# Practice Final Solutions 

1. (a) $\frac{1}{2}\cancel{m_1}v_1^2 = \cancel{m_1}gR \Rightarrow v_1 = \sqrt{2gR}$

(b) time it takes the tiny mass to fall distance $h/2$:

$$\frac{1}{2}gt^2 = \frac{1}{2}h \Rightarrow t = \sqrt{\frac{h}{g}}$$

then, final velocity $u_1$ of $m_1$ must satisfy, for small mass to hit it:

$$0 < u_1 t < h/2$$

$$0 < u_1 < \frac{1}{2}\frac{h}{\sqrt{\frac{h}{g}}} = \frac{1}{2}\sqrt{gh}$$

now find $u_1$: Center of mass motion is:

$$V = \frac{m_1 v_1 + m_2 \cdot 0}{m_1 + m_2} = \frac{m_1}{m_1+m_2}v_1$$

in center of mass frame:

$$v_1' = v_1 - V = \left(1 - \frac{m_1}{m_1+m_2}\right)v_1 = \frac{m_2}{m_1+m_2}v_1$$

after collision:

$$u_1' = -v_1' = -\frac{m_2}{m_1+m_2}v_1$$

$$u_1 = u_1' + V = \frac{m_1 - m_2}{m_1+m_2}v_1$$

so, need $0 < \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 < \frac{1}{2}\sqrt{gh}$

The lower limit means $m_1 > m_2$

upper: $\dfrac{m_1 - m_2}{m_1 + m_2} < \dfrac{\frac{1}{2}\sqrt{gh}}{\sqrt{2gR}} = \sqrt{\dfrac{h}{8R}}$

$$\rho \equiv \sqrt{\frac{h}{8R}}$$

$$m_1 - m_2 < \rho \cdot (m_1 + m_2)$$

$$m_1 (1-\rho) < m_2 (1+\rho)$$

$$\boxed{m_2 < m_1 < \left(\frac{1+\rho}{1-\rho}\right) m_2 = \frac{1+\sqrt{\frac{h}{8R}}}{1-\sqrt{\frac{h}{8R}}}\, m_2}$$

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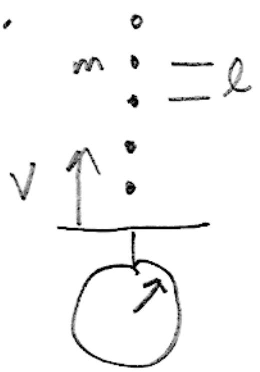
2.




In time $\Delta t$,

$\dfrac{v\Delta t}{l}$ particles hit.

$$\Delta p = \left(\frac{v\Delta t}{l}\right) \times (mv)$$

$$F = \frac{\Delta p}{\Delta t} \rightarrow \frac{dp}{dt} = \frac{mv^2}{l}$$

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3. (a) $\left.\dfrac{dU}{dx}\right|_{x_0} = -\dfrac{2A}{x_0^3} + \dfrac{B}{x_0^2} = 0$

$$\boxed{\frac{2A}{B} = x_0 = \frac{2 \cdot 5}{10} = 1 \text{ meter}}$$

$$\left.\frac{d^2U}{dx^2}\right|_{x_0} = \frac{6A}{x_0^4} - \frac{2B}{x_0^3} = \frac{6A}{16\frac{A^4}{B^4}} - \frac{2B}{8\frac{A^3}{B^3}} = \underbrace{\frac{1}{8} \cdot \frac{B^4}{A^3} > 0}_{\text{so it's stable.}}$$

(b) $k = \frac{1}{8}\frac{B^4}{A^3} = \frac{1}{2^3}\frac{2^4\cdot 5^4}{2^3\cdot 5^3} = \frac{5}{4}$

$$\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5^2}{2^2}} = \frac{5}{2}\ \text{rad/s}}$$

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4. For beam: $I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \left(\frac{1+3}{12}\right)ML^2$

$= \frac{1}{3}ML^2$

$L = I_0\omega_0 + I\cdot 0 = (I_0 + I)\omega_f$

$$\boxed{\omega_f = \frac{I_0}{I_0+I}\omega_0 = \frac{I_0}{I_0+\frac{1}{3}ML^2}\omega_0}$$

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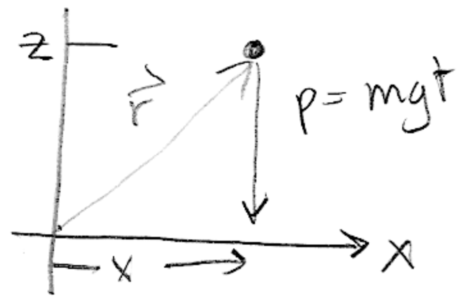
5. $v = gt$

$p = mv = mgt$




direction of $\vec{r}\times\vec{p}$: into the page, aka, y-direction.

$$\boxed{\begin{array}{l}\vec{L}\ \text{magnitude: } r_\perp \cdot p = xmgt \\ \vec{L}\ \text{direction: into page, y direction.}\end{array}}$$

