4.11 – non conservative forces – read

4.12 – actually non-conservative forces are situations where mechanical energy turns into heat

Heat ... energy put into random motion of particles (atoms)

M.E. $\rightarrow$ Heat happens

Heat $\rightarrow$ M.E. not really

Power:

$$dW = \vec{F} \cdot d\vec{r}$$

$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$\frac{\text{Joules}}{\text{second}} = \frac{\text{Newton-meters}}{\text{second}}$$

"Watts"

746 Watts = 1 horse power

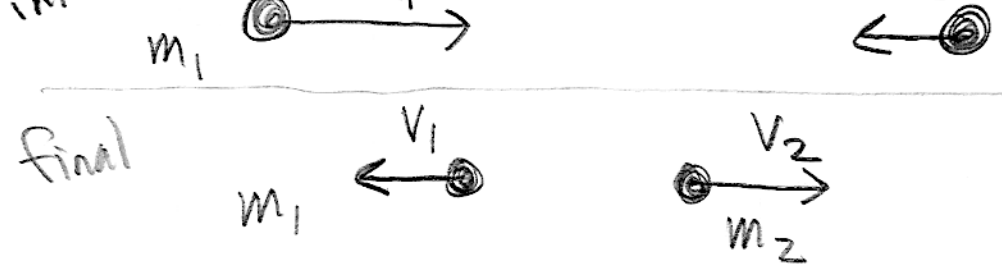
1 kilocalorie = 4200 Joules

diet: = 2000-3000 kcal/day

1000 W/m^2 = sunlight on earth

Collisions (1-d, initially)

m_1, m_2
v_1, v_2 given



find v_1, v_2

$$V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

center of mass velocity

or

$$\underbrace{m_1 u_1 + m_2 u_2}_{\text{initial momentum}} = \underbrace{m_1 V_1 + m_2 V_2}_{\text{final momentum}}$$

1 equation with 2 unknowns.

Second equation

Elastic → all energy stays mechanical
no heat, no internal excitement

Inelastic → mechanical energy is converted.

$$\underbrace{\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2}_{\substack{\text{initial energy} \\ \text{all kinetic}}} = \underbrace{\tfrac{1}{2} m_1 V_1^2 + \tfrac{1}{2} m_2 V_2^2}_{\substack{\text{final energy} \\ \text{that is kinetic}}} + Q$$

amount of energy deposited (usually) or rarely withdrawn..

$Q > 0$ ... inelastic

$Q = 0$ elastic

$Q < 0$ "superelastic"...
a "cocked gun" that releases

Elastic

$Q=0$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2$$

$$v_2 = \frac{m_1 u_1 + m_2 u_2 - m_1 v_1}{m_2}$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 \frac{(m_1 u_1 + m_2 u_2 - m_1 v_1)^2}{m_2^2}$$

yuck!

$$m_1\left(1 + \frac{m_1}{m_2}\right) v_1^2 - 2\,\frac{m_1 (m_1 u_1 + m_2 u_2)}{m_2} v_1 + \frac{(m_1 u_1 + m_2 u_2)^2}{m_2} - m_1 u_1^2 - m_2 u_2^2 = 0$$

$$a v_1^2 + b v_1 + c = 0$$

$$a = \left(1 + \frac{m_1}{m_2}\right) m_1$$

$$b = -2 \cdot \frac{m_1}{m_2}(m_1 u_1 + m_2 u_2)$$

$$c = \frac{1}{m_2}\left(m_1^2 u_1^2 + 2 m_1 m_2 u_1 u_2 + m_2^2 u_2^2\right) - m_1 u_1^2 - m_2 u_2^2$$

$$= m_1\left(\frac{m_1}{m_2} - 1\right) u_1^2 + 2 m_1 u_1 u_2$$

$$v_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = 4\left(\frac{m_1}{m_2}\right)^2 (m_1 u_1 + m_2 u_2)^2 - 4\left(1 + \frac{m_1}{m_2}\right) m_1 \left[m_1\left(\frac{m_1}{m_2} - 1\right) u_1^2 + 2 m_1 u_1 u_2\right]$$

