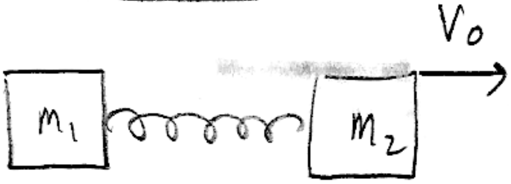
Initial Conditions



$t = 0$

note: $u = x_1' - x_2' + \ell = x_1 - x_2 + \ell$

$$\dot{u} = \dot{x}_1' - \dot{x}_2' = \dot{x}_1 - \dot{x}_2 = v_1 - v_2$$

$$\dot{u}(0) = v_1(0) - v_2(0) = -v_0$$

$$\dot{u}(t) = \omega A \cos\omega t - \omega B \sin\omega t$$

$$\dot{u}(0) = \omega A = -v_0 \quad , \quad A = -v_0/\omega$$

B? $\quad u(0) = x_1' - x_2' + \ell = 0 = B$

$$\dot{X} = \frac{m_1 \dot{x}_1 + m_2 \dot{x}_2}{m_1 + m_2} = \frac{m_2}{m_1 + m_2} v_0 \quad \text{(constant!)}$$

$$x_1 = X + \frac{\mu}{m_1}(x_1 - x_2)$$

$$\dot{x}_1 = \dot{X} + \frac{\mu}{m_1}(\dot{x}_1 - \dot{x}_2) = \dot{X} + \frac{\mu}{m_1}\dot{u}$$

$$= \frac{m_2}{m_1 + m_2}\Big(\underset{\uparrow \atop \dot{R}}{v_0} - \omega\frac{v_0}{\omega}\cos\omega t\Big)$$

$$\boxed{\dot{x}_1 = \frac{\mu}{m_1} v_0 (1 - \cos\omega t)}$$

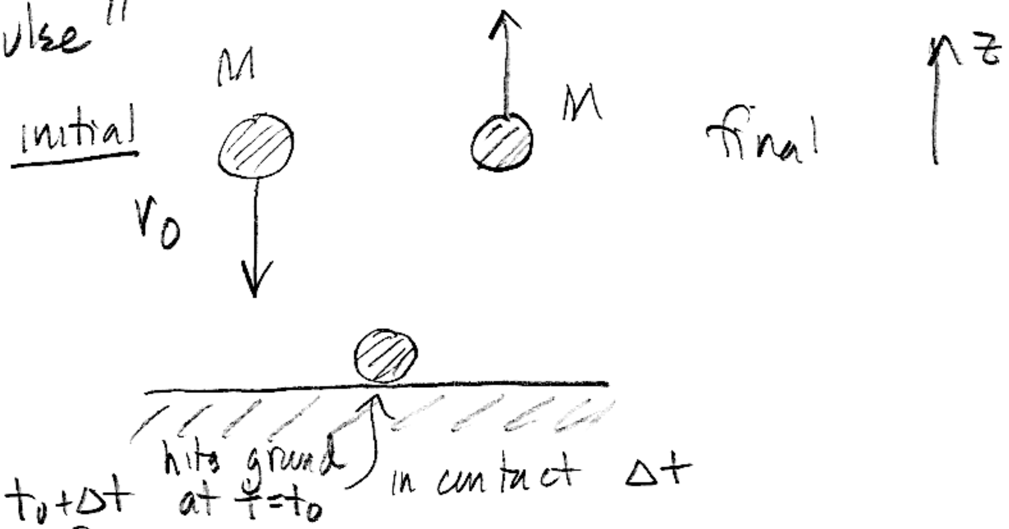
$$\dot{x}_2 = \dot{X} - \frac{\mu}{m_2}(\dot{x}_1 - \dot{x}_2) = \frac{m_2}{m_1 + m_2} v_0 + \frac{m_1}{m_1 + m_2} v_0 \cos\omega t$$

$$\boxed{\dot{x}_2 = v_0 \mu \left(\frac{1}{m_1} + \frac{\cos\omega t}{m_2}\right)}$$

Impulse

$$\vec{F} = \frac{d\vec{P}}{dt} \qquad \text{point particle}$$

$$\underbrace{\int_0^t \vec{F}\,dt}_{\text{"Impulse"}} = \int_0^t \frac{d\vec{P}}{dt}\,dt = \underbrace{\vec{P}(t) - \vec{P}(0)}_{\text{change in momentum}}$$



$$\int_{t_0}^{t_0+\Delta t} \vec{F}\,dt = \vec{P}(\text{final}) - \vec{P}(\text{initial})$$

$$= +Mv_0\hat{k} - (-Mv_0\hat{k})$$

$$\simeq \vec{F}\cdot\Delta t = 2Mv_0\hat{k}$$

$$\vec{F} \simeq \frac{2Mv_0}{\Delta t}\hat{k}$$

Ball...
$M = 0.2\,kg$
$v_0 = 8\,m/s$
$\Delta t \simeq 10^{-3}\,s$

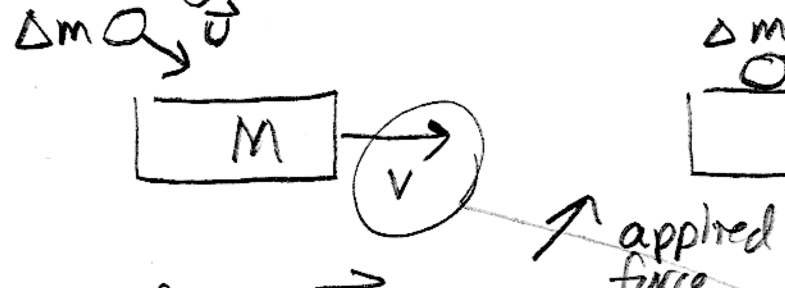
$$|\vec{F}| \simeq \frac{2 \times 0.2 \times 8}{10^{-3}} \simeq \underline{\underline{3.2 \cdot 10^3\,N}}$$

