Physics 21 Problem Set 1

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Due Monday, Jan. 10 in class

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. K&K 1.5
2. Fig. 1 shows a triangle, with an arbitrary point in the middle. Dotted and dashed lines which are parallel to respective sides are drawn through the point. Denote by r_i the ratio of the length of one of the dotted/dashed lines to the length of the side it is parallel to. Show that:

$$\sum_{i=1}^{3} r_i = 2$$

   Hints: Use similar triangles to show that two of the ratios r_i are equal to $1-x$ and $1-y$, where x and y are defined in Fig. 1. Then find two different vector sums that give a resultant that goes from the lower left corner to the arbitrary point; one of the sums is $x\vec{S}+y\vec{T}$, and use the second vector beneath $x\vec{S}$ to form the second. Use this second vector to determine the third r_i.
3. K&K 1.6. The law of sines states that if α, β, and γ are the angles in a triangle, and a, b, and c are respectively the lengths of the sides that are opposite the angles, then:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

4. K&K 1.11
5. K&K 1.2
6. Three vectors are given by $\mathbf{a} = 3\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} - 2\hat{\boldsymbol{k}}$, $\mathbf{b} = -\hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}} + 2\hat{\boldsymbol{k}}$, and $\mathbf{c} = 2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$. Find
   (a) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$
   (b) $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$
   (c) $\mathbf{a}\times(\mathbf{b}+\mathbf{c})$
7. A graph of x versus t is given in Fig. 2, for a particle moving along a straight line. Make a table with a horizontal row for each of the intervals OP, PQ, etc., and with three columns: 1)Displacement from O, 2)velocity v_x, and 3)acceleration a_x. Then, fill the table with entries $+$, $-$, or 0, which describe whether the displacement, velocity, and acceleration are positive, negative, or zero in the respective intervals.

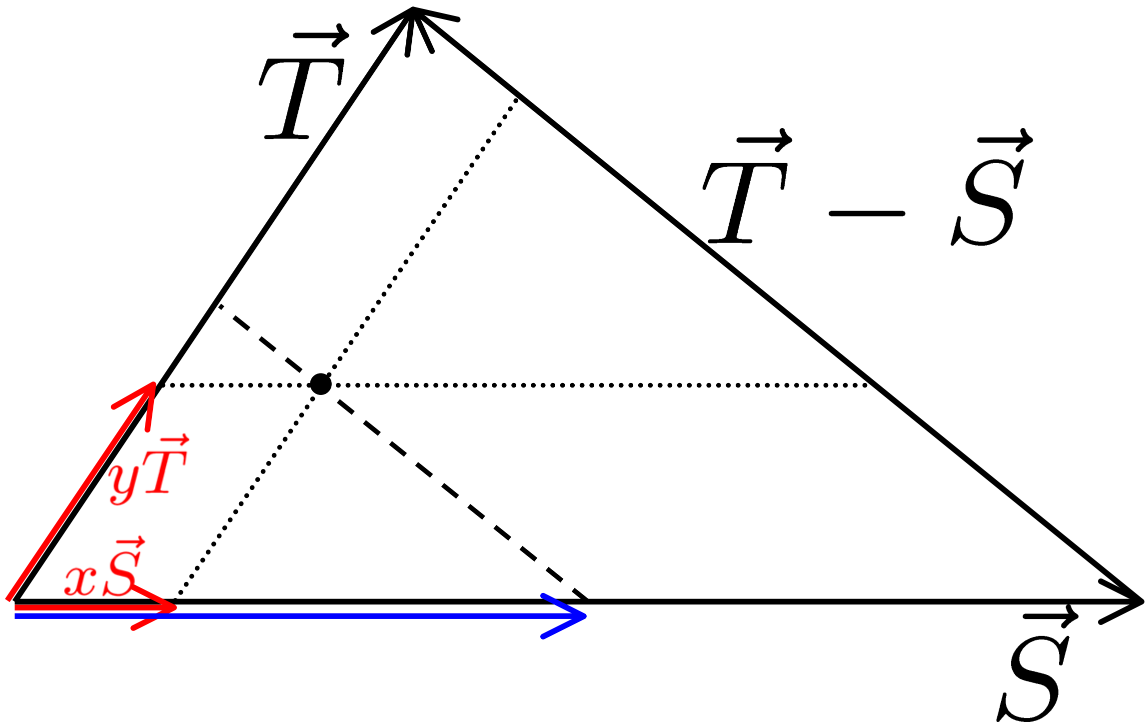


Figure 1: For use in problem 2.

