$$0 = M(\vec{v}+\Delta\vec{v}) + \Delta m(\vec{v}+\Delta\vec{v}+\vec{u}) - (M+\Delta m)\vec{v}$$

$$= M\Delta\vec{v} + \Delta m\,\vec{u}$$

or $M\dfrac{\Delta\vec{v}}{\Delta t} = -\dfrac{\Delta m}{\Delta t}\vec{u}$

so $\boxed{M\dfrac{d\vec{v}}{dt} = -\dfrac{dm}{dt}\vec{u}}$ $\underbrace{\dfrac{dm}{dt} = -\dfrac{dM}{dt}}$

mass lost = loss of mass from rocket.

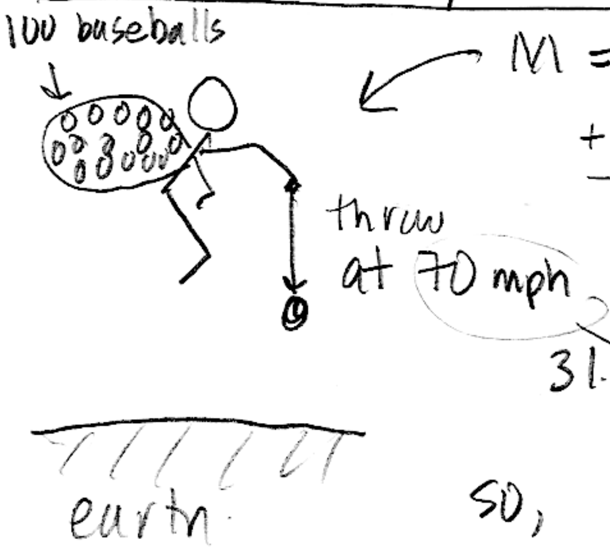
in gravitational field..

$$M\vec{g} = M\frac{d\vec{v}}{dt} - \frac{dM}{dt}\vec{u}$$

$$\boxed{\text{or } \frac{d\vec{v}}{dt} = \vec{g} + \frac{\vec{u}}{M}\frac{dM}{dt}}$$

Levitation By Throwing Baseballs

$\dfrac{dv}{dt} = 0$



$M = 60$ kg (body)
$+ 14.5$ kg (100 baseballs)
74.5 kg.

31.3 m/s.

so, $\dfrac{dM}{dt} = \left|\dfrac{g}{u/M}\right| = \left|\dfrac{9.8}{31.3/74.5}\right|$

$$\frac{dM}{dt} = 23.3 \text{ kg/s}$$

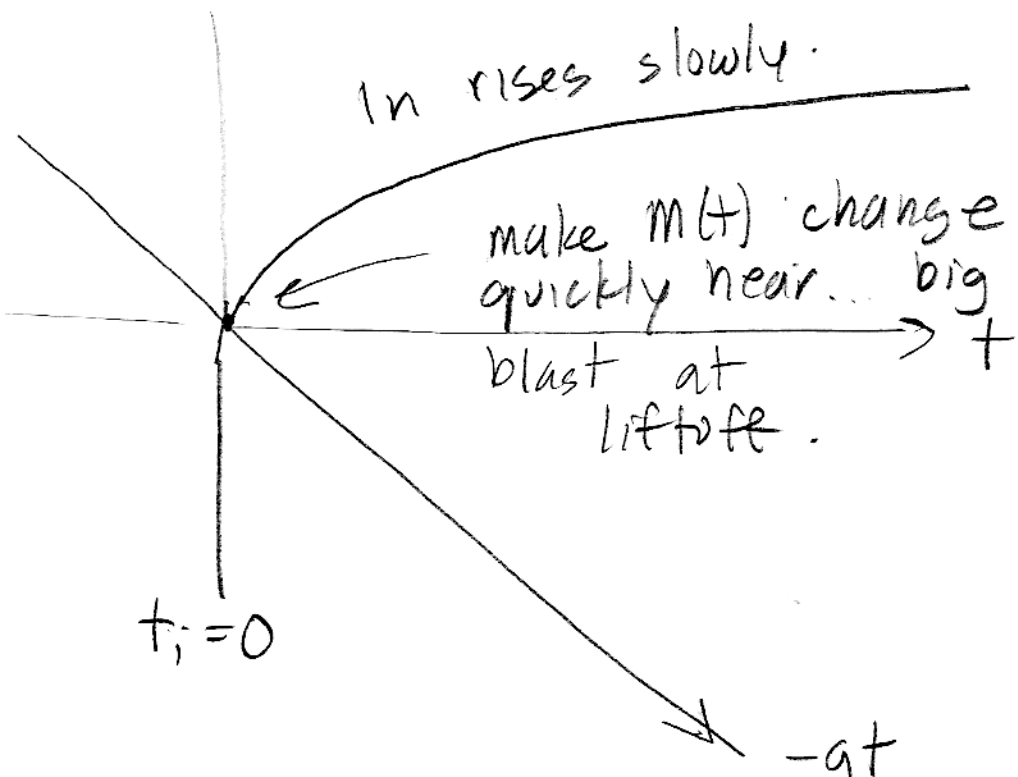
1 baseball $\simeq$ 0.145 kg $\nearrow$ $\dfrac{23.3}{0.145} = 161 \dfrac{\text{baseballs}}{\text{second}}$

