Extended Bodies : motion can be decomposed into :

① Translation of the center of mass

② Rotation about the center of mass — time to begin.

Initial Focus :

find analog of $\vec{F} = \frac{d\vec{p}}{dt}$

net external (↑ $\vec{F}$) — linear momentum (↖ $\vec{p}$).

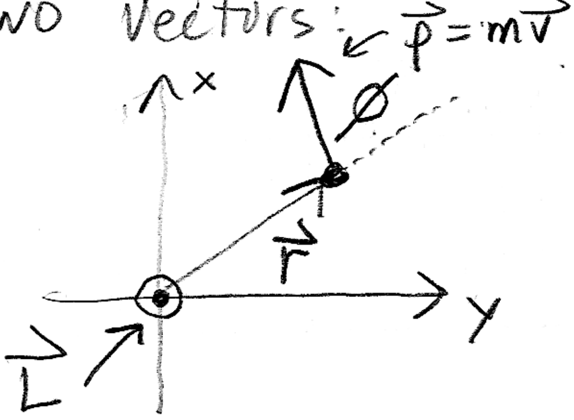
Other types of "forces" and "momenta" can be defined....

$$\vec{r} \times \vec{F} \equiv \vec{\tau} \qquad \vec{r} \times \vec{p} \equiv \vec{L}$$

and, turns out, $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

but before deriving this, let's talk about each of these.

Can make a plane that encompasses any two vectors:

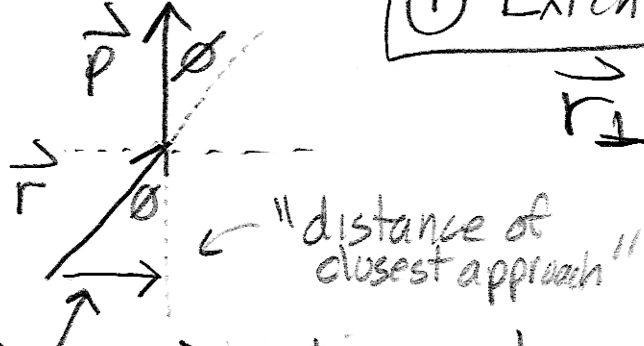


$\vec{L} = \vec{r} \times \vec{p}$:

- as shown, ∥ to $\hat{z}$
- z component $|\vec{r}||\vec{p}| \sin\phi$

Cross Product Visualizations

$\boxed{\text{① Extend } \vec{p}}$



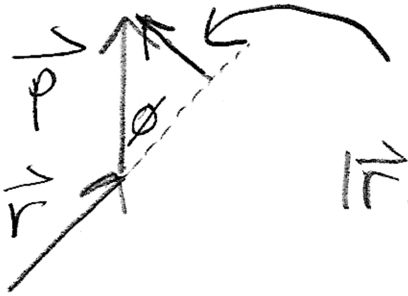
$\vec{r}_\perp$ itself has a 90° angle w/r to $\vec{p}$

$\vec{r}_\perp$, $|\vec{r}_\perp| = |r\sin\phi|$

$$|\vec{r}\times\vec{p}| = |\vec{r}_\perp||\vec{p}|$$
$$= |\vec{r}||\vec{p}||\sin\phi|$$

get direction by RHR

Or, equivalently: $\boxed{\text{② Extend } \vec{r}}$



$\vec{p}_\perp$, $|\vec{p}_\perp| = |p\sin\phi|$

$$|\vec{r}\times\vec{p}| = |\vec{r}||\vec{p}||\sin\phi|$$

directions - RHR

Both visualizations useful

Sometimes must do....

$$\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

when $z = p_z = 0$ (motion in x-y plane)

simplifies a lot....

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ p_x & p_y & 0 \end{vmatrix} = \cancelto{0}{\hat{i}\begin{vmatrix} y & 0 \\ p_y & 0 \end{vmatrix}} - \cancelto{0}{\hat{j}\begin{vmatrix} x & 0 \\ p_x & 0 \end{vmatrix}}$$
$$+ \hat{k}\begin{vmatrix} x & y \\ p_x & p_y \end{vmatrix} = \hat{k}(xp_y - yp_x)$$