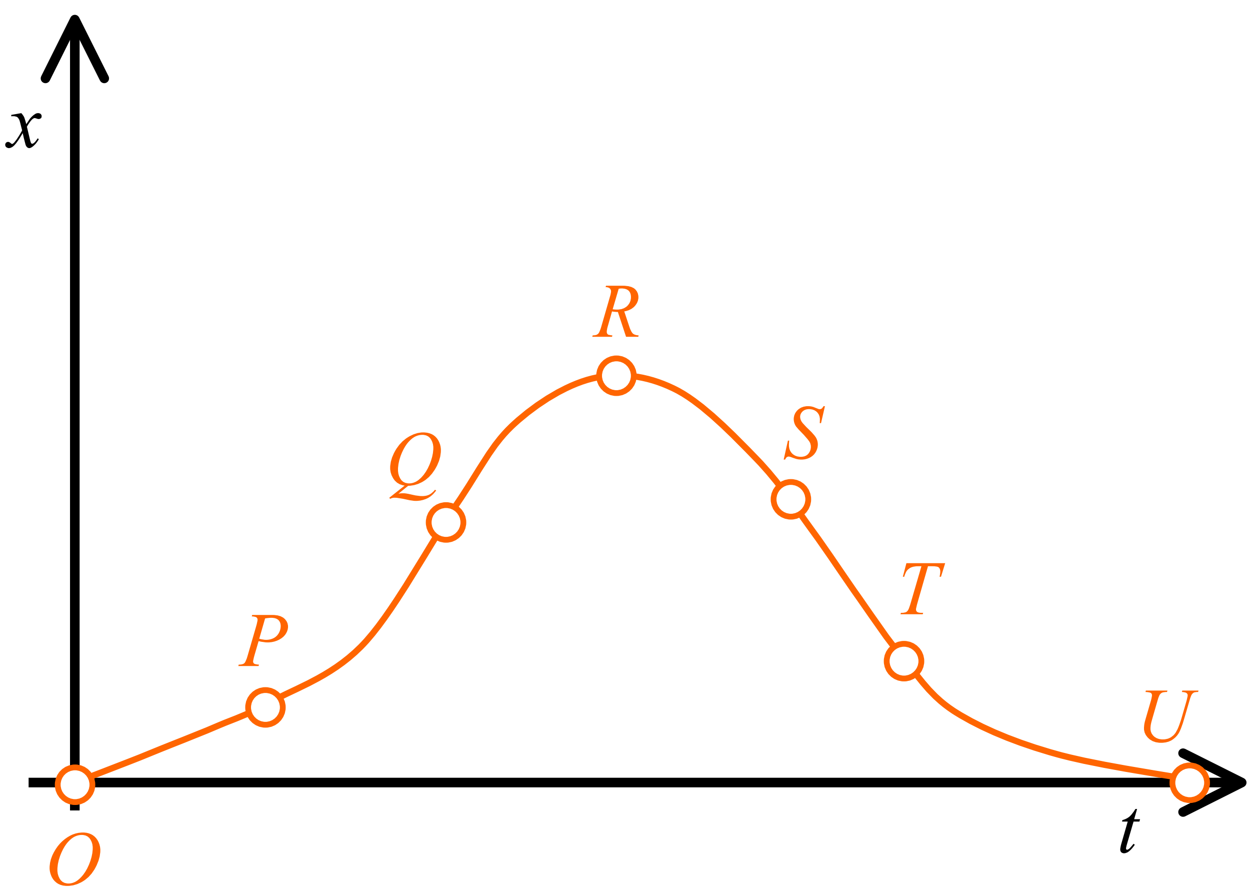


Figure 2: For use in problem 7.

8. A particle moves in the x-y plane with position vector:

$$\mathbf{r}(t) = \frac{t}{2\pi}(\hat{\boldsymbol{\imath}}\cos[2\pi t] - \hat{\boldsymbol{\jmath}}\sin[2\pi t])$$

   (a) Plot the trajectory of the particle in the x-y plane starting at $t = 0$ through $t = 3$.

