Physics 21 Practice Final - 3 hours

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Friday, March 18

Write your answers in a blue book. Calculators and two pages of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if numbers provided) after a symbolic answer is available. You can take the acceleration of gravity near the earth as $g = 10\,\mathrm{m/s^2}$, to simplify calculations. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

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1. You release a mass m_1 from rest at the top of a ramp shaped like a quarter-circle of radius R on to a horizontal table, where m_1 eventually has an elastic collision with mass m_2, as shown in Figure 1. The mass m_1 is a cube of length $h/2$ on its sides, while m_2 is a cube of length h on each side. The two masses are made of materials with very different densities. A tiny mass of negligible size is sitting on the top of mass m_2, just at the left edge, as shown in the figure. Everything is frictionless.

   (a) What is the velocity, v_1, of the mass m_1 just before it collides with m_2, in terms of the masses, R, and g?

   (b) For what range of masses m_1 will the tiny mass fall on to m_1 after the collision with m_2?

2. A weight scale is moving through interstellar space with speed v, where it encounters a line of dust particles, each of mass m and separated by distance ℓ. The line is exactly perpendicular to the pan on the weight scale, and the line hits the pan dead center. When each dust particle hits the weight scale pan, the particle sticks to the pan. What is the reading on the weight scale? Neglect any 'jerkiness' that arises because the dust particles hit the pan one at a time.

3. A particle of mass $m = 1/5\,\mathrm{kg}$ moves in one dimension from the origin to ∞ and is subject to the potential energy:

$$U(x) = \frac{A}{x^2} - \frac{B}{x} \tag{1}$$

   where $A = 5\,\text{Joule-meter}^2$, $B = 10\,\text{Joule-meter}$.

   (a) Is there a stable equilibrium point for the particle, and if so, at what value of x does it occur (symbolically and numerically)?

   (b) Determine the circular frequency ω of small oscillations about any stable equilibrium point (both symbolically and numerically).

4. An ice skater with moment of inertia I_0 spins with angular velocity ω_0, as shown in Figure 2. A wooden beam, initially not rotating at all, drops from above and is caught by the skater. One end of the beam is on the rotation axis of the skater. No net external force or torque acts on the system. What is the final angular velocity of the combined skater plus beam?

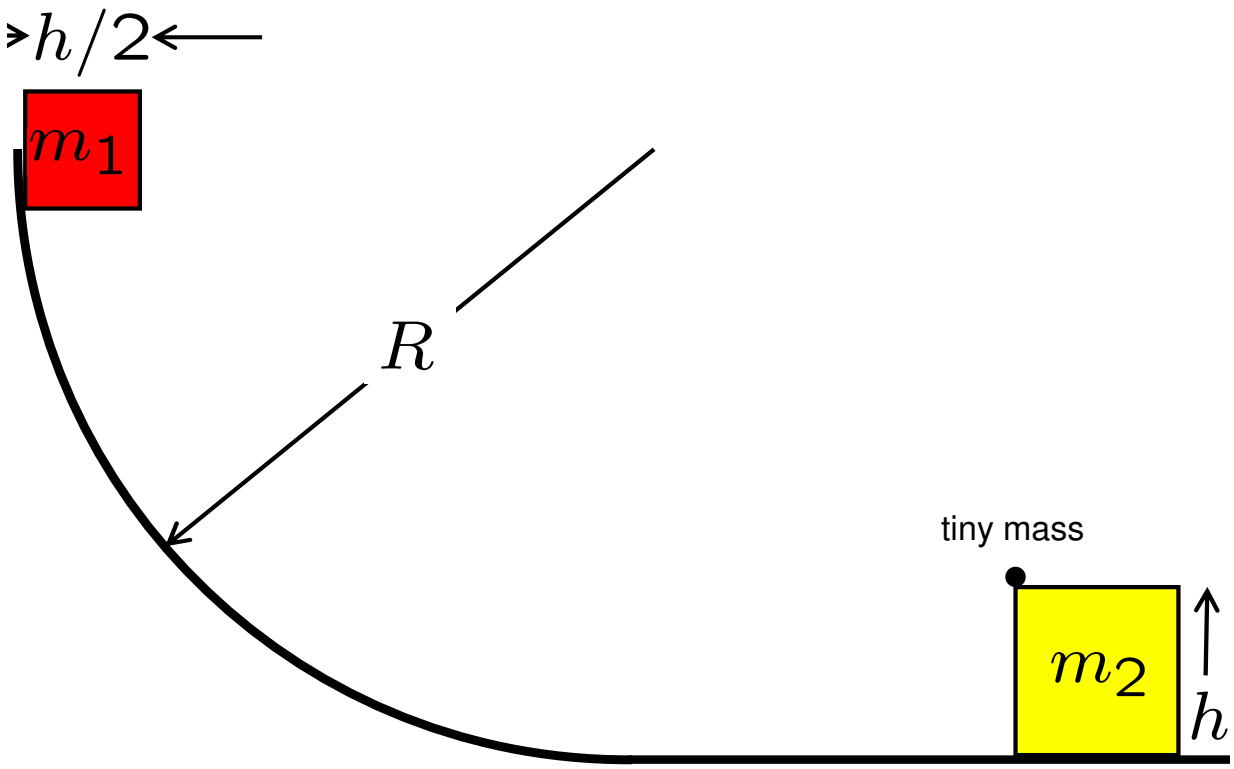

Figure 1: For use in Problem 1.