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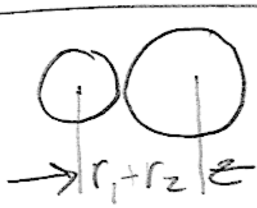
6.




after explosion:

$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1m_2}{r_1+r_2} = 0$

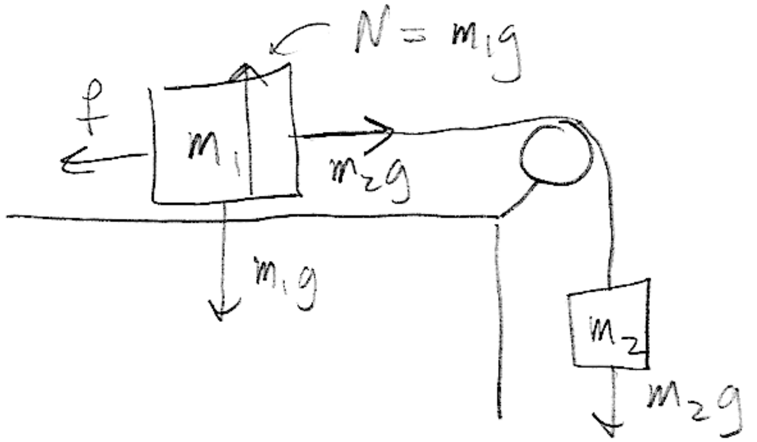
escape.

momentum: $m_1 v_1 = -m_2 v_2$

$\frac{1}{2}\cancel{m_1}\left(1+\frac{m_1}{m_2}\right)v_1^2 = \frac{G\cancel{m_1}m_2}{r_1+r_2}$

$$\boxed{v_1 = \sqrt{\frac{2Gm_2}{(r_1+r_2)\left(1+\frac{m_1}{m_2}\right)}}}$$

7.



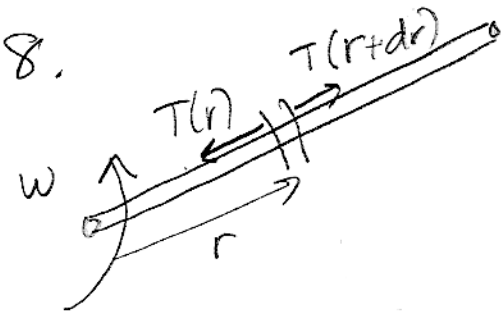

motion when

$$m_2 \cancel{g} = \mu \cdot m_1 \cancel{g}$$

$$m_2 = \mu \cdot m_1 = \frac{1}{2} \cdot 10$$

$$\boxed{m_2 = 5 kg}$$

8.




$$T(r+dr) - T(r) = -(\lambda \cdot dr) w^2 r$$

$$\frac{dT}{dr} = -\lambda w^2 r dr$$

$$T(r) = -\frac{1}{2}\frac{M}{R} w^2 r^2 + T_0$$

$R = 10 m$
$M = 1 kg$

$$\lambda = \frac{M}{R} = \frac{1}{10}\frac{kg}{m}$$

$w = 2\pi/s$

when $r = R$, $T(R) = 0$

so $T_0 = \frac{1}{2}\frac{M}{R} w^2 R^2$

and $T(0) = \frac{1}{2} M w^2 R$

$$= \frac{1}{2} \cdot 1 \cdot (2\pi)^2 \cdot 10$$

$$\boxed{T(0) = 20\pi^2 \text{ Newtons}}$$

9. (a) $V_{z0} = 20 \cdot \frac{1}{\sqrt{2}}$, $\cancel{m} g h = \frac{1}{2} \cancel{m} V_{z0}^2$

$$\boxed{h = \frac{V_{z0}^2}{2g} = \frac{400 \cdot \frac{1}{2}}{2 \cdot 10} = \frac{100}{10} = 10 \text{ meters}}$$

(b) 1 way up or down $\frac{1}{2} g t^2 = h \Rightarrow t = \sqrt{\frac{2h}{g}}$

$$= \sqrt{\frac{2 \cdot 10}{10}} = \sqrt{2}\, s$$

so, round trip is: $\boxed{2 \cdot \sqrt{2}\ s}$
up, back down

(c) $\boxed{v_{0x} \cdot t = \frac{20}{\sqrt{2}} \cdot 2 \cdot \sqrt{2} = 40 \text{ meters}}$

10. $$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{M}\left(\frac{1}{f} + \frac{1}{1-f}\right) = \frac{1}{Mf(1-f)}$$

$$\omega' = \sqrt{\frac{k}{Mf(1-f)}} = \frac{\omega}{\sqrt{f(1-f)}}$$

$$\frac{d\omega'}{df} = -\frac{1}{2} \frac{\omega}{\sqrt{f(1-f)}} \cdot 1-2f = 0$$

$\boxed{f = \frac{1}{2} \quad \text{minimum}}$