(2) $U(\infty) = 0$

$$U(\infty) = \underbrace{\frac{-G M_e m}{\infty}}_{0} + \alpha = 0$$

$\alpha = 0$

(use in astrophysics

Springs

$\ell =$ equilibrium length of spring

$$U(x) = -\int \left(-k(x-\ell)\right) dx + \alpha$$

$x' = x - \ell$

$dx' = dx$

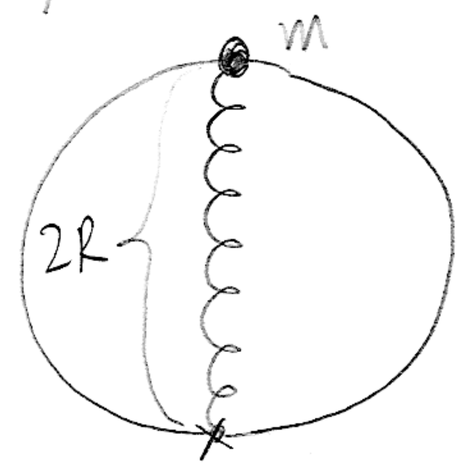
$$= +\int kx' \, dx' + \alpha$$

$$= \frac{1}{2} k x'^2 + \alpha$$

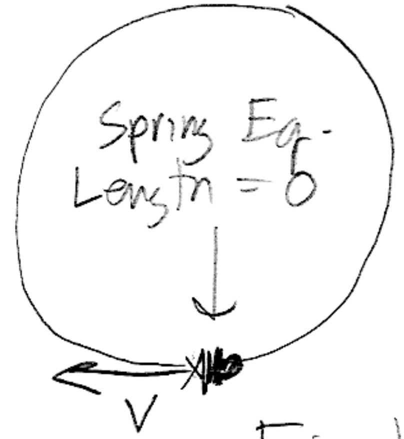
$$= \frac{1}{2} k (x-\ell)^2 + \alpha$$

usually: $U(\ell) = 0$, so $\alpha = 0$

$$\boxed{U(x) = \frac{1}{2} k (x-\ell)^2}$$



Initial



Final

Initial:

$$U = 2mgR + \frac{1}{2}k(2R)^2$$

$$K = 0$$

Final:

$$U = 0$$

$$K = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = 2mgR + 2kR^2$$

$$\boxed{v = 2\sqrt{gR + \underbrace{\frac{k}{m}}_{\omega^2}R^2}}$$

Inverse of Potential Energy

$$E = K + U(\vec{r})$$

constant $\quad K = \frac{1}{2}m|\vec{v}|^2 + U(\vec{r})$

$$U(\vec{r}) = -\int \vec{F}(\vec{r}) \cdot d\vec{r}$$

go to 1-d

$$U(x) = -\int F(x)\, dx$$

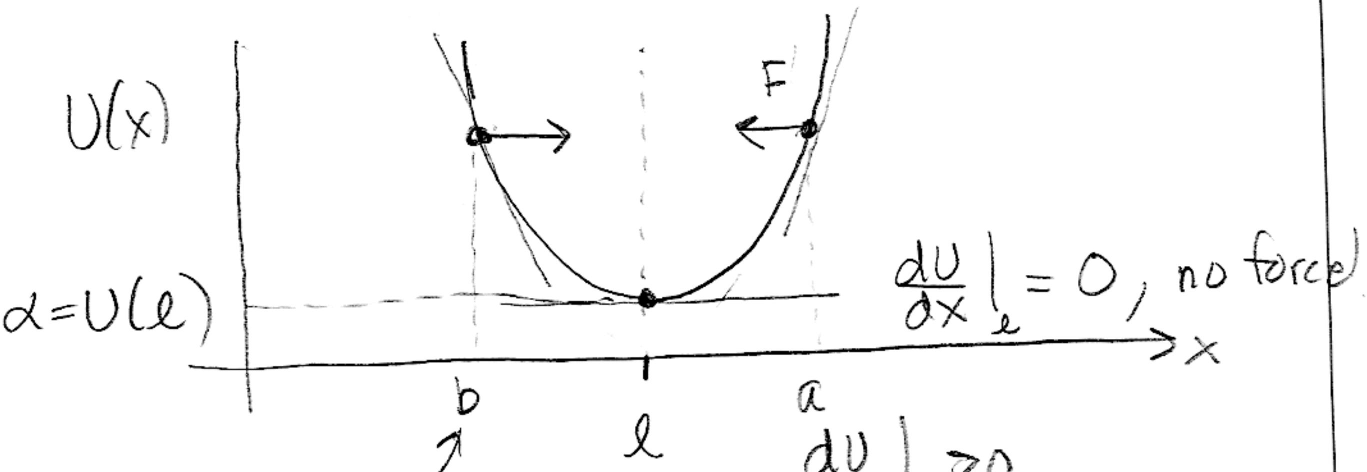
so,
$$-\frac{dU}{dx} = F(x)$$

3-d

$$\underbrace{-\left(\hat{\imath}\frac{\partial U}{\partial x} + \hat{\jmath}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)}_{-\vec{\nabla} U = \vec{F}} = \vec{F}$$

