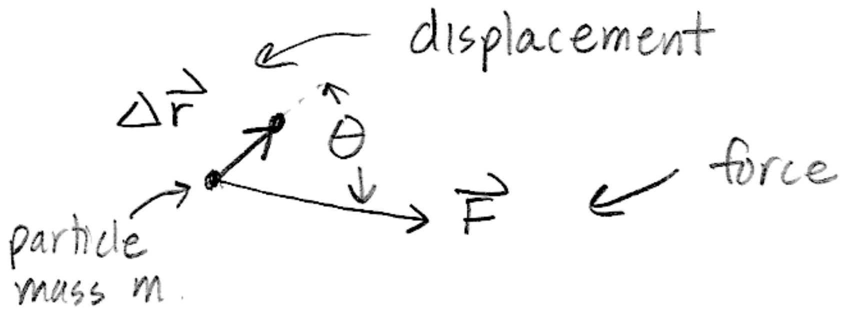
Work - Energy in > 1 Dimension

In 1 dimension, F + displacement always either parallel (+ sign) or antiparallel (- sign)

$$\underbrace{\int F(x)\,dx}_{\text{either}}$$

+	$\xrightarrow{dx}$	$\xrightarrow{F}$
0	$\rightarrow$	0
-	$\rightarrow$	$\xleftarrow{}$ F

In 2 or 3 dimensions, something new pops up:



Look at $\vec{F}\cdot\Delta\vec{r}$ as replacement in > 1 d.

note: $\theta = 90°$, no work!

Turns out : $\vec{F}\cdot\Delta\vec{r} = \frac{1}{2} m \frac{d}{dt}(v^2)\,\Delta t$

$v^2 = \vec{v}\cdot\vec{v}$ (where dot product goes).

$$\vec{F}\cdot\Delta\vec{r} = m\frac{d\vec{v}}{dt}\cdot\Delta\vec{r} = m\frac{d\vec{v}}{dt}\cdot\vec{v}\,\Delta t$$

$$\frac{d(v^2)}{dt} = \frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt} = 2\frac{d\vec{v}}{dt}\cdot\vec{v}$$ chain rule

or, $$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d}{dt}(v_x^2+v_y^2+v_z^2) = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt} + 2v_z\frac{dv_z}{dt} = 2\frac{d\vec{v}}{dt}\cdot\vec{v}$$

$$\boxed{\vec{F}\cdot\Delta\vec{r} = m\cdot\frac{1}{2}\frac{d}{dt}(v^2)\,\Delta t}$$

to go forward, need concepts of

(1) a <u>Force Field</u>

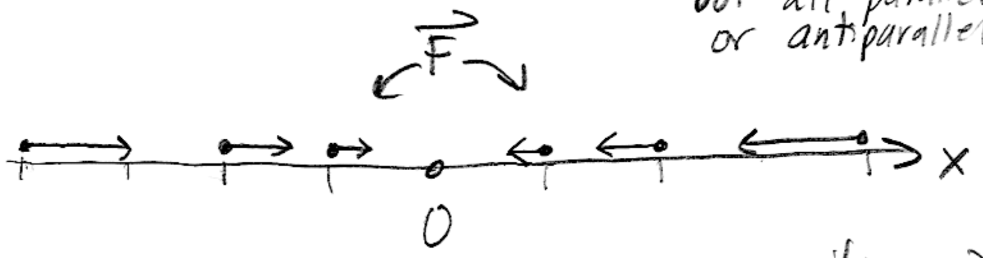
(2) a <u>line integral</u>

<u>Force Field</u>: at every point in space, associate a force.

<u>Examples</u>:

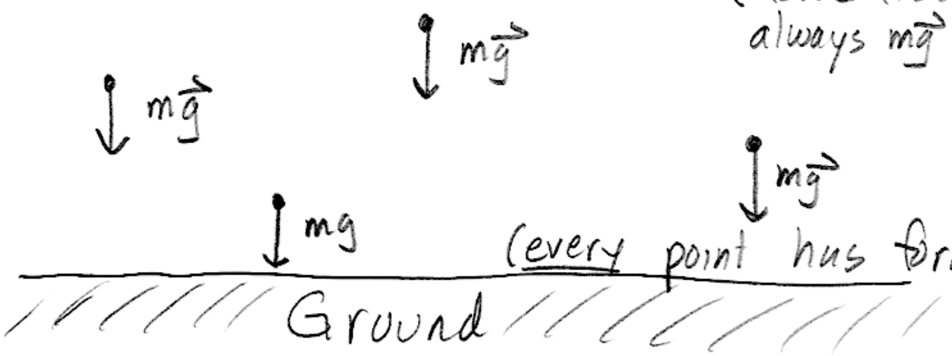
<u>spring</u> (1-d)