   (b) Plot the speed of the particle starting at $t = 0$ through $t = 3$.

   (c) Plot the magnitude of the acceleration of the particle starting at $t = 0$ through $t = 3$.

9. Two blocks, mass m_1 and m_2, rest on a frictionless table and are connected by a spring of negligible mass. They are pulled apart and released, and subsequently bounce back and forth. Find the ratio of their accelerations a_1 and a_2.

10. (a) Two 10-lb weights are attached to a spring scale as shown in Fig. 3. What is the reading on the scale?

    (b) A single 10-lb weight is attached to the spring scale which itself is attached to a wall, as shown in Fig. 4. What is the reading on the scale?

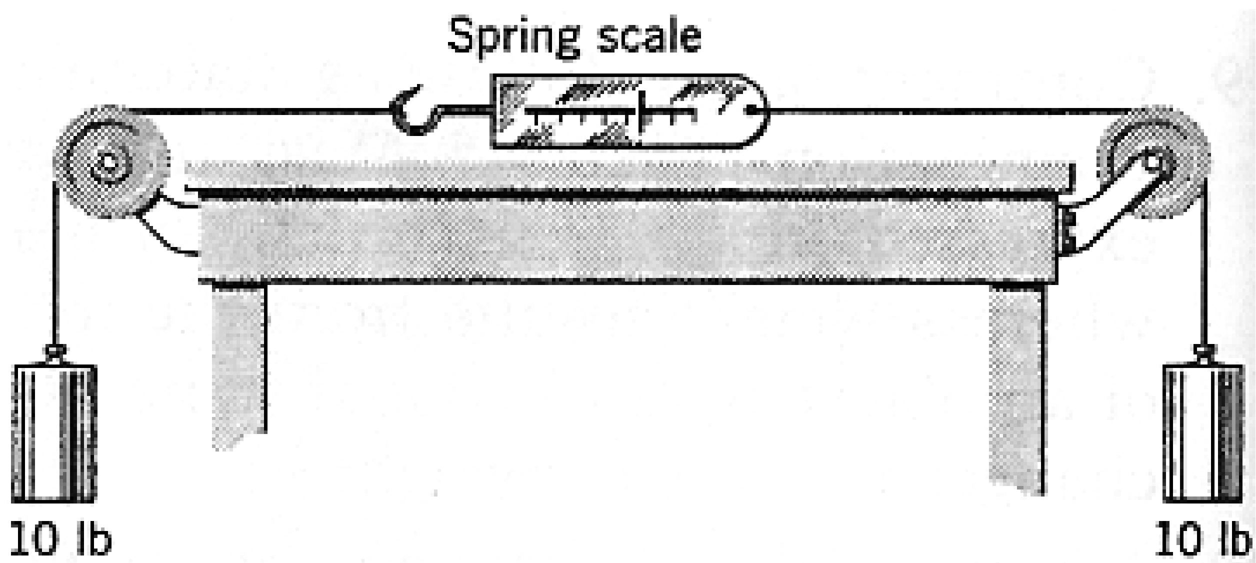


Figure 3: For use in problem 10(a).

11. Two blocks are in contact on a frictionless table. A horizontal force is applied to one block, as shown in Fig. 5.

    (a) If $m_1 = 2.0\,\text{kg}$, $m_2 = 1.0\,\text{kg}$, and $\boldsymbol{F} = 3.0\,\text{N}$, find the force of contact between the two blocks.

    (b) Now remove the force from part (a) and apply a force $-\boldsymbol{F}$ to the right side of mass m_2, and find the force of contact between the blocks.