Interpreting Plots of U(x)

Simple harmonic oscillator:



$$-\left.\frac{dU}{dx}\right|_b > 0$$

positive force
back toward minimum

$$\left.\frac{dU}{dx}\right|_a > 0$$

$$-\left.\frac{dU}{dx}\right|_a < 0 \quad \ldots \text{ negative force (restoring) back toward minimum}$$

"Restoring": when

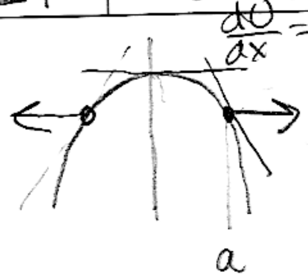
$$\left.\frac{dU}{dx}\right|_e = 0, \quad \text{what about } \left.\frac{d^2U}{dx^2}\right|_e ?$$

visually: $\cup$ $+$ at e

$$\left.\frac{d^2U}{dx^2}\right|_e > 0 \quad \text{local minimum}$$

"stable, restoring force"

look at



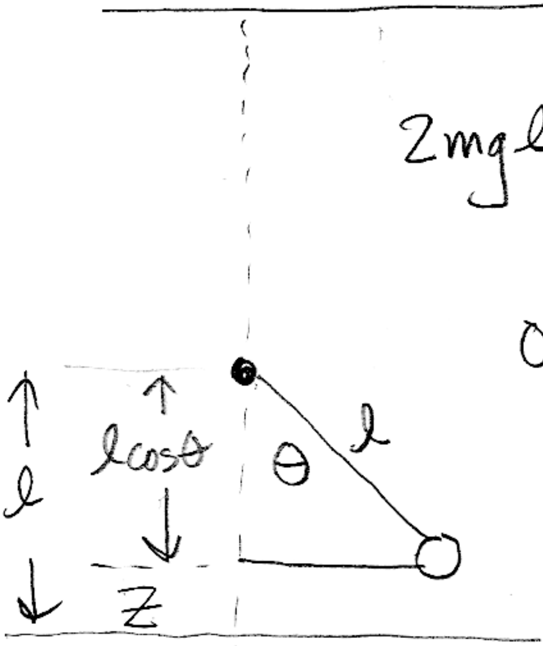
$-\frac{dU}{dx}\Big|_a > 0$ ... positive force <u>away</u> from minimum

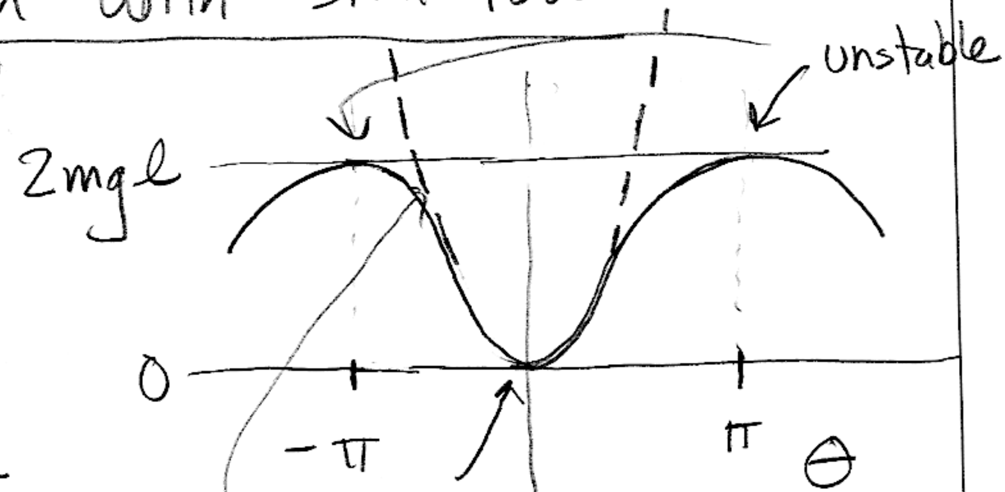
$-\frac{dU}{dx} < 0$ <u>away</u> from minimum
negative

This time, moving away from point where $\frac{dU}{dx}=0$ gives rise to force that pushes you <u>away</u> from $\frac{dU}{dx}=0$ point

$\frac{d^2U}{dx}\Big|_l < 0$ ... local energy maximum, "unstable".