$$= 4 m_1^2 \left[\left(\frac{m_1}{m_2}\right)^2 u_1^2 + \frac{2 m_1}{m_2} u_1 u_2 + u_2^2 - \left(\left(\frac{m_1}{m_2}\right)^2 - 1\right) u_1^2 - 2\left(1 + \frac{m_1}{m_2}\right) u_1 u_2\right]$$

$$= 4m_1^2\left[u_1^2 - 2u_1u_2 + u_2^2\right] = \left(2m_1(u_1-u_2)\right)^2$$

$$V_1 = \frac{\cancel{2}\cdot\frac{m_1}{m_2}(m_1u_1+m_2u_2) \pm \cancel{2}\cancel{m_1}(u_1-u_2)}{\cancel{2}\left(1+\frac{m_1}{m_2}\right)\cancel{m_1}}$$

$$V_1 = \frac{m_1u_1+m_2u_2 \pm \left(m_2(u_1-u_2)\right)}{m_1+m_2}$$

$$V_1 = \frac{(m_1+m_2)u_1 + m_2u_2 - m_2u_2}{m_1+m_2}, \quad \frac{(m_1-m_2)u_1 + 2m_2u_2}{m_2+m_1}$$

$$\boxed{V_1 = \overset{①}{u_1}, \quad \overset{②}{\frac{(m_1-m_2)u_1 + 2m_2u_2}{m_1+m_2}}}$$

$$\boxed{V_2 = \frac{m_1u_1+m_2u_2 - m_1u_1}{m_2} = u_2 \qquad ①}$$

① : nothing happened!,

$$= \frac{m_1u_1+m_2u_2}{m_2} - \frac{m_1}{m_2}, \quad \frac{(m_1-m_2)u_1 + 2m_2u_2}{m_1+m_2}$$

$$= \frac{(m_1u_1+m_2u_2)(m_1+m_2) - m_1(m_1-m_2)u_1 - 2m_1m_2u_2}{m_2(m_1+m_2)}$$

$$= \frac{m_1(m_1+m_2)u_1 + m_2(m_1+m_2)u_2 - m_1(m_1-m_2)u_1 - 2m_1m_2u_2}{m_2(m_1+m_2)}$$

$$= \frac{2m_1\cancel{m_2}u_1 + \cancel{m_2}(m_2-m_1)u_2}{\cancel{m_2}(m_1+m_2)}$$

$$\boxed{V_2 = \frac{(m_2-m_1)u_2 + 2m_1u_1}{m_1+m_2} \qquad ②}$$

$$\boxed{\text{There is an easier way!}}$$

no need for quadratic equation!

$\Rightarrow$ Work in a new reference frame.

AKA "Center of Mass" or "Center of Momentum" frame.

"Lab Frame"

$$V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$u_{1c} = u_1 - V \qquad\qquad u_{2c} = u_2 - V$$

$$= \frac{\cancel{m_1 u_1} + m_2 u_1 - \cancel{m_1 u_1} - m_2 u_2}{m_1 + m_2} \qquad \frac{m_1 u_2 + \cancel{m_2 u_2} - m_1 u_1 - \cancel{m_2 u_2}}{m_1 + m_2}$$

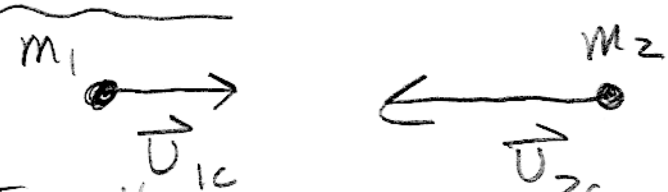
$$u_{1c} = \frac{m_2}{m_1 + m_2}(u_1 - u_2) \qquad u_{2c} = -\frac{m_1}{m_1 + m_2}(u_1 - u_2)$$

$$\boxed{\text{works in 3-d!}}$$

$$\vec{u}_{1c} = \frac{m_2}{m_1 + m_2}\underbrace{(\vec{u}_1 - \vec{u}_2)}_{\vec{u}} \qquad \vec{u}_{2c} = -\frac{m_1}{m_1 + m_2}\underbrace{(\vec{u}_1 - \vec{u}_2)}_{\vec{u}}$$

note: $$m_1 \vec{u}_{1c} = \underbrace{\frac{m_1 m_2}{m_1 + m_2}}_{\mu} \vec{u} = -m_2 \vec{u}_{2c} = \underbrace{\frac{-m_1 m_2}{m_1 + m_2}}_{\mu} \vec{u}$$

in CM frame



(as drawn, $m_2 < m_1$)

