Homework 1

Correction to solutions

Feraz Azhar

Question 7

The "displacement from O" for interval TU should be + (only @ the point U is the displacement O)

[Though I accepted both answers]

Question 9

The result should be:

$$\frac{|\vec{a}_1|}{|\vec{a}_2|} = \frac{m_2}{m_1}$$

[no minus sign]

PHYSICS 21

PROBLEM SET 1

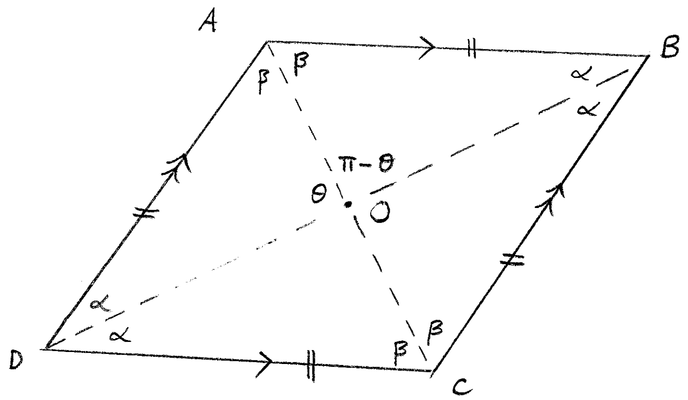
SOLUTIONS

Feraz Azhar

QUESTION 1

METHOD 1

By "equilateral parallelogram" I assume they mean a rhombus;



Now Δ's ABD and BCD are isosceles and $AB \parallel DC$ so all angles marked "α" are the same

Δ's ACD and ABC are also isosceles and $AD \parallel BC$ so all angles marked "β" are the same

Let $\angle AOD = \theta$

Then $\angle AOB = \pi - \theta$

So from ΔAOD,

$$\alpha + \beta + \theta = \pi \qquad (1)$$

and from ΔAOB,

$$\alpha + \beta + \pi - \theta = \pi \qquad (2)$$

Solving (1) & (2) simultaneously gives the required result:

$$\theta = \frac{\pi}{2}$$

METHOD 2 (The Easier way)

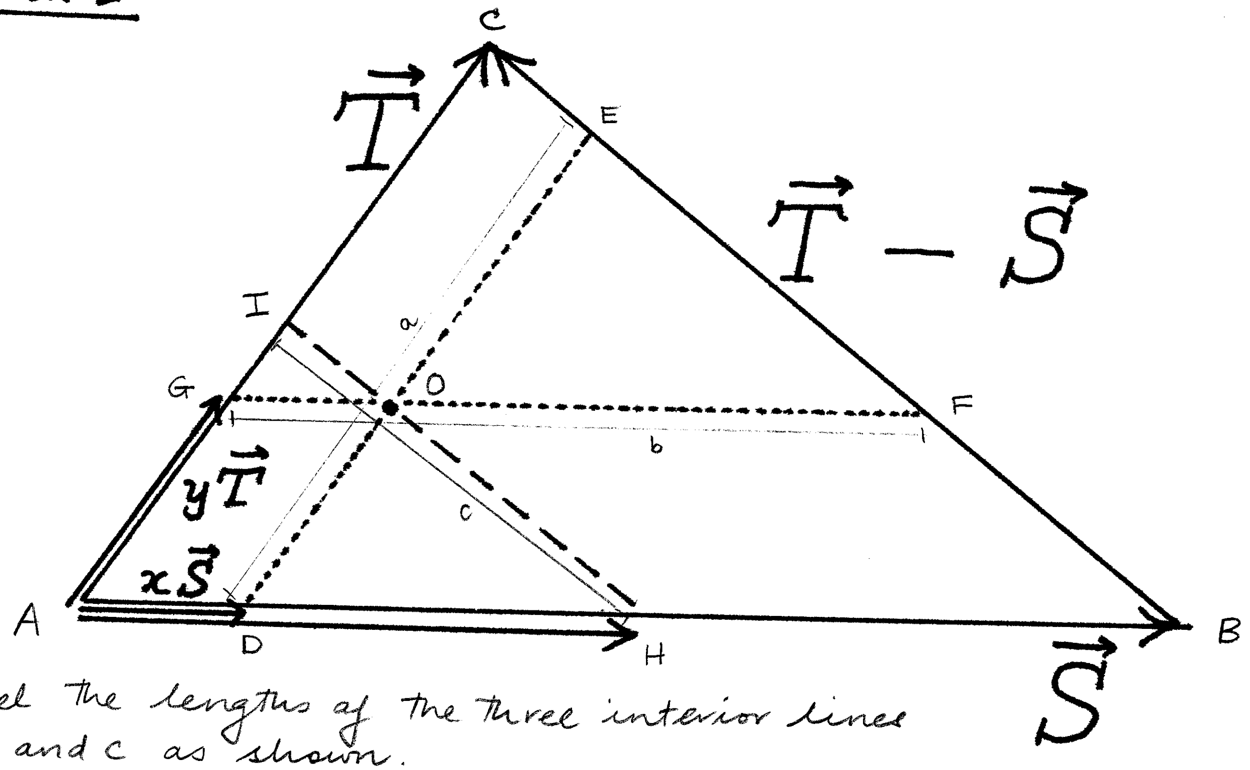
Note that diagonal $\vec{DB} = \vec{DC} + \vec{CB}$ and diagonal $\vec{CA} = \vec{CB} - \vec{DC}$

