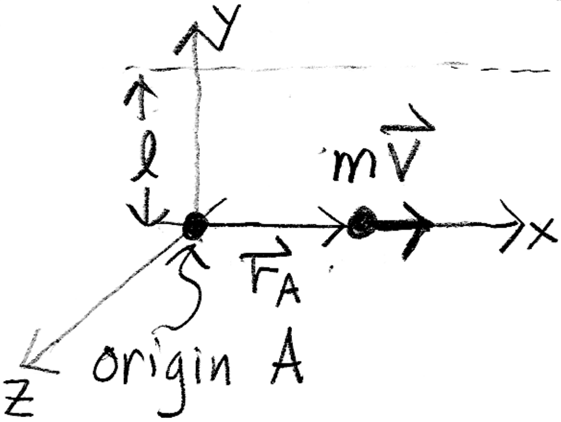
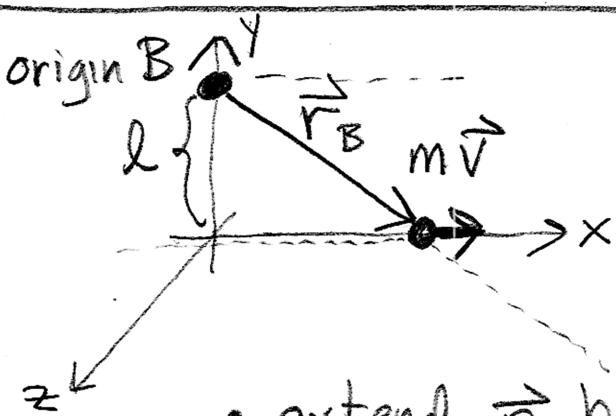
The value of the angular momentum depends on where the origin of your coordinates is (unlike linear momentum)!!!





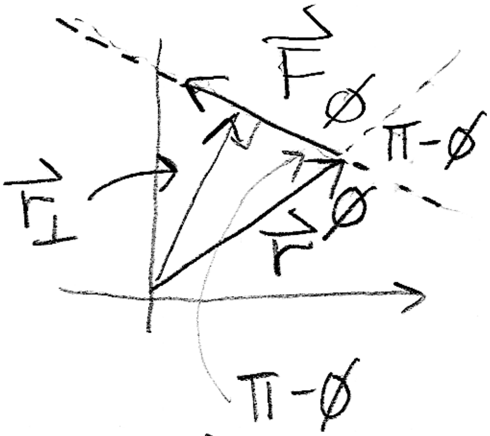
$\vec{r}_A \parallel \vec{V}$

so $\vec{r}_A \times \vec{V} = 0$

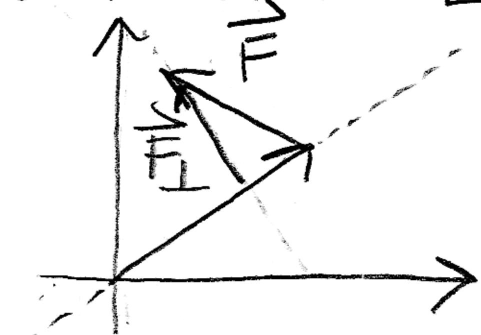
- extend $\vec{p}$ back.
- "distance of closest approach" is ℓ
- $\vec{L} = mv\ell \cdot \hat{k}$ ← RHR (right hand rule)

For $\dfrac{d\vec{L}}{dt} = \vec{\tau}$,

$\vec{\tau} = \vec{r} \times \vec{F}$, Must use same origin as used to evuate $\vec{L}$



or



$\sin(\pi-\phi) = \sin\phi$

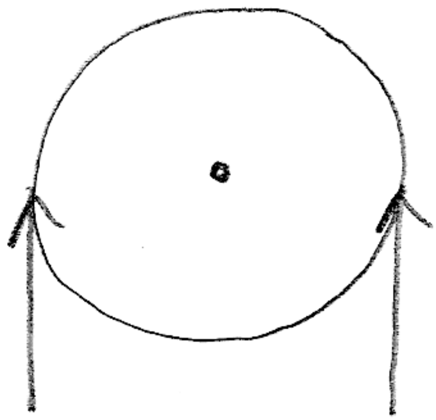
$|\vec{r}_\perp| \rightarrow$ "lever arm"

$$|\vec{\tau}| = |\vec{r}_\perp||\vec{F}| = |\vec{r}||\vec{F}_\perp|$$
$$= |\vec{r}||\vec{F}|\sin\theta$$

