Physics 21 Problem Set 10 (Three Pages)

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Due Friday, March 11 9:00pm
under Prof. Nelson's Door (5103 Broida)

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. K&K 6.1. I suggest you imagine evaluating the total angular momentum and the total torque about two distinct origins, which are located at displacements relative to the origin of the coordinate system $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. Also, denote the difference between the displacements to the origins $\boldsymbol{r}_{BA} = \boldsymbol{r}_B - \boldsymbol{r}_A$. Then, relate the total angular momentum (or torque) about $\boldsymbol{r}_B$, $\boldsymbol{L}_B$ (or $\boldsymbol{\tau}_B$):

$$\boldsymbol{L}_B = \sum_{i=1}^{N} (\boldsymbol{r}_{Bi} \times \boldsymbol{p}_i) \tag{1}$$

$$\boldsymbol{\tau}_B = \sum_{i=1}^{N} (\boldsymbol{r}_{Bi} \times \boldsymbol{F}_i) \tag{2}$$

to the total angular momentum (or torque) about $\boldsymbol{r}_A$, $\boldsymbol{L}_A$, through a substitution that involves $\boldsymbol{r}_{BA}$. In Equation 1 and Equation 2 the index i goes over the N particles, and the vectors $\{\boldsymbol{r}_{Bi}\}$ are the displacements from the point B to each of the particles.

What happens to the terms that involve $\boldsymbol{r}_{BA}$, particularly when the total linear momentum (or torque) is zero?

2. In this problem, analyze a trajectory near the earth's surface in terms of torque and the time derivative of the angular momentum. See Figure 1. A mass starts at $x_0 = z_0 = 0$, with initial velocity $\boldsymbol{v}_0 = v_{0x}\hat{\boldsymbol{\imath}} + v_{0z}\hat{\boldsymbol{k}}$. Assume that the trajectory is described by the equations on Page 21 of your text.

   (a) Evaluate the torque $\boldsymbol{\tau}_g$ by the force of gravity about the point labeled (x_p, z_p) in Figure 1 as a function of time. For this problem, I suggest that you put the vector between the point (x_p, z_p) and the trajectory of the mass $(x(t), z(t))$ in the form:

$$\boldsymbol{r}_p = (x(t) - x_p)\hat{\boldsymbol{\imath}} + (z(t) - z_p)\hat{\boldsymbol{k}} \tag{3}$$

   and do the same for the force of gravity. Then evaluate the cross product through distributing the cross projects of $\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\jmath}} = 0$ and $\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = -\hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{k}}$.

   (b) Evaluate the angular momentum $\boldsymbol{L}_p(t)$ about the point labeled (x_p, z_p) as a function of time, by explicitly using the trajectory. I suggest that you put $\boldsymbol{p}$ in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{k}}$. Put your result for $\boldsymbol{L}_p(t)$ in the form:

$$\boldsymbol{L}_p(t) = (a + b \times t + c \times t^2)\hat{\boldsymbol{\jmath}}. \tag{4}$$