<u>Pendulum with stiff rod</u>





stable $= \theta_0$

"parabolic approximation" ...

$z = l - l\cos\theta$
$= l(1-\cos\theta)$

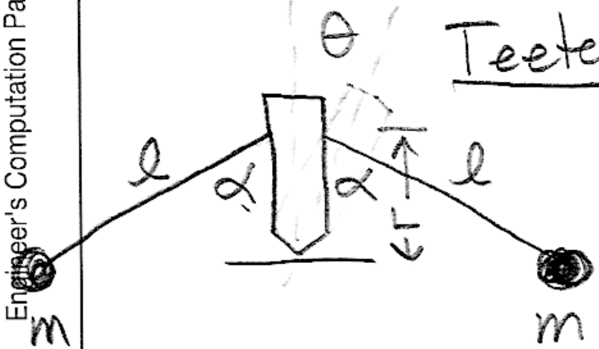
$U(\theta) = mgl(1-\cos\theta)$

$U(z) = mgz$

$$U(\theta) \simeq U(\theta_0) + (\theta-\theta_0)\frac{dU}{d\theta}\Big|_{\theta=\theta_0} + \frac{1}{2}(\theta-\theta_0)^2\frac{d^2U}{d\theta^2}\Big|_{\theta=0^\circ} + (\text{ignore})$$

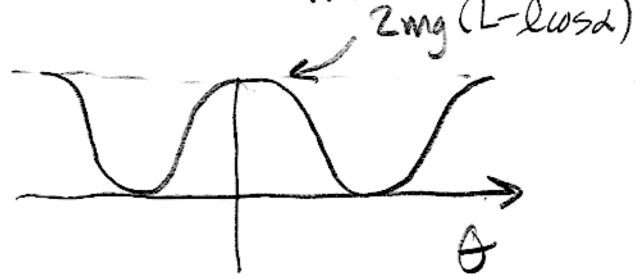
Can approximate, for small excursions about a stable equilibrium, any potential as a simple harmonic oscillator

Teeter Toy (see p. 175)

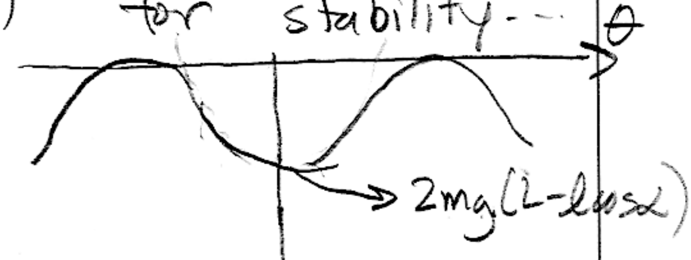


$$U(\theta) = 2mg\cos\theta\,\underbrace{(L - l\cos\alpha)}_{\text{must be } -}$$

for stability...



when $L - l\cos\alpha > 0$



when $L - l\cos\alpha < 0$

In either case...

$$\cos\theta \simeq \cos\theta_0 + (\theta-\theta_0)(-\sin\theta_0) + \frac{1}{2}(\theta-\theta_0)^2(-\cos\theta_0)$$

$$\theta_0 = 0 \quad \simeq 1 - \frac{1}{2}\theta^2$$

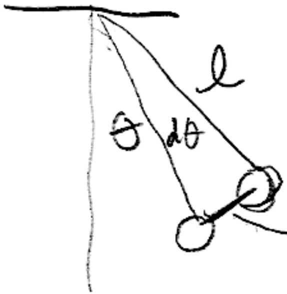
<u>Pendulum</u>: $U(\theta) \simeq mgl(1-\cos\theta) \simeq \frac{1}{2}mgl\,\theta^2$

<u>Teeter Toy</u>: $U(\theta) \simeq 2mg(L - l\cos\alpha)\left(1 - \frac{1}{2}\theta^2\right)$

< 0 constant

Energy Diagram

Since Total Energy E constant, following diagram graphically represents situation...



distance $= \ell \, d\theta$

speed $= \ell \dot{\theta}$

$$K = \frac{1}{2} m \cdot (\text{speed})^2 = \frac{1}{2} m \ell^2 \dot{\theta}^2, \quad B = m\ell^2$$

$$U \qquad\qquad = \frac{1}{2} m g \ell \theta^2, \quad A = mg\ell$$

$$\boxed{\omega = \sqrt{\frac{A}{B}} = \sqrt{\frac{mg\ell}{m\ell^2}} = \sqrt{\frac{g}{\ell}}}$$