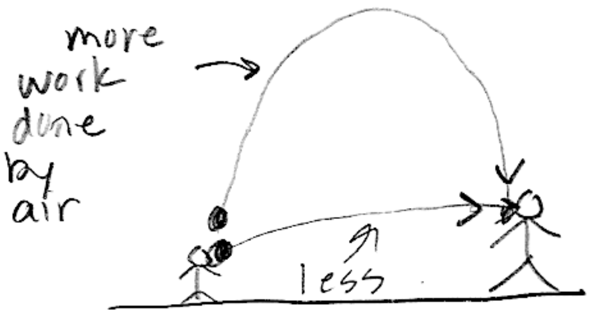
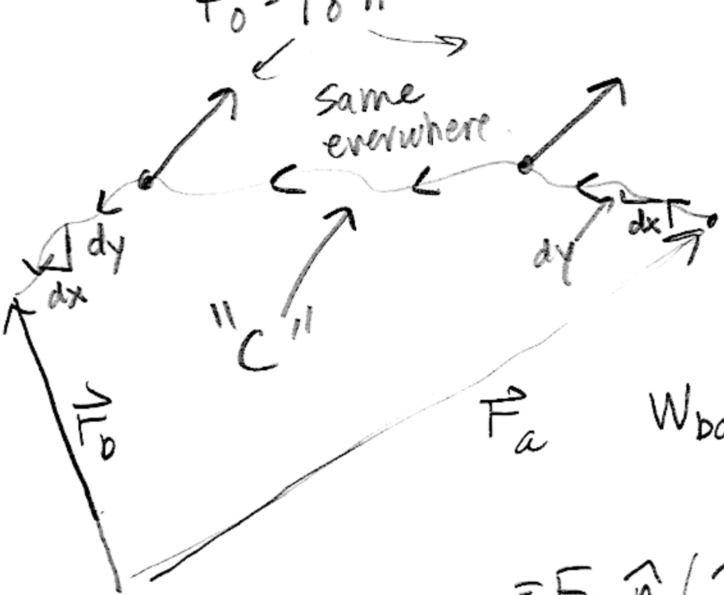
This second case is very important. In many interesting cases, the line integral does not depend on the specific line ... just its endpoints. The class of force fields that give rise to "line independence" are known as "conservative" force fields.

Conservative	Non-Conservative
Gravity	Friction
Normal Forces	Viscosity
Electrostatic	Air Resistance.
(magnetism) (Steady State)	
Weak (")	
Strong (")	



Constant Force

$\vec{F}_0 = F_0 \hat{n}$ $\hat{n}$



$$W_{ba} = \int_{\vec{r}_a}^{\vec{r}_b} \underbrace{F_0 \hat{n}}_{\text{constant!}} \cdot d\vec{r}$$

$$= F_0 \hat{n} \cdot \int_{\vec{r}_a}^{\vec{r}_b} d\vec{r}$$

$$d\vec{r} = \hat{\imath} \cdot dx + \hat{\jmath}\, dy + \hat{k}\, dz$$

$$W_{ba} = F_0 \hat{n} \left(\hat{\imath} \int_{\vec{r}_a}^{\vec{r}_b} dx + \hat{\jmath} \int_{\vec{r}_a}^{\vec{r}_b} dy + \hat{k} \int_{\vec{r}_a}^{\vec{r}_b} dz \right)$$

$$= F_0 \hat{n} \cdot \left(\hat{\imath} \cdot (x_b - x_a) + \hat{\jmath} (y_b - y_a) + \hat{k} (z_b - z_a) \right)$$

$$\boxed{W_{ba} = F_0\,\hat{n}\cdot(\vec{r}_b - \vec{r}_a)}$$

depends only on end points!

Gravitational Force

$$W_{ba} = \int_{\vec{r}_a}^{\vec{r}_b}\left(-G\frac{M_e m}{r^2}\right)dr = \underbrace{G M_e}_{=gR_e^2} m\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$\boxed{W_{ba} = mgR_e^2\left(\frac{1}{r_b} - \frac{1}{r_a}\right)}$$

when $r_b > r_a$, $W_{ba} < 0$.

path independent

Path-dependent work is hard to evaluate! (at this point, not high priority).

Friction on table ...

When $\vec{F}(\vec{r})$ is conservative (can't tell yet when! Just Gravity is so far) then....

$$\int_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r})\cdot d\vec{r} \equiv \underbrace{\text{function of } \vec{r}_b}_{\text{call } -U(\vec{r}_b)} - \underbrace{\text{function of } \vec{r}_a}_{\text{call } -U(\vec{r}_a)}$$

$$= \underbrace{-U(\vec{r}_b) + U(\vec{r}_a)}_{-\text{ sign annoying now!}}$$

$$= K_b - K_a$$

$$K_a + U(\vec{r}_a) = K_b + U(\vec{r}_b)$$

$$\underbrace{\tfrac{1}{2}mv_a^2 + U(\vec{r}_a)}_{\text{depends on } a} = \underbrace{\tfrac{1}{2}mv_b^2 + U(\vec{r}_b)}_{\text{depends on } b} = E \text{ total energy}$$

$U(\vec{r}) \rightarrow$ called the potential energy.