(not uniform, but all parallel or antiparallel)



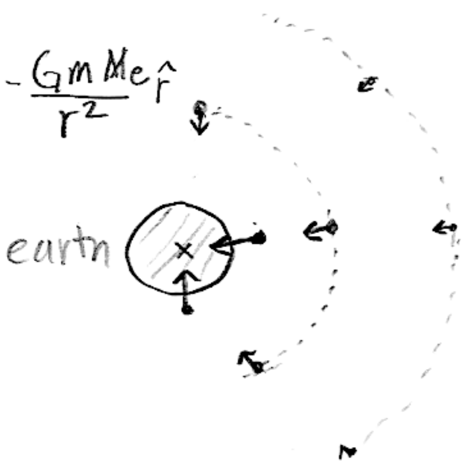
gravity near earth (actually 3-d, draw 2-d)

(uniform force field always $m\vec{g}$)



(<u>every</u> point has force)

gravity far from earth (3-d, draw 2-d)



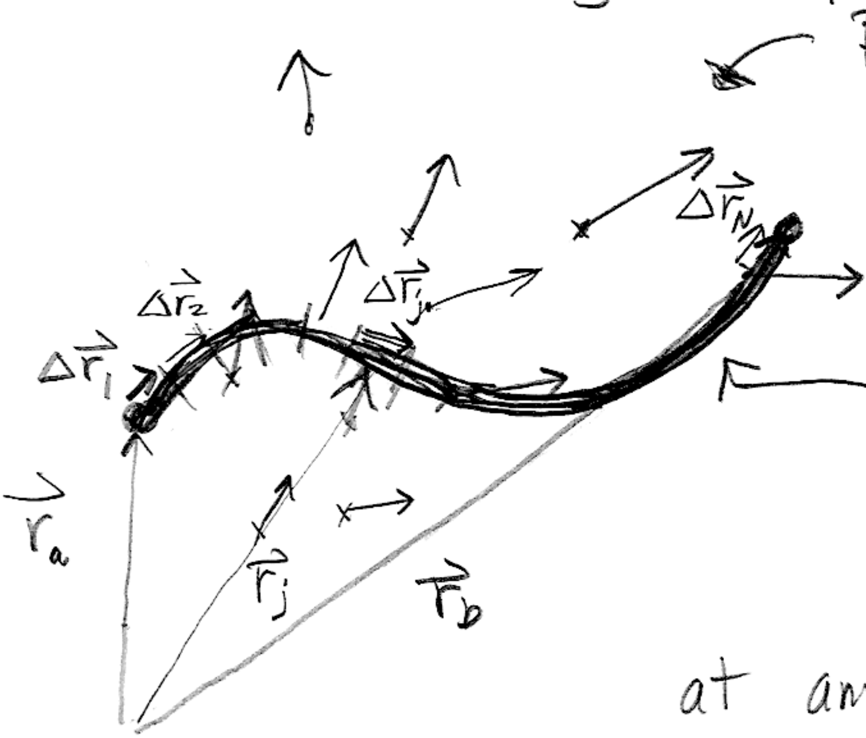
now varies both in magnitude and direction

Line Integral: you are familiar with a special case... the "normal" integral.... x-axis is the "line" and whatever function is what you integrate!

Generalize this:

(A) Choose a line (not necessarily straight!) in 2-d or 3-d.