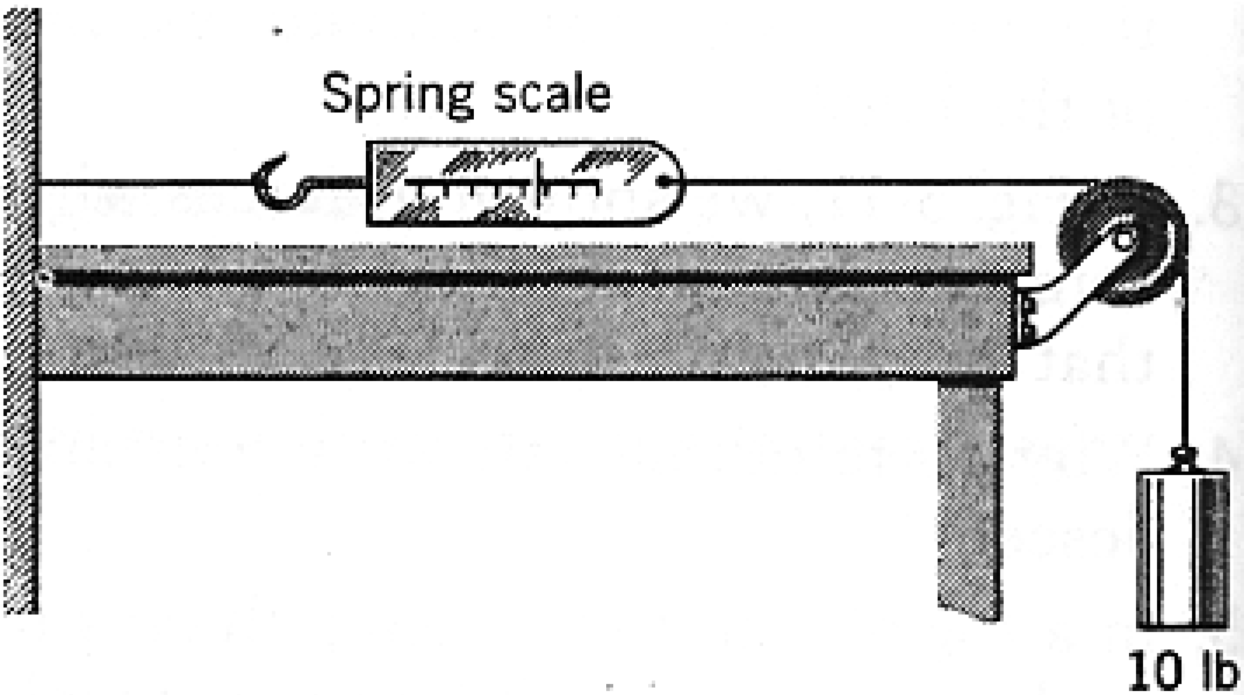


Figure 4: For use in problem 10(b).

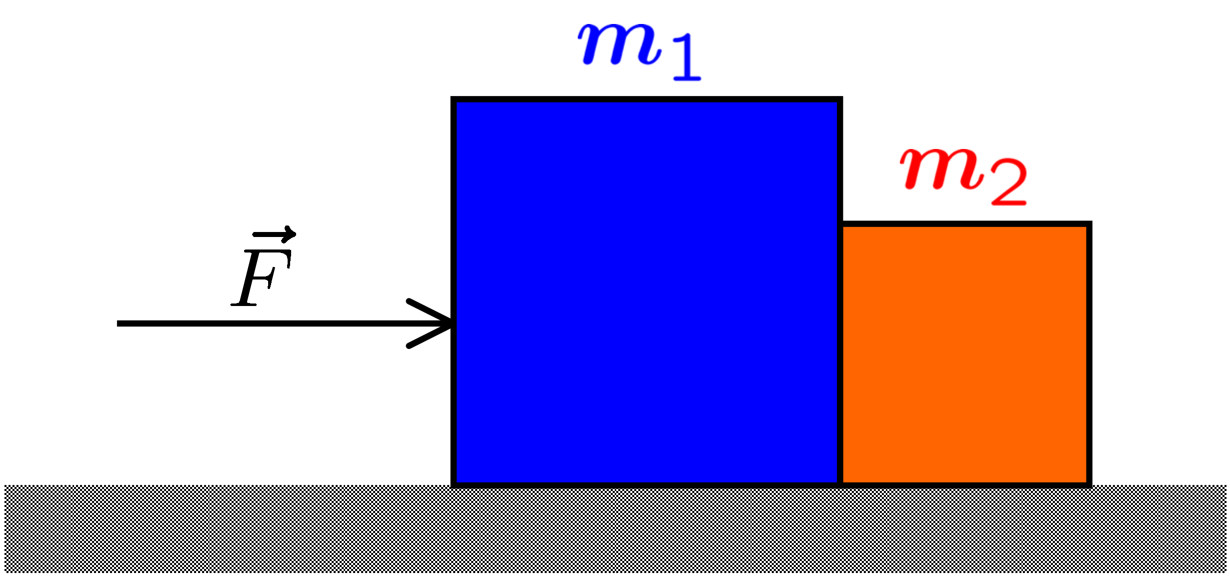


Figure 5: For use in problem 11.