"CM Frame"

$$m_1 \vec{u}_{1c} + m_2 \vec{u}_{2c} = 0$$

After collision,

$$m_1\vec{V}_{1c} + m_2\vec{V}_{2c} = 0 \quad \text{still!}$$

Look at energy equation

$$\frac{1}{2}m_1|\vec{U}_{1c}|^2 + \frac{1}{2}m_2|\vec{U}_{2c}|^2 = \frac{1}{2}m_1|\vec{V}_{1c}|^2 + \frac{1}{2}m_2|\vec{V}_{2c}|^2 + Q$$

In center of mass frame

$$|\vec{U}_{2c}| = \left(\frac{m_1}{m_2}\right)|\vec{U}_{1c}| \qquad |\vec{V}_{2c}| = \left(\frac{m_1}{m_2}\right)|\vec{V}_{1c}|$$

$$\frac{1}{2}m_1\left(1+\frac{m_1}{m_2}\right)|\vec{U}_{1c}|^2 = \frac{1}{2}m_1\left(1+\frac{m_1}{m_2}\right)|\vec{V}_{1c}|^2 + Q$$

need to know to proceed.

elastic: $Q = 0$

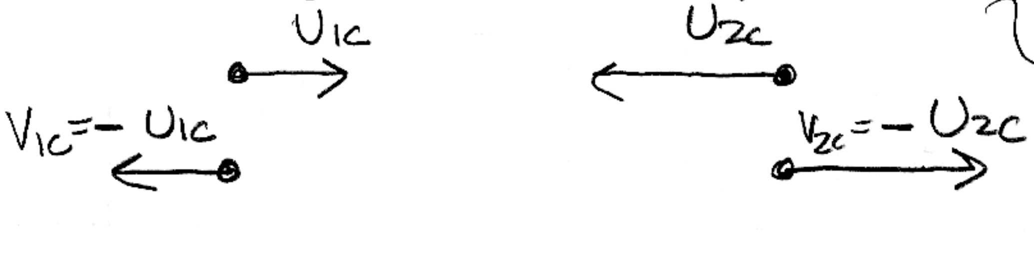
$$\boxed{\text{then } \quad |\vec{U}_{1c}| = |\vec{V}_{1c}| \qquad |\vec{U}_{2c}| = |\vec{V}_{2c}|}$$

1-d $\Rightarrow$

	something	nothing
	$V_{1c} = -U_{1c}$,	$+U_{1c}$
	$V_{2c} = -U_{2c}$,	$+U_{2c}$

2-d, 3-d ... deal with angles

1-d: gets main point.



Center of mass frames.

Back to Lab

$$V_1 = -U_{1c} + V \qquad\qquad V_2 = -U_{2c} + V$$

$$= -\frac{m_2}{m_1+m_2}(U_1 - U_2) + \frac{m_1 U_1 + m_2 U_2}{m_1+m_2}$$

$$\boxed{V_1 = \frac{(m_1-m_2)U_1 + 2m_2 U_2}{m_1+m_2}}$$

$$V_2 = \frac{m_1}{m_1+m_2}(U_1 - U_2) + \frac{m_1 U_1 + m_2 U_2}{m_1+m_2}$$

$$\boxed{V_2 = \frac{(m_2-m_1)U_2 + 2m_1 U_1}{m_1+m_2}}$$

Important case: $m_2 \gg m_1$ (2 problems on PS#9)

$$V_1 \Rightarrow \frac{-m_2 U_1 + 2m_2 U_2}{m_2} \qquad V_2 \Rightarrow \frac{m_2 U_2}{m_2} = U_2$$

$$= \underbrace{-U_1 + 2U_2}$$

but $U_2 = -|U_2|$ as shown..