So $\vec{DB} \cdot \vec{CA} = \vec{DC} \cdot \vec{CB} - |\vec{DC}|^2 + |\vec{CB}|^2 - \vec{DC} \cdot \vec{CB}$

$= 0$ since $|\vec{DC}| = |\vec{CB}|$

So the angle between the diagonals must be $90°$ as required.

QUESTION 2



Label the lengths of the three interior lines a, b and c as shown.

Now ΔEDB is similar to ΔABC so the lengths of their sides are in the same ratio. In particular;

$$\frac{a}{|\vec{T}|} = \frac{|\vec{S}| - x|\vec{S}|}{|\vec{S}|} = 1 - x \equiv r_1$$

Again, ΔCGE is similar to ΔABC so;

$$\frac{b}{|\vec{S}|} = \frac{|\vec{T}| - y|\vec{T}|}{|\vec{T}|} = 1 - y \equiv r_2$$

We need the hint to find the third ratio r_3;

One vector sum that goes from A to O is $x\vec{S} + y\vec{T}$; another goes from A to H then H to O; the latter we can represent as $u\vec{S} + v(\vec{T} - \vec{S})$ noting that $\vec{HO} \parallel \vec{T} - \vec{S}$. Since both of these vectors represent the same vector;

$$x\vec{S} + y\vec{T} = u\vec{S} + v(\vec{T} - \vec{S})$$

we can solve this to show $(u, v) = (x + y, y)$ i.e., $\vec{AH} = (x+y)\vec{S}$

* (see later) Thus since ΔAHI is similar to ΔABC,

$$\frac{c}{|\vec{T} - \vec{S}|} = \frac{(x+y)|\vec{S}|}{|\vec{S}|} = x + y \equiv r_3$$

Hence $\displaystyle \underline{\sum_{i=1}^{3} r_i = 1 - x + 1 - y + x + y = 2}$

Another Method (suggested in the discussion section) is to note that ΔDOH is similar to ΔACB. This implies that

$$\frac{|\overrightarrow{DH}|}{|\overrightarrow{AB}|} = \frac{|\overrightarrow{AG}|}{|\overrightarrow{AC}|}$$

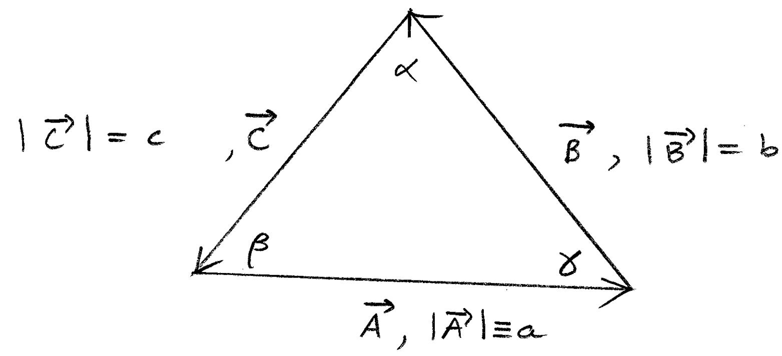
i.e., $$|\overrightarrow{DH}| = \frac{y\,|\vec{T}|}{|\vec{T}|} \cdot |\vec{s}| = y\,|\vec{s}|$$

$\Rightarrow$ $\overrightarrow{AH} = (x+y)\,|\vec{s}|$. Then we repeat the same argument following * on page 2 to obtain the required result.

I think this way is rather more elegant than the one suggested in the Hint.