Some References to Calculus Texts

A very popular textbook is G. B. Thomas, Jr., "Calculus and Analytic Geometry," 4th ed., Addison-Wesley Publishing Company, Inc., Reading, Mass.

The following introductory texts in calculus are also widely used:
M. H. Protter and C. B. Morrey, "Calculus with Analytic Geometry," Addison-Wesley Publishing Company, Inc., Reading, Mass.
A. E. Taylor, "Calculus with Analytic Geometry," Prentice-Hall, Inc., Englewood Cliffs, N.J.
R. E. Johnson and E. L. Keokemeister, "Calculus With Analytic Geometry," Allyn and Bacon, Inc., Boston.

A highly regarded advanced calculus text is R. Courant, "Differential and Integral Calculus," Interscience Publishing, Inc., New York.

If you need to review calculus, you may find the following helpful: Daniel Kleppner and Norman Ramsey, "Quick Calculus," John Wiley & Sons, Inc., New York.

Problems

1.1 Given two vectors, $\mathbf{A} = (2\hat{\imath} - 3\hat{\jmath} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\imath} + \hat{\jmath} + 2\hat{\mathbf{k}})$, find: (*a*) $\mathbf{A} + \mathbf{B}$; (*b*) $\mathbf{A} - \mathbf{B}$; (*c*) $\mathbf{A} \cdot \mathbf{B}$; (*d*) $\mathbf{A} \times \mathbf{B}$.

Ans. (a) $7\hat{\imath} - 2\hat{\jmath} + 9\hat{\mathbf{k}}$; (c) 21

1.2 Find the cosine of the angle between

$$\mathbf{A} = (3\hat{\imath} + \hat{\jmath} + \hat{\mathbf{k}}) \qquad \text{and} \qquad \mathbf{B} = (-2\hat{\imath} - 3\hat{\jmath} - \hat{\mathbf{k}}).$$

Ans. -0.805

1.3 The direction cosines of a vector are the cosines of the angles it makes with the coordinate axes. The cosine of the angles between the vector and the x, y, and z axes are usually called, in turn α, β, and γ. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$, using either geometry or vector algebra.

1.4 Show that if $|\mathbf{A} - \mathbf{B}| = |\mathbf{A} + \mathbf{B}|$, then $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

1.5 Prove that the diagonals of an equilateral parallelogram are perpendicular.

1.6 Prove the law of sines using the cross product. It should only take a couple of lines. (*Hint*: Consider the area of a triangle formed by $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, where $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$.)

1.7 Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors in the xy plane making angles θ and ϕ with the x axis, respectively. Show that $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$, $\hat{\mathbf{b}} = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$, and using vector algebra prove that

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$

1.8 Find a unit vector perpendicular to

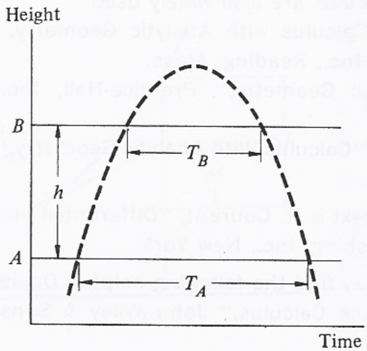
$$\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \quad \text{and} \quad \mathbf{B} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}).$$

Ans. $\hat{\mathbf{n}} = \pm(2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}})/\sqrt{38}$

1.9 Show that the volume of a parallelepiped with edges **A**, **B**, and **C** is given by $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$.

1.10 Consider two points located at $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Find a vector **A** from the origin to a point on the line between $\mathbf{r}_1$ and $\mathbf{r}_2$ at distance xr from the point at $\mathbf{r}_1$, where x is some number.

1.11 Let **A** be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\mathbf{A} = (\mathbf{A}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}} + (\hat{\mathbf{n}}\times\mathbf{A})\times\hat{\mathbf{n}}$.



