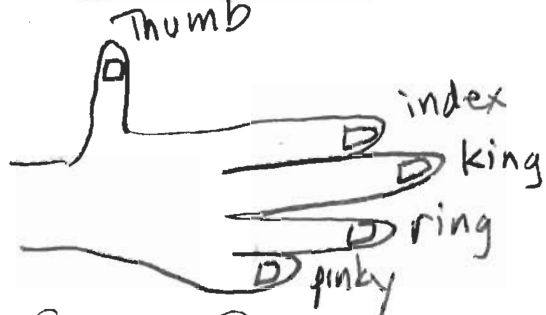



This one is different... $x''''$ to $y''''$ is clockwise, not counterclockwise

We use ... "right handed" coordinate systems
(arbitrary but universal convention)

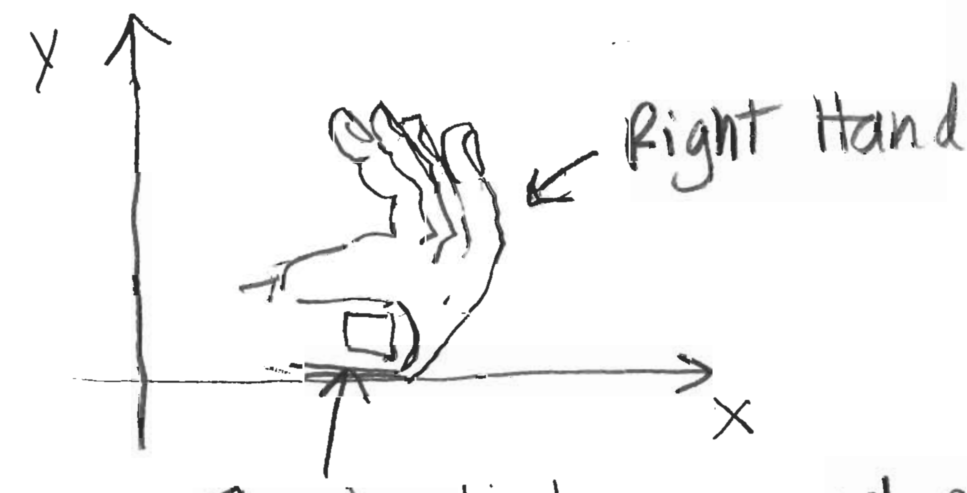
→ x rotated to y goes $\boxed{\text{counterclockwise}}$

→ take right hand, make it flat, thumb up:




Right Hand

→ align four fingers in x-direction, so that when you bend them, they bend toward y-axis




Thumb sticks up, out of page... that is a right-handed coordinate system... had you used left hand, then your thumb would have poked down into page.

(If you look at a left handed coordinate system from <u>under the page</u>, it looks right handed! Hmm... handedness is related to 3 dimensions!)