QUESTION 3

Consider the area of a triangle formed by $\vec{A}$, $\vec{B}$, $\vec{C}$ where $\vec{A}+\vec{B}+\vec{C}=0$;



Label the angles α, β, γ as shown. Then the area of the triangle can be written in three separate ways;

$$A = \frac{1}{2}|\vec{A}\times\vec{C}| = \frac{1}{2}|\vec{B}\times\vec{A}| = \frac{1}{2}|\vec{C}\times\vec{B}|$$

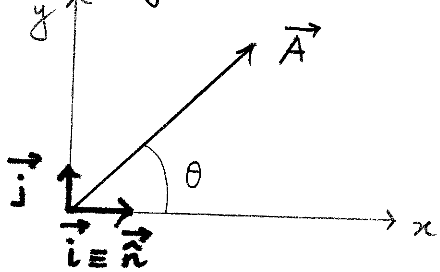
$$\Longrightarrow \quad \frac{1}{2}ac\sin\beta = \frac{1}{2}ab\sin\gamma = \frac{1}{2}bc\sin\alpha$$

Then dividing by $\frac{1}{2}abc$, you get the required result;

$$\frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{\sin\alpha}{a}$$

QUESTION 4

Without loss of generality we can set up coordinate axes such that $\hat{n}$ lies along the x-axis



where $\vec{i}$ & $\vec{j}$ are unit vectors pointing along the x and y axes respectively, and $\hat{n} = \vec{i}$

Then $\vec{A} = |\vec{A}|\cos\theta \, \vec{i} + |\vec{A}|\sin\theta \, \vec{j}$

But

$$\vec{A} \cdot \vec{\hat{n}} = |\vec{A}|\,|\vec{\hat{n}}|\cos\theta = |\vec{A}|\cos\theta$$

and

$$(\vec{\hat{n}} \times \vec{A}) \times \vec{\hat{n}} = \left[\vec{i} \times \left(|\vec{A}|\cos\theta\,\vec{i} + |\vec{A}|\sin\theta\,\vec{j}\right)\right] \times \vec{i}$$

$$= |\vec{A}|\sin\theta\,(\vec{i} \times \vec{j}) \times \vec{i}$$

$$= |\vec{A}|\sin\theta\,\vec{j}$$

Thus;

$$\underline{\underline{\vec{A} = (\vec{A} \cdot \vec{\hat{n}})\,\vec{\hat{n}} + (\vec{\hat{n}} \times \vec{A}) \times \vec{\hat{n}}}}$$

QUESTION 5

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$\Rightarrow \quad \underline{\underline{\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{-6-3-1}{\sqrt{3^2+1^2+1^2}\,\sqrt{2^2+3^2+1^2}} = \frac{-10}{\sqrt{11}\,\sqrt{14}}}}$$

QUESTION 6

(a) $$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -4 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -8\vec{i} + 5\vec{j} + 6\vec{k}$$

So $$\underline{\underline{\vec{a} \cdot (\vec{b} \times \vec{c}) = 3(-8) + 3(5) + (-2)(6) = -21}}$$

(b) $$\vec{b} + \vec{c} = \vec{i} - 2\vec{j} + 3\vec{k}$$

So $$\underline{\underline{\vec{a} \cdot (\vec{b} + \vec{c}) = 3(1) + 3(-2) + (-2)(3) = -9}}$$

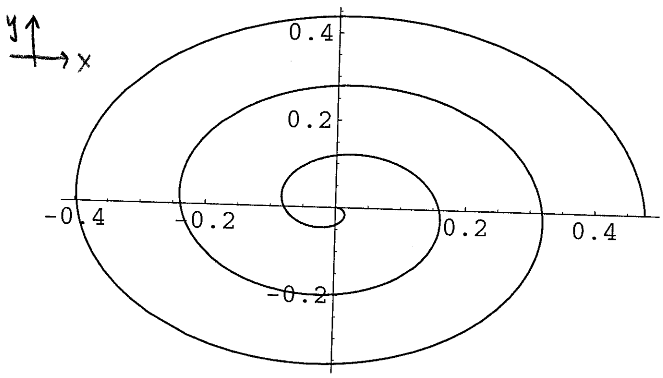
(c) $$\underline{\underline{\vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix} = 5\vec{i} - 11\vec{j} - 9\vec{k}}}$$

QUESTION 7

	Displacement from O	Velocity v_x	Acceleration a_x
OP	+	+	O
PQ	+	+	+
QR	+	+	−
RS	+	−	−
ST	+	−	O
TU	O	−	+

QUESTION 8

(a) The trajectory of the particle is;