(B) Specify some function that depends on stuff along the path.



the line: trajectory of a particle!

at any spot, can compute:

$$\vec{F}(\vec{r}_j)\cdot\Delta\vec{r}_j = \frac{1}{2}m\frac{d}{dt}(v_j^2)\Delta t_j$$

$$\sum_{j=1}^{N}\vec{F}(\vec{r}_j)\cdot\Delta\vec{r}_j = \frac{1}{2}m\sum_{j=1}^{N}\frac{d}{dt}(v_j^2)\Delta t_j$$

let $\Delta \to 0$

C reminds a specific "curve" →

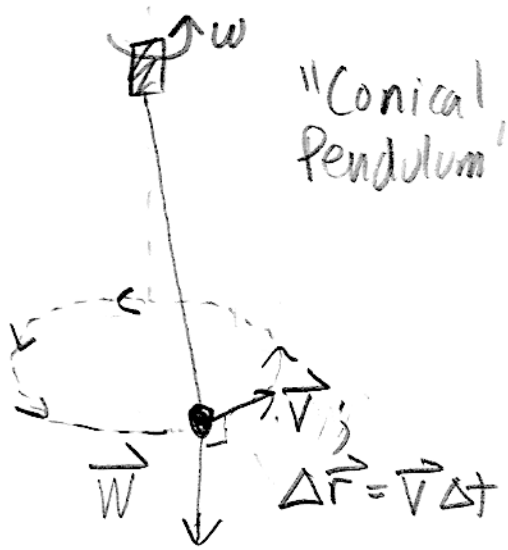
$$\int_{C\,\vec{r}_a}^{\vec{r}_b}\vec{F}(\vec{r})\cdot d\vec{r} = \frac{1}{2}m\int_{t_a}^{t_b}\frac{d}{dt}(v^2)\,dt$$

$$\underbrace{\oint_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) \cdot d\vec{r}}\; = \;\underbrace{\tfrac{1}{2} m \left(v_b^2 - v_a^2\right)}$$

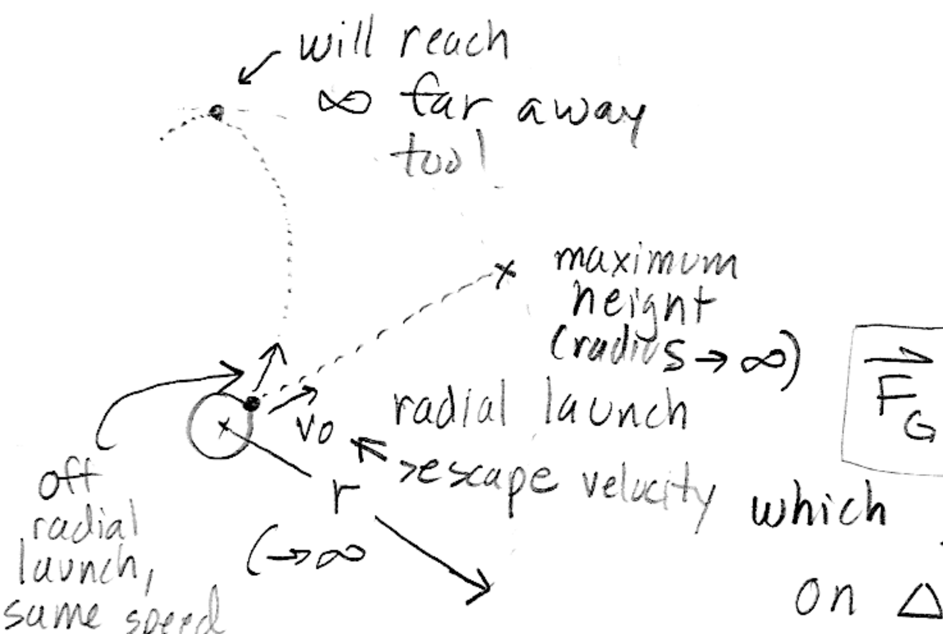
- depends on (curve) line you pick
- vectors important (Hard! in general)
- net force.

EASY

$$W_{ba} = K_b - K_a$$



In going any amount around the circle, does gravity do any work? No: $\vec{W} \cdot \Delta\vec{r}$
$= \vec{W} \cdot \vec{v}\, \Delta t$
$= 0$



$$\vec{F}_G = -\frac{G M_e m}{r^2}\,\hat{r}$$

(really need conservation of angular momentum) hold

$$\boxed{\vec{F}_G \cdot \Delta\vec{r} = -\frac{G M_e m}{r^2}\,\Delta r}$$

which does not depend on $\Delta\theta$! (does depend on Δr)