Torque and Force Independent

Imagine a disk that can pivot about its center

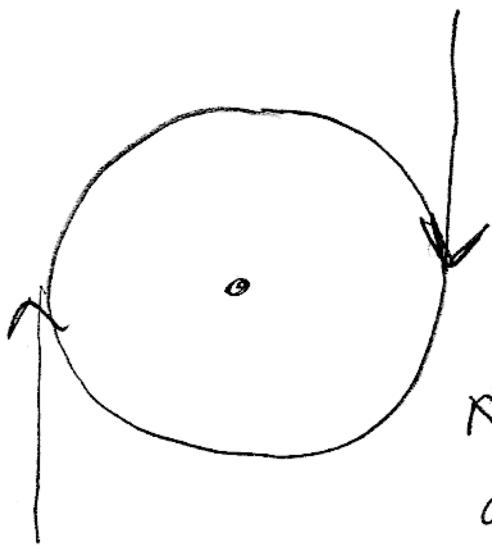
some net Force (not net Torque about pivot)



$\vec{F}_{net} \neq 0$

$\vec{\tau}_{net} = 0$

no net Force (some net Torque about pivot)

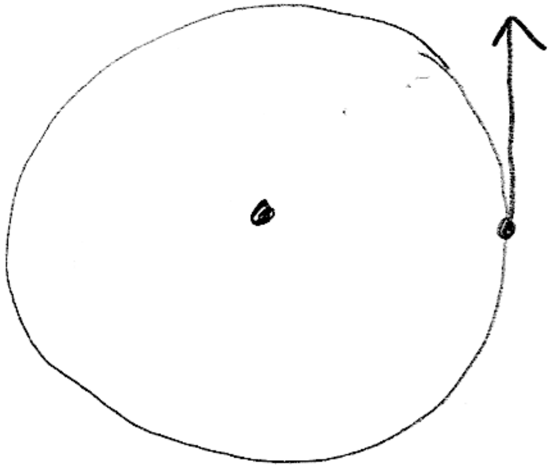


location irrelevant

$\vec{F}_{net} = 0$

$\vec{\tau}_{net} \neq 0$

called a "couple"



$\vec{F}_{net} \neq 0$

$\tau_{net} \neq 0$

$$\boxed{\text{Why} \quad \vec{\tau} = \frac{d\vec{L}}{dt}}$$

$$\frac{d}{dt}\vec{L} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \underbrace{\frac{d\vec{r}}{dt}\times\vec{p}}\, + \vec{r}\times\overset{\vec{F}}{\overset{\parallel}{\frac{d\vec{p}}{dt}}}$$

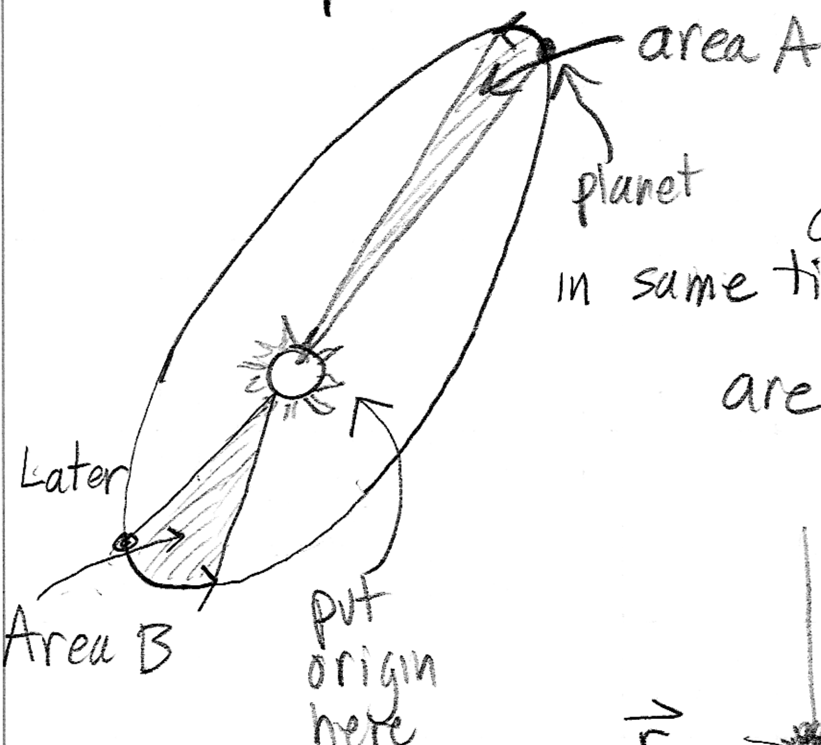
$$\frac{d\vec{r}}{dt}\times\vec{p} = m\left(\frac{d\vec{r}}{dt}\times\frac{d\vec{r}}{dt}\right) \rightarrow 0$$