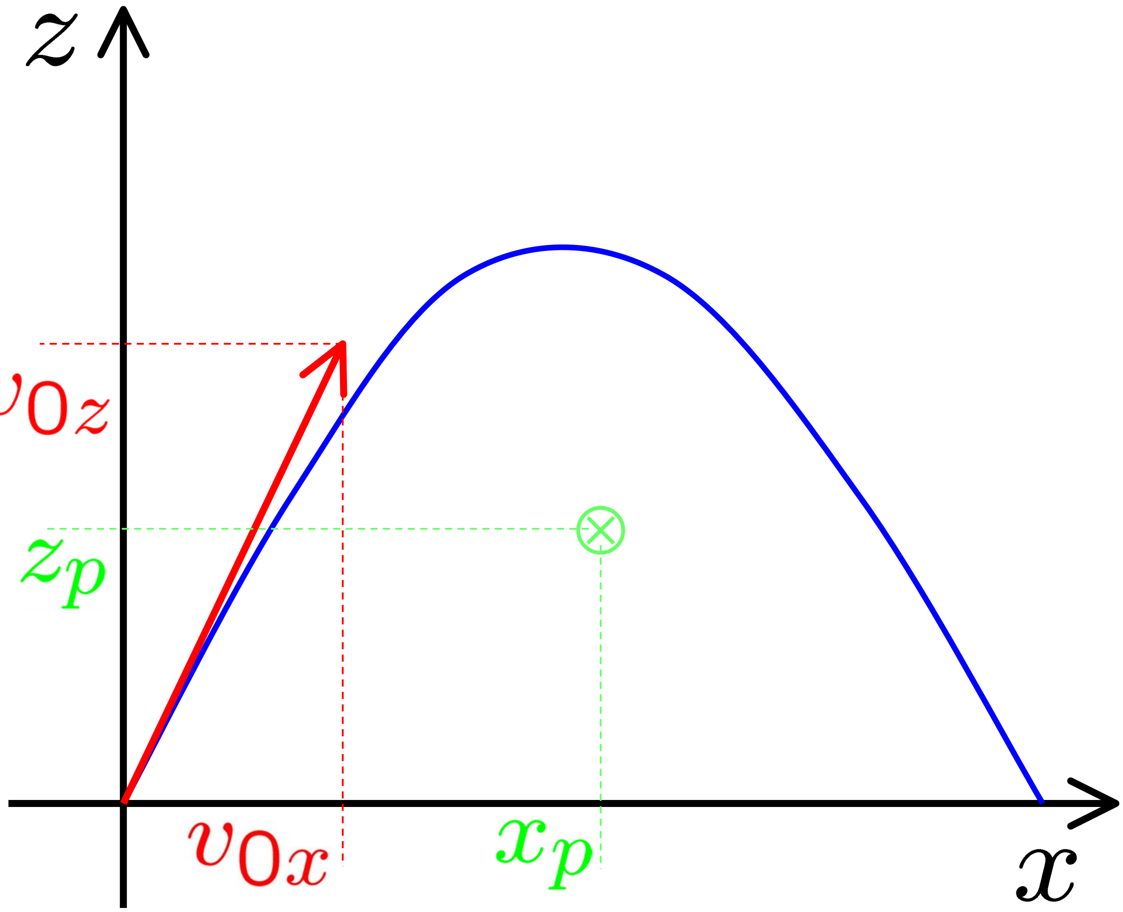

Figure 1: For use in Problem 2.

(c) Explicitly verify by computing the derivative of $\boldsymbol{L}_p(t)$ that:

$$\frac{d\boldsymbol{L}_p(t)}{dt} = \boldsymbol{\tau}_g. \tag{5}$$

3. K&K 6.3, only simplify this problem by assuming that the pivot point is in the *center* of the ring; instead of doing (a) and (b) find ω for all times. For extra credit (equivalent to 1/2 of a whole problem set!) do the original 6.3, and find the rotational velocity of the ring $\omega(t)$ for *all* times. Figure 2 will help you.

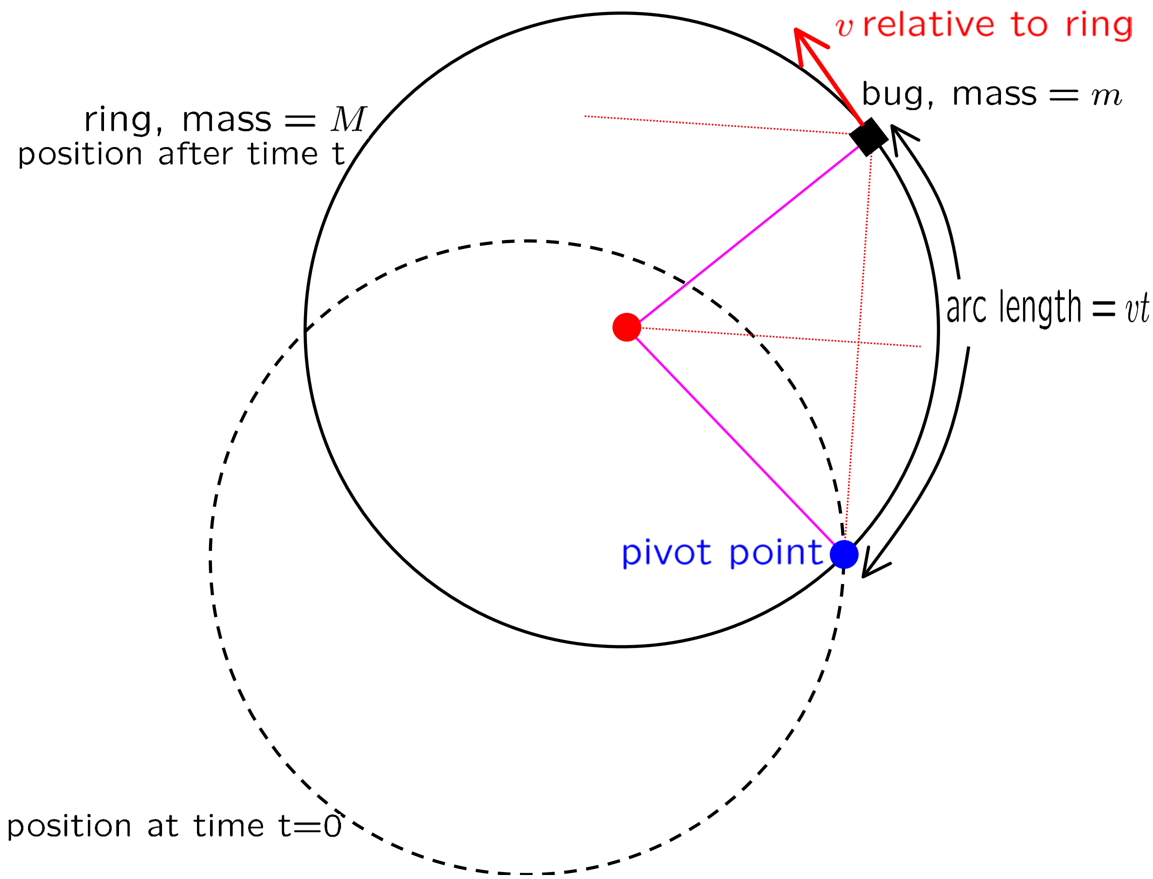

Figure 2: For use in Problem 3.