Physics 21 Problem Set 8

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Due Monday, Feb. 28 in class. You can turn in Problem Set 7 at the same time, if you haven't already.

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. A pendulum, shown in Figure 1, has a peg that interrupts the swing, as was demonstrated in class on Wednesday, Feb. 23. To what distance h above the peg, in terms of g and R (the length of the string as the bob circles the peg) must the pendulum bob be raised so that circular part of the bob's trajectory about the peg make a complete circle, without loss of tension in the rope?

   A good way to approach this problem is to carefully analyze the trajectory of the bob as it passes directly above the peg; this situation is emphasized in the drawing. It is pretty obvious that there must be enough energy for the bob to still have some velocity when it reaches this point. The question is... how much velocity is necessary for the string to still have tension as the bob goes beyond this point, and falls toward the ground?

   So, a good approach is to compare is the 'free' trajectory that the bob would have, were the string removed just as the bob passes directly above the peg, with the circular trajectory that the string tries to enforce. You know how to compute the 'free' trajectory that the bob would take without the string, but you must remember to eliminate the time t and get an expression relating y(vertical) to x(horizontal). With the string, it is useful to pick a coordinate system centered on the peg, and then perform the expansion of the equation for a circle, for small $x << R$ (x is horizontal):

   $$\sqrt{R^2 - x^2} \approx R + a_2 x^2 + a_4 x^4 + ...$$

   where you must determine a_2 and a_4. Then compare with the 'free' trajectory... how must the two trajectories compare for the tension to stay in the string?

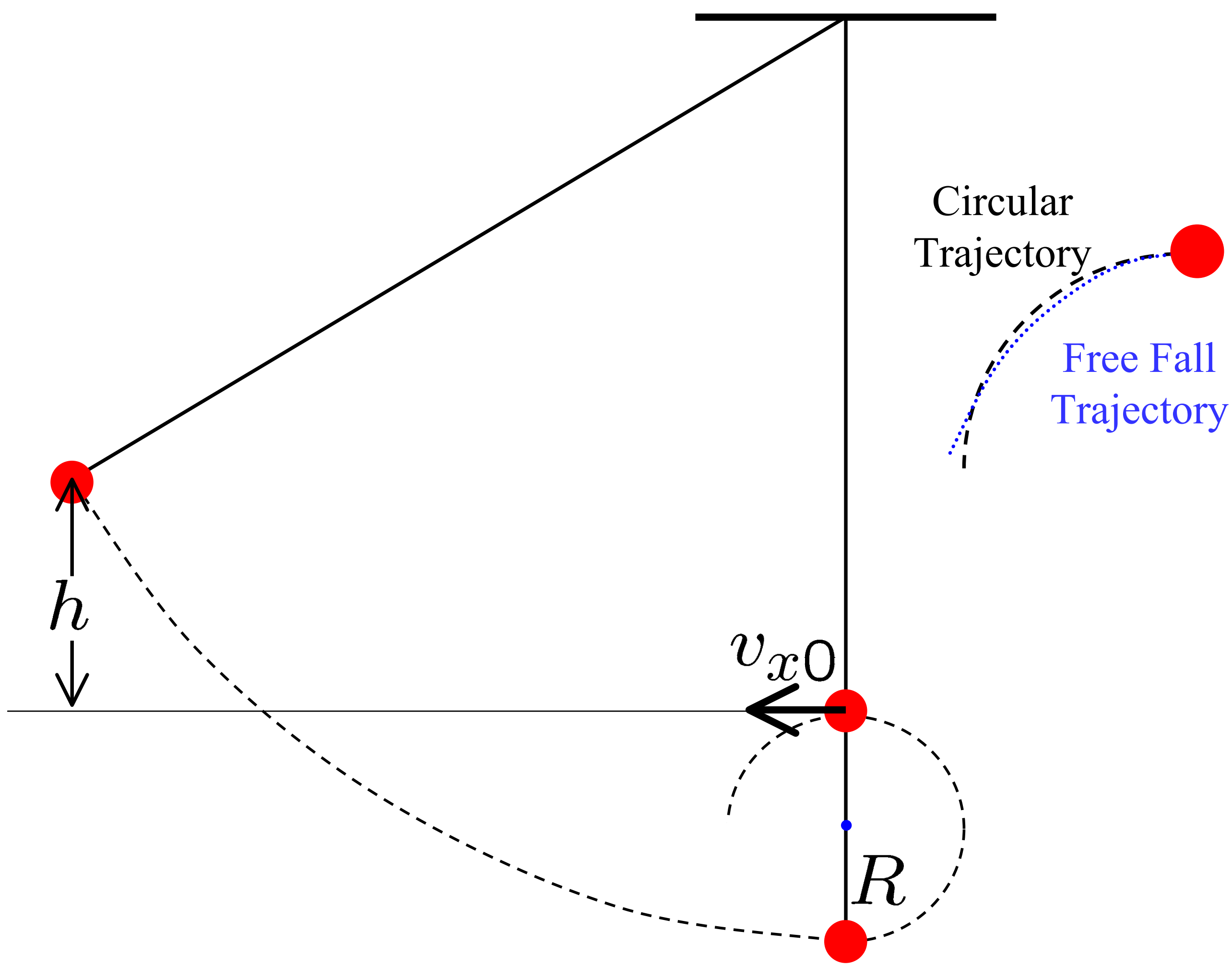

Figure 1: For use in Problem 1. The free fall trajectory shown on the upper right pertains for a specific value of v_{x0}; a larger v_{x0} could lead to a trajectory that goes (for a while) outside the circular trajectory.