so $$\boxed{\frac{dL}{dt} = \vec{r}\times\vec{F}}$$

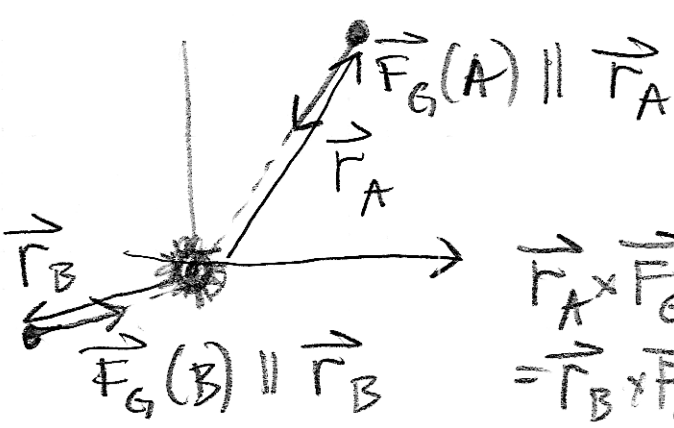
Most important case : $\vec{F} \parallel \vec{r}$, so
$$\frac{d\vec{L}}{dt} = 0$$

Orbits about the sun

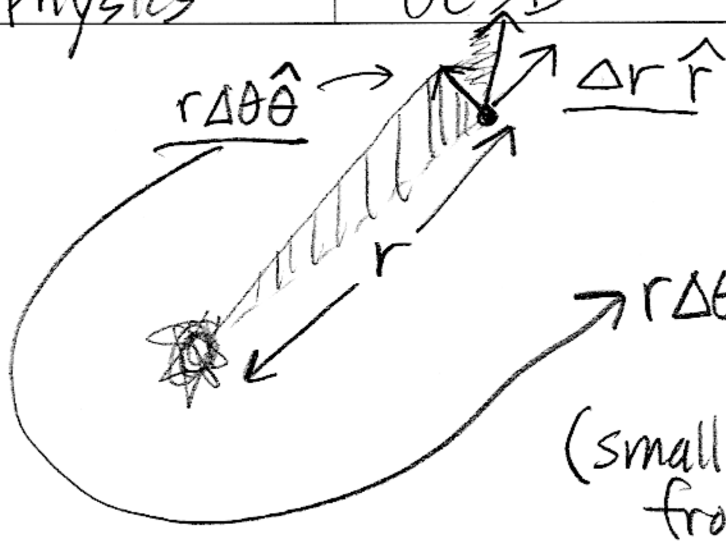
Kepler's Second Law...