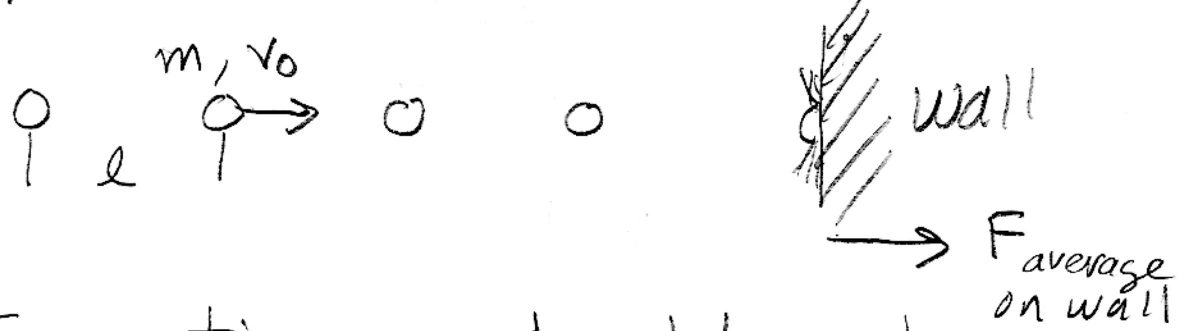
<u>Rocket</u>: vertical...

$$\frac{dv_y}{dt} = -g + \frac{u_y}{M}\frac{dM}{dt}$$

$$dv_y = -g\,dt + u_y \frac{dM}{M}$$

$$\int_{v_i(t)}^{v_f(t)} dv_y = v_f - v_i = \underbrace{-g(t_f - t_i)}_{-\text{ linear}} + \underbrace{u_y \ln\left(\frac{M_f(t)}{M_i(t)}\right)}_{+\text{ logarithmic}}$$





Every time a drop hits, there is a momentum transfer to drop of:

$$\Delta P = m \cdot 0 - m \cdot v_0 = -mv_0$$

This happens every how often?

$$\text{time between drops} = T = \frac{l}{v_0}$$

On average, the momentum transfer per unit time, which is force on the drops, is:

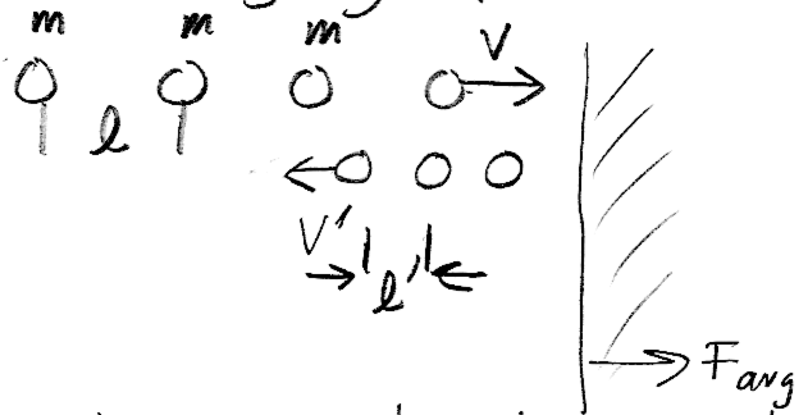
$$\frac{\Delta P}{T} = \frac{-mv_0}{(l/v_0)} = -\frac{m}{l} v_0^2$$

by newton's third law, this is − the average force on the wall

$$F_{average} = \frac{m}{l} v_0^2 \quad \propto v_0^2$$

One power: momentum
second power: rate

What if outgoing speed $\neq 0$?



<u>Key point</u>: mass leaving in time Δt must equal mass arriving.

so $$\cancel{m}\left(\frac{v\,\cancel{\Delta t}}{\ell}\right) = \cancel{m}\left(\frac{v'\,\cancel{\Delta t}}{\ell'}\right)$$

$$\frac{v}{\ell} = \frac{v'}{\ell'} \quad \text{or} \quad \frac{\ell}{v} = \frac{\ell'}{v'}$$

$$F_{avg} = F_{incoming} + F_{outgoing}.$$

$$= \underbrace{\frac{mv}{(\ell/v)}}_{\text{slow to stop.}} + \underbrace{\frac{mv'}{(\ell'/v' = \ell/v)}}_{\text{push off}}$$

$$\boxed{F_{avg} = \frac{m}{\ell}v(v+v')}$$

v' ranges from 0 to v