1.12 The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

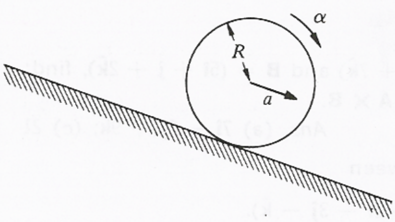
Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A{}^2 - T_B{}^2},$$

where h is the height of line B above line A.

1.13 An elevator ascends from the ground with uniform speed. At time T_1 a boy drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8\ \text{m/s}^2$, and hits the ground T_2 seconds later. Find the height of the elevator at time T_1.

Ans. clue. If $T_1 = T_2 = 4$ s, $h = 39.2$ m

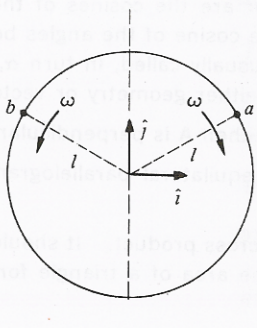


1.14 A drum of radius R rolls down a slope without slipping. Its axis has acceleration a parallel to the slope. What is the drum's angular acceleration α?

1.15 By *relative velocity* we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)

a. A point is observed to have velocity $\mathbf{v}_A$ relative to coordinate system A. What is its velocity relative to coordinate system B, which is displaced from system A by distance **R**? (**R** can change in time.)

Ans. $\mathbf{v}_B = \mathbf{v}_A - d\mathbf{R}/dt$



b. Particles a and b move in opposite directions around a circle with angular speed ω, as shown. At $t = 0$ they are both at the point $\mathbf{r} = l\hat{\mathbf{j}}$, where l is the radius of the circle.

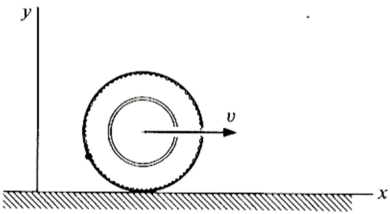
Find the velocity of a relative to b.

1.16 A sportscar, Fiasco I, can accelerate uniformly to 120 mi/h in 30 s. Its *maximum* braking rate cannot exceed $0.7g$. What is the minimum time required to go $\frac{1}{2}$ mi, assuming it begins and ends at rest? (*Hint:* A graph of velocity vs. time can be helpful.)

1.17 A particle moves in a plane with constant radial velocity $\dot{r} = 4$ m/s. The angular velocity is constant and has magnitude $\dot{\theta} = 2$ rad/s. When the particle is 3 m from the origin, find the magnitude of (*a*) the velocity and (*b*) the acceleration.

Ans. (*a*) $v = \sqrt{52}$ m/s

1.18 The rate of change of acceleration is sometimes known as "jerk." Find the direction and magnitude of jerk for a particle moving in a circle of radius R at angular velocity ω. Draw a vector diagram showing the instantaneous position, velocity, acceleration, and jerk.



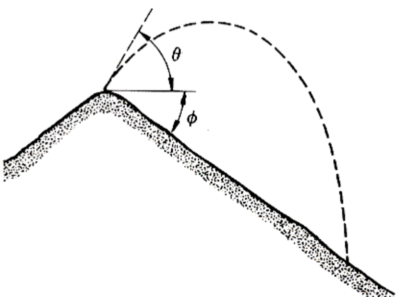
1.19 A tire rolls in a straight line without slipping. Its center moves with constant speed V. A small pebble lodged in the tread of the tire touches the road at $t = 0$. Find the pebble's position, velocity, and acceleration as functions of time.

1.20 A particle moves outward along a spiral. Its trajectory is given by $r = A\theta$, where A is a constant. $A = (1/\pi)$ m/rad. θ increases in time according to $\theta = \alpha t^2/2$, where α is a constant.

a. Sketch the motion, and indicate the approximate velocity and acceleration at a few points.

b. Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.

c. At what angles do the radial and tangential accelerations have equal magnitude?



1.21 A boy stands at the peak of a hill which slopes downward uniformly at angle ϕ. At what angle θ from the horizontal should he throw a rock so that it has the greatest range?

Ans. clue. If $\phi = 60°$, $\theta = 15°$