areas A & B are swept out in same time Δt, small, are EQUAL



$$\vec{r}_A\times\vec{F}_G(A) = \vec{r}_B\times\vec{F}_G(B) = 0$$



$\Delta r\,\hat{r}$ by itself, does not alter area

$r\Delta\theta\,\hat{\theta}$ by itself, does alter area

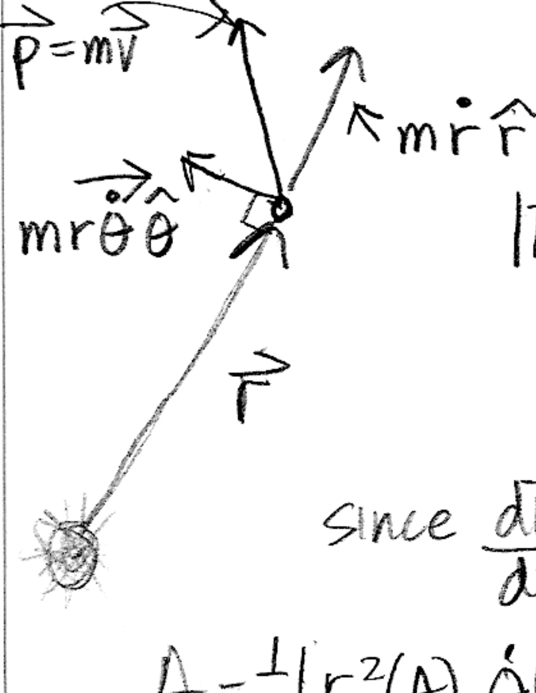
(small area increase comes from both together) $O(\Delta\theta\cdot\Delta r) \to$ ignore.

$$A \simeq \frac{1}{2} r^2 \Delta\theta$$

triangle — r altitude — $r\Delta\theta$ base

$$A \simeq \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}\,\Delta t = \frac{1}{2} r^2 \dot{\theta}\,\Delta t$$

<u>point</u>: $|r^2\dot{\theta}| = \dfrac{|\vec{L}|}{m}$



$$\vec{p}_{\perp} = mr\dot{\theta}\,\hat{\theta}$$

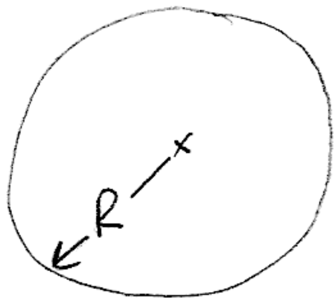
$$|\vec{L}| = |\vec{p}_{\perp}||\vec{r}| = mr^2\dot{\theta}$$

$$\frac{|\vec{L}|}{m} = |r\dot{\theta}^2|$$

since $\dfrac{d\vec{L}}{dt} = 0$, $|\vec{L}|$ is constant

$$A = \frac{1}{2}|r^2(A)\,\dot{\theta}(A)|\Delta t = \frac{1}{2}|r^2(B)\dot{\theta}(B)|\Delta t = B$$

