Physics 21 Problem Set 9 (Two Pages)

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Due Monday, March 7 in Class

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. K&K 4.10. Notes:

   (1) Call the new period T_1, and relate it to T_0, M, and m.

   (2) Call the original mechanical energy (which is just the total energy) E_0, and the new mechanical energy E_1, and then determine the change in mechanical energy $\Delta E = E_1 - E_0$.

   A key concept in both parts (a) and (b) is whether or not the some energy is expended to get m 'up to speed'.

2. K&K 4.11. Notes: The heart of this problem involves reasoning out the momentum transfer per unit time, as a function of time, applied by the scale to the chain. The force causing this momentum transfer, and consequently the reaction force (by Newton's Third Law) on the scale, start out being 0, just after the chain is released. By the time the last bit of chain hits the scale, the forces reach a maximum. Reasoning the precise formula for the momentum transfer as a function of time is what you need to due to solve the problem. This is mainly an exercise in differential calculus.

   I suggest you start by analyzing a small element $dm = (M/l)dx$ of the chain, a distance x above the scale. What is the relationship between the time t it takes for this element dm to hit the scale and x? Then how much time dt is required for dm to fall on to the pan, after the time t has passed? During that time dt, how much momentum dp is removed from the chain? Assume that the chain comes to rest upon hitting the scale. What force F, equal to dp/dt, is required to bring the element dm to rest in time dt? What is the resulting force on the scale? It is very useful to put your result for dp/dt in terms of t_{max}, which is the time it takes for the very top of the chain to hit the scale, M, and g. Then don't forget that the force needed to stop the chain links is not the only force on the scale: gravity always acts on the chain links that are already at rest on the scale. Finally, you need to write down Newton's Second law for the scale, and analyze why assuming that the scale itself is at rest is appropriate.

3. K&K 4.15. Notes:

   a. Remember the - sign relating $U(x)$ and $\int F(x)dx$.

   c. Evaluate x_0 for $A = 2\,\text{N-m}^2$ and $B = 1/2\,\text{N}$.

   d. Evaluate for the same A and B of part c., and take the mass $m = 2\,\text{kg}$.

4. K&K 4.16. Convert everything to meters, seconds, kilograms, and watts.

5. K&K 4.21. Notes: this problem resembles 4.11; the applied force must both counteract gravity and sustain the increase in momentum per unit time by elements of rope that leave the table. Once you find the applied force, consider the kinetic energy of the mass of rope that is off the table, and the potential energy of that portion of rope, and differentiate with respect to time.

6. K&K 4.23. Notes: like all collision problems, this one is best viewed in the center of mass frame; since the superball is of mass $M >> m$, the center of mass frame is that of the superball.

7. K&K 4.29. Notes: this is a very important problem for thinking about ideal gases. It is also a hard problem, at least starting at part b. For part b, I suggest working through the following steps:

   (α) Find the change in velocity experienced by the superball when it bounces off the *moving* surface.

   (β) In time dt, determine *how many* collisions with the moving surface occur, assuming that the distance between the surfaces is x.

   (γ) Relate the change in velocity of the superball, dv, to the time dt by combining (α) and (β), which then leads you to a differential equation relating dv/dt to V, v, and x.

   (δ) Relate x to l, V, and t. This allows you to state dv/dt in terms of constants, v, and t.

   (ϵ) Solve the differential equation for $v(t)$. It is not hard... you can do it simply by inspection, or by separating variables and integrating. Put the result in a form that relates v to l, v_0, and x.

   (ζ) Now the answer for part b follows from the same reasoning as used for part a (which is not so tough), but be sure to replace l by x and v_0 by v, and then simplify.

8. K&K 6.4.