5. A mass m near the earth's surface is initally at rest at coordinates (x, z) where the origin is at the level of the ground, z is the elevation above the ground, and x is positive and is the horizontal distance from the z axis. The mass m is released; find its angular momentum vector with respect to the origin of the coordinates as a function of time, from the time of release to the time it hits the ground.

6. Two spheres, of mass m_1 and r_2 and radii r_1 and r_2, respectively, are initially just in contact with one another, and they are deep in intergalactic space, so they feel only each other's gravity. There is an explosion, after which the masses move with velocities u_1 and u_2 along the x-axis. What is the minimum u_1 necessary to insure that the masses escape to infinity?

7. A 10 kg mass sits on a horizontal surface, with which it has a coefficient of friction $\mu = 0.5$. A rope of negligible mass is attached to the mass; the rope goes over a pulley of negligible mass and connects to a second mass, m_2, that is hanging, vertically, by the rope. What is the minimum value of m_2 that will cause the 10 kg mass to move?

8. A rope of length 10 meters and mass 1 kg has one end fixed, and the rest of the rope is swung in a circle about the center, so that it completes one revolution every second. Neglect the earth's gravity. What is the tension in the rope at the center?

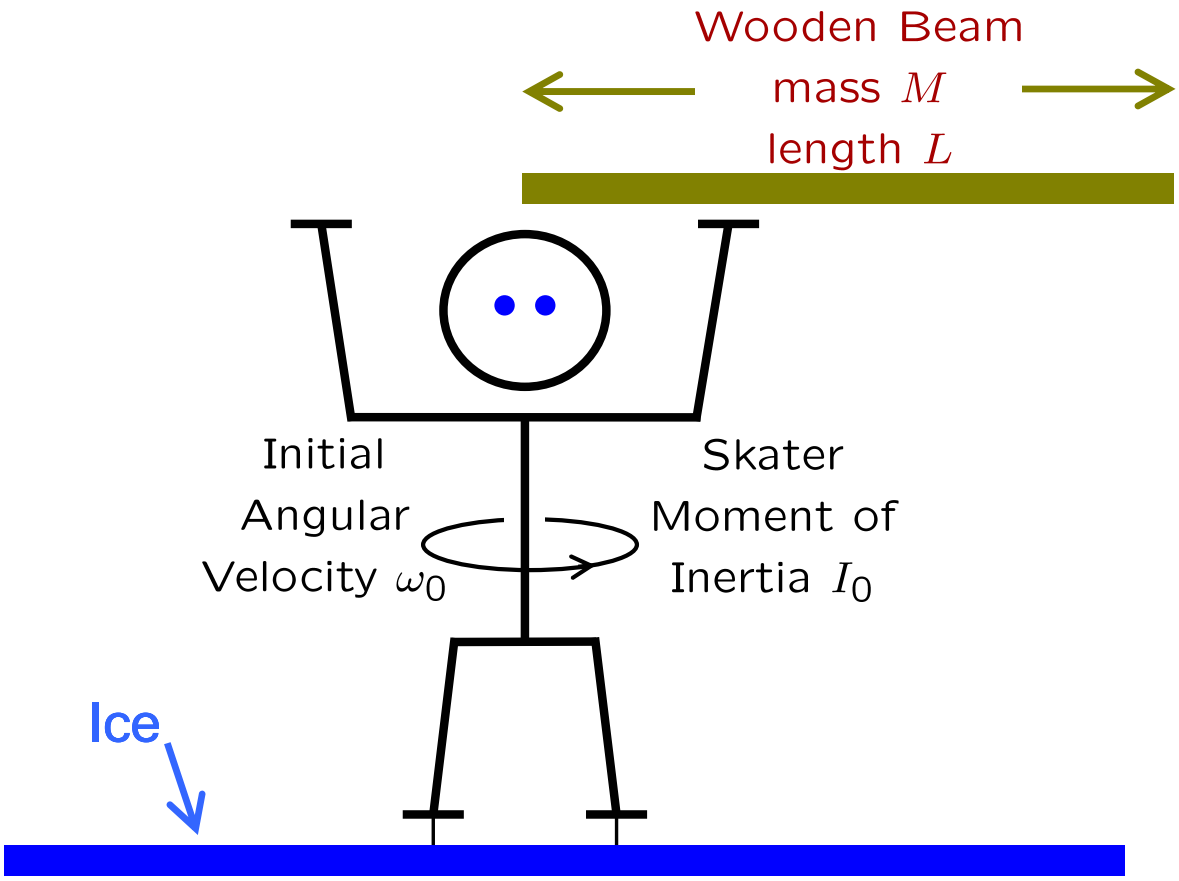

Figure 2: For use in Problem 4.

9. You launch a dense ball from ground level in a direction 45° above the horizontal, with an initial speed of 20 m/s (about 40 miles per hour). Ignore air resistance.

   (a) How high does the ball go?

   (b) How much time elapses before it returns to the ground?

   (c) What is the horizontal distance that the ball travels?

10. When a mass M is hung with a particular spring from the ceiling, its circular frequency of oscillation is ω_0. The mass M is then split into two masses, $m_1 = f \times M$, and $m_2 = (1 - f) \times M$, so note that $m_1 + m_2 = M$. The spring is then taken down, and the two masses put on either end of the spring, on a frictionless horizontal surface. Find the new circular frequencey ω_1 in terms of ω_0 and f. What value of f minimizes ω_1?