<u>Differences</u> in U are meaningful

$$U(\vec{r}_a) - U(\vec{r}_b) = K_b - K_a$$

$$= \underbrace{U(\vec{r}_a) + \alpha}_{\text{could call } U(\vec{r}_a) \text{ too!}} - U(r_b) - \alpha$$

$$U(\vec{r}) = -\int \vec{F}(\vec{r})\cdot d\vec{r} + \underbrace{\text{arbitrary constant}}_{\text{choose convenience.}}$$

Gravity

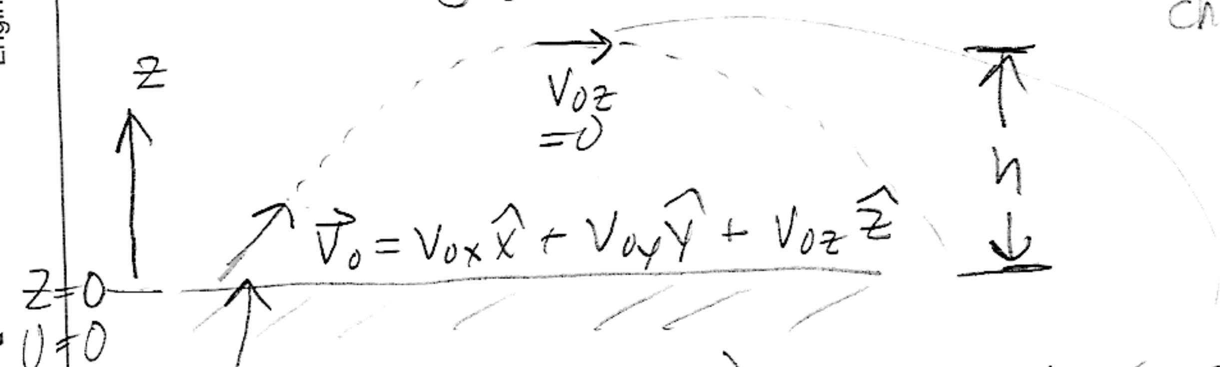
Near Earth's Surface

$$\vec{F} = -mg\,\hat{z}$$

$$U = -\int(-mg\,\hat{z})\cdot(\cancel{\hat{x}\cdot dx} + \cancel{\hat{y}\cdot dy} + \hat{z}\cdot dz)$$

$$= mg\int dz = mgz + \boxed{\text{constant}}$$

choose for convenience



initial $\quad E = \frac{1}{2}m|\vec{v}_0|^2 + mg\cancel{z}_{0} = \frac{1}{2}m(\cancel{v_{0x}^2} + \cancel{v_{0y}^2} + v_{0z}^2)$

"final" $\quad E = \frac{1}{2}m(\cancel{v_{0x}^2} + \cancel{v_{0y}^2} + \cancel{v_z^2(h)}^{\,0}) + mg(z=h)$

$$\frac{1}{2}mv_{0z}^2 = mgh$$

$$h = \frac{v_{0z}^2}{2g}$$

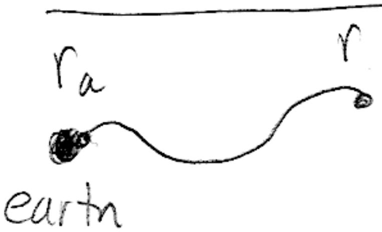
also

$$\frac{1}{2}\cancel{m}(v_{0z}^2 - v_z^2) = \cancel{m}gz$$

$$\frac{1}{2}v_{0z}^2 - gz = \frac{1}{2}v_z^2$$

$$v_z = \sqrt{v_{0z}^2 - 2gz}$$

Far From Earth



$$U = -\int \vec{F}(\vec{r}) \cdot dr$$

$$= G M_e m \int \frac{dr}{r^2} =$$

$$= -\frac{G M_e m}{r} + \alpha$$

2 choices of α

① $U(R_e) = 0$ (as in last problem)

$$-\frac{G M_e m}{R_e} + \alpha = 0 \qquad \alpha = \frac{G M_e m}{R_e}$$

$$U(r) = G M_e m \left(\frac{-1}{r} + \frac{1}{R_e} \right)$$

$$r = R_e + h \qquad h \ll R_e$$

$$\frac{1}{R_e + h} = \frac{1}{R_e} \left(\frac{1}{1 + h/R_e} \right)$$

$$\simeq \frac{1}{R_e} \left(1 - \left(\frac{h}{R_e}\right) + \left(\frac{h}{R_e}\right)^2 \cdots \right)$$

$$= \frac{1}{R_e} - \frac{h}{R_e^2} + \frac{h^2}{R_e^3} \cdots$$

$$-\frac{1}{r} + \frac{1}{R_e} = -\cancel{\frac{1}{R_e}} + \frac{h}{R_e^2} - \frac{h^2}{R_e^3} + \cancel{\frac{1}{R_e}}$$

$$U(r) \cong \frac{G M_e m}{R_e^2} \left[h - \frac{h^2}{R_e} \right]$$

$$U(r) \cong mgh \left[1 - \frac{h}{R_e} \right]$$