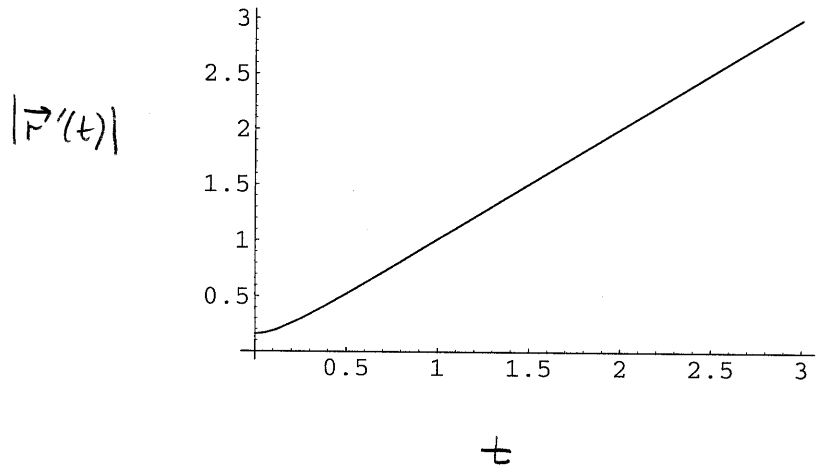


(b)

$$\vec{r}\,'(t) = \left(\frac{1}{2\pi} \cos[2\pi t] - t \sin[2\pi t], \frac{1}{2\pi} \sin[2\pi t] + t\cos[2\pi t] \right)$$

So the speed is;

$$|\vec{r}\,'(t)| = \frac{\sqrt{1+4\pi^2 t^2}}{2\pi}$$

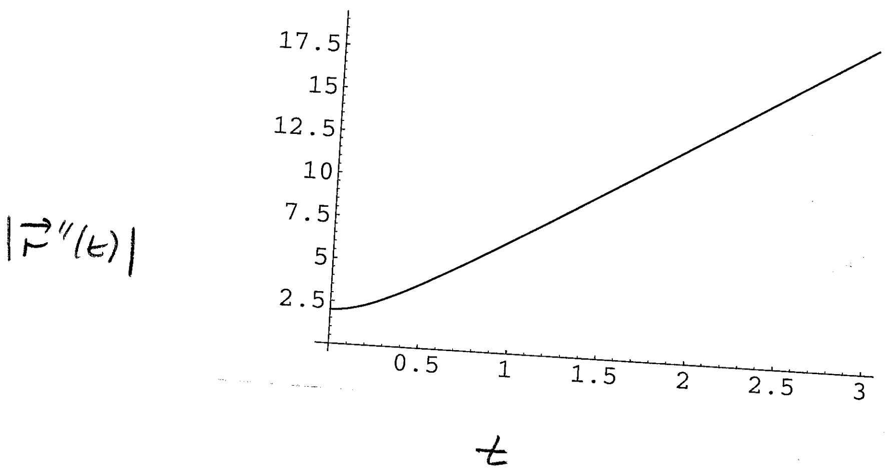


(C)

$$\vec{r}\,''(t) = 2\left(-\sin[2\pi t]-\pi t\cos[2\pi t],\ \cos[2\pi t]-\pi t\sin[2\pi t]\right)$$

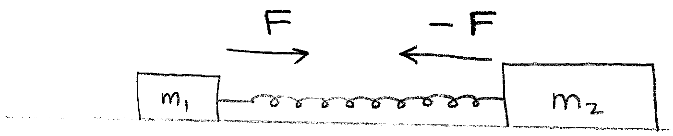
And the magnitude of the acceleration is given by

$$|\vec{r}\,''(t)| = 2\sqrt{1+\pi^2 t^2}$$



The last three figures were plot using Mathematica

QUESTION 9



Both masses are subject to the same force but in opposite directions, so;

$$\frac{a_1}{a_2} = \frac{\frac{F}{m_1}}{\frac{-F}{m_2}} = -\frac{m_2}{m_1}$$

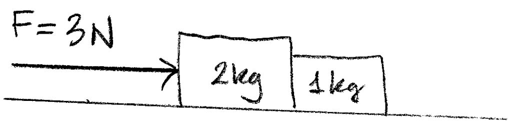
QUESTION 10

(a) The spring scale measures the tension in the rope. The system is at rest (and remains so) thus the scale must read 10 lb

(b) 10 lb, since the mass isn't moving

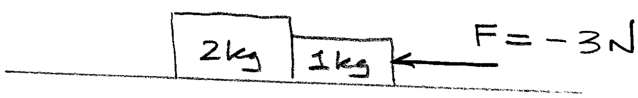
QUESTION 11

(a)



The acceleration of the blocks is $1\,m/s^2$ $\left(= \frac{3N}{3kg}\right)$ and so the 2kg block must be pushing on the 1kg block with a force of 1N and vice versa

(b)



Again the blocks' acceleration is $1\,m/s^2$ (to left) and so the 1 kg block must be exerting a force with magnitude 2N & vice versa.