Cross Section of a Planet or Sun

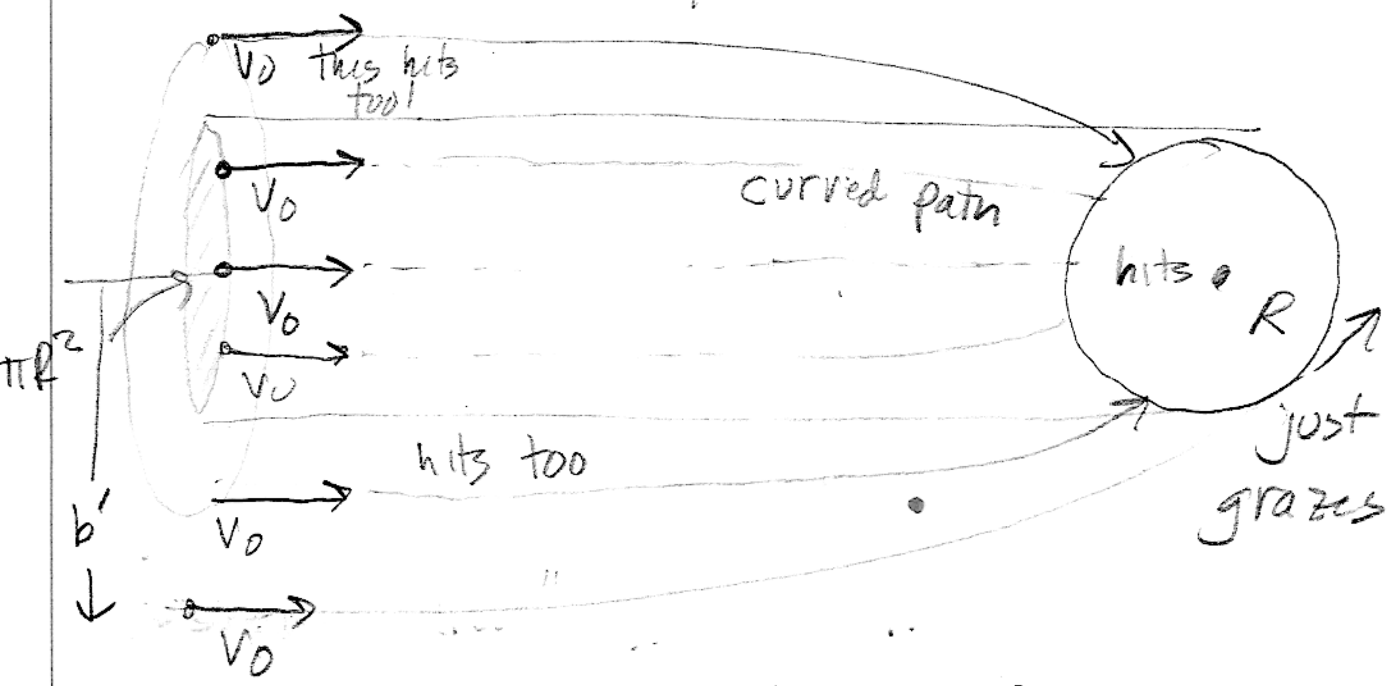


Moon
Sun or Planet: view with light, which goes (essentially) straight.

$A = \text{Area} = \pi R^2$

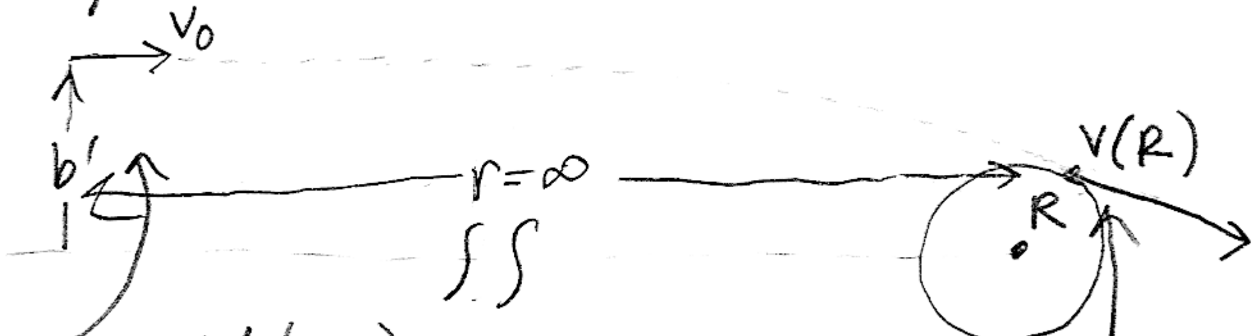
Reverse Viewpoint: Shoot a laser pointer AT the moon... looks like area $A = \pi R^2$ to successfully hit moon.

<u>Now</u>: shoot little rocks at the moon, launch velocity v_0



"Capture Cross Section" $= \pi b'^2 > \pi R^2$

Let's calculate $\pi b'^2$.



$L = -b' \cdot (m v_0)$

into page

$L = R m v(R)$

$\cancel{m} v_0 b' = \cancel{m} v(R) R$ 2 unknowns

$$E = \frac{1}{2}\cancel{m} v_0^2 - \underbrace{\frac{GM\cancel{m}}{r}}_{r \to \infty,\ \to 0} = \frac{1}{2}\cancel{m} v^2(R) - \frac{GM\cancel{m}}{R}$$

$$v^2(R) = v_0^2 + \frac{2GM}{R}$$

$$\left(v_0 \cdot \frac{b'}{R}\right)^2 = v_0^2 + \frac{2GM}{R}$$

$$b'^2 = R^2\left(1 + \frac{2GM}{v_0^2}\right)$$

$$\pi b'^2 = \pi R^2 \left(1 + \frac{2GM}{v_0^2}\right)$$

$$\boxed{\sigma(v_0) = \sigma(0)\left(1 + \frac{2GM}{v_0^2}\right)}$$

$v_0 \to 0$ can't miss!