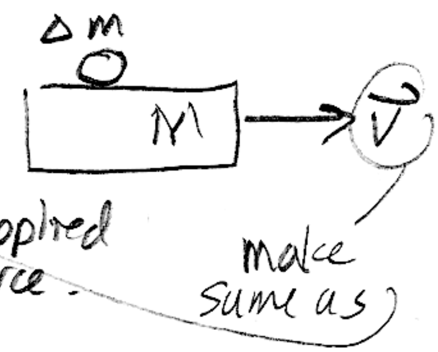
$|\vec{F}|$ causes broken bones.

When you fall... make Δt as large as possible!, Reduces risk of injury.

Getting hit....

Δt later





need force $\vec{F}$ to keep

$\vec{v}$ same...

initial: $\vec{P}(t) = M\vec{v} + \Delta m \vec{u}$

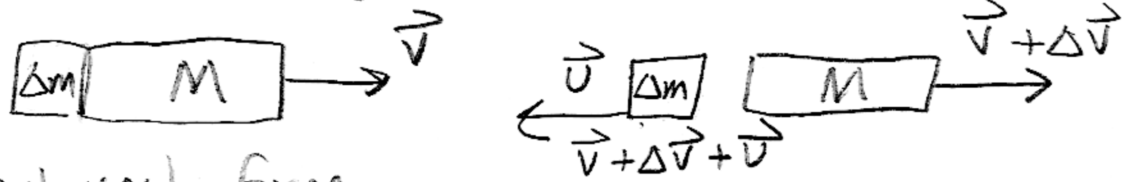
final $\vec{P}(t+\Delta t) = M\vec{v} + \Delta m \vec{v}$

$$\Delta\vec{P} = \vec{P}(t+\Delta t) - \vec{P}(t) = \Delta m(\vec{v}-\vec{u})$$

$$\text{so} \quad \vec{F} = \frac{\Delta \vec{P}}{\Delta t} = \frac{\Delta m}{\Delta t}(\vec{V}-\vec{U}) \rightarrow \frac{dm}{dt}(\vec{V}-\vec{U})$$

$$= 0 \text{ when } \vec{V} = \vec{U}$$

More interesting...



<u>no external force.</u>

$$0 = \vec{P}(\text{final}) - \vec{P}(\text{initial})$$

$$0 = M(\vec{v}+\Delta\vec{v}) + \Delta m(\vec{v}+\Delta\vec{v}+\vec{u}) - (M+\Delta m)\vec{v}$$

$$= M\Delta\vec{v} + \Delta m\,\vec{u}$$

or $$M\frac{\Delta\vec{v}}{\Delta t} = -\frac{\Delta m}{\Delta t}\vec{u}$$

so $$\boxed{M\frac{d\vec{v}}{dt} = -\frac{dm}{dt}\vec{u}}$$

$$\underbrace{\frac{dm}{dt} = -\frac{dM}{dt}}$$

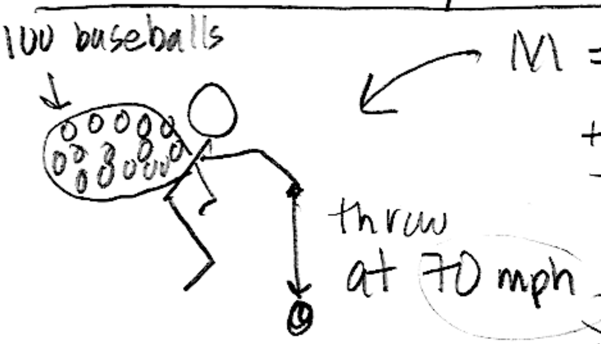
mass lost = loss of mass from rocket.

in gravitational field..

$$M\vec{g} = M\frac{d\vec{v}}{dt} - \frac{dM}{dt}\vec{u}$$

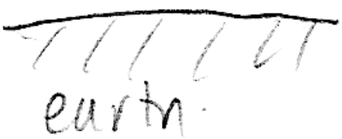
$$\boxed{\text{or}\quad \frac{d\vec{v}}{dt} = \vec{g} + \frac{\vec{u}}{M}\frac{dM}{dt}}$$

Levitation By Throwing Baseballs $\frac{dv}{dt} = 0$



$M = 60$ kg (body)
$+\ 14.5$ kg (100 baseballs)
74.5 kg.

31.3 m/s.



so, $$\frac{dM}{dt} = \left|\frac{g}{u/M}\right| = \left|\frac{9.8}{31.3/74.5}\right|$$

$$\frac{dM}{dt} = 23.3 \text{ kg/s}$$

1 baseball ≈ 0.145 kg $\nearrow$ $$\frac{23.3}{0.145} = 161 \frac{\text{baseballs}}{\text{second}}$$