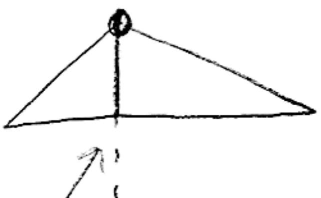
$$\lambda(x) = 2\,\frac{M}{L}\cdot\frac{x}{L}$$

(2) $$x_{cm} = \frac{1}{M}\int_0^L x\,\lambda(x)\,dx = \frac{1}{M}\int_0^L x\cdot 2\,\frac{M}{L}\,\frac{x}{L}\,dx$$

$$= \frac{2}{L^2}\int_0^L x^2\,dx = \frac{2}{3L^2}\,x^3\Big|_0^L$$

$$\boxed{x_{cm} = \frac{2}{3}L}$$

<u>2-d</u>: best way is to hang & intersect



c.m. along line.

(3-d

c.m.

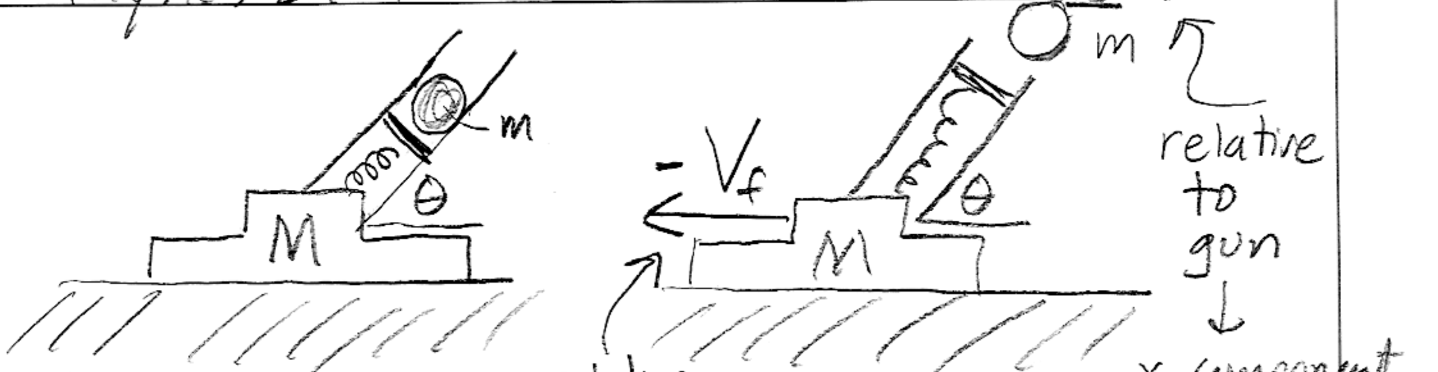
Conservation of Momentum

<u>System</u>: $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$

then $$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext,i}$$

what happens when $\sum \vec{F}_{ext,i} = 0$

$$\frac{d\vec{P}}{dt} = 0$$ } may happen for only 1 component



Initial

Final

is $v_0 \cos\theta - V_f$ (computed earlier)

Vertical : $\sum F_{ext,y} \neq 0$, unless you include the earth in the system

$\sum F_{ext,x} = 0$ however. (spring is an internal force).

$$\frac{dP_x}{dt} = 0 \Rightarrow P_x = \text{constant}$$

Initial : $P_x = M \cdot 0 + m \cdot 0 = 0$

Final : $-MV_f + m(v_0 \cos\theta - V_f) = 0$

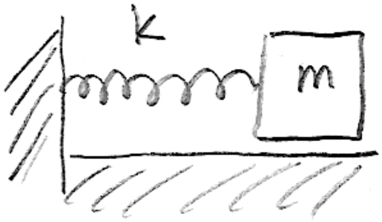
$$m v_0 \cos\theta = (M+m) V_f$$

$$\boxed{V_f = \frac{m}{M+m} v_0 \cos\theta} \to 0 \text{ as } M \to \infty$$

$$v_0 \cos\theta - V_f = \frac{(M+m) v_0 \cos\theta}{(M+m)} - \frac{m v_0 \cos\theta}{M+m}$$

$$= \frac{M}{M+m} v_0 \cos\theta \to v_0 \cos\theta \text{ as } M \to \infty$$

Two Bodies Connected to a Spring

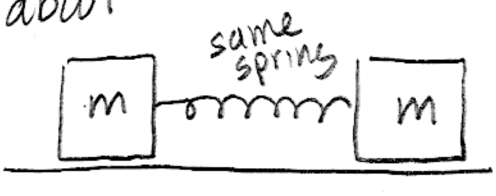


$\omega = \sqrt{\frac{k}{m}}$

$x(t) = A \sin \omega t + B \cos \omega t$

$A + B$ from initial conditions

How about



what is ω now?

or even more interesting...

above make up 2 cases...

(1) $m_1 = m$, $m_2 \to \infty$, $\omega = \sqrt{\frac{k}{m_1}}$

(2) $m_1 = m = m_2$, $\omega = ???$

(3) $m_2 \ll m_1 \to \omega = \sqrt{\frac{k}{m_2}}$

<u>a good guess</u>: $\omega = \sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = \sqrt{\frac{k}{\mu}}$

reduced mass $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$ ... gets limiting cases right

$\frac{1}{\mu} = \frac{m_1 + m_2}{m_1 m_2}$

would predict for equal masses...

$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$\omega = \sqrt{\frac{2k}{m}}$ <u>higher</u> frequency
less time for complete cycle.

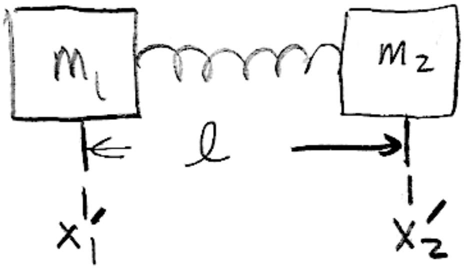
$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_1' \equiv x_1 - X = \frac{(m_1 + m_2)x_1 - m_1 x_1 - m_2 x_2}{m_1 + m_2} = \frac{m_2}{m_1 + m_2}(x_1 - x_2) = \frac{\mu}{m_1}(x_1 - x_2$$

$$x_2' = x_2 - X = \frac{(m_1+m_2)x_2 - m_1x_1 - m_2x_2}{m_1+m_2} = \frac{-m_1}{m_1+m_2}(x_1-x_2) = -\frac{\mu}{m_2}(x_1-x_2)$$

Equation of motion:

equilibrium



when $x_2' - x_1' = l$, no force.

$\rightarrow$ force caused when

$$x_2' - x_1' - l \neq 0$$

$x_2' - x_1' - l > 0$, m_2 feels $-$ force

so $$m_2\ddot{x}_2' = -k(x_2' - x_1' - l)$$

N3 $$m_1\ddot{x}_1' = +k(x_2' - x_1' - l)$$

so $$\ddot{x}_2' = -\frac{k}{m_2}(x_2' - x_1' - l)$$

$$\ddot{x}_1' = +\frac{k}{m_1}(x_2' - x_1' - l)$$

so $$\ddot{x}_1' - \ddot{x}_2' = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(x_1' - x_2' + l)$$

$$u \equiv x_1' - x_2' - l = x_1 - x_2 - l$$

$$\ddot{u} = \ddot{x}_1' - \ddot{x}_2'$$

$$\ddot{u} = -\frac{k}{\mu}u \rightarrow \omega = \sqrt{\frac{k}{\mu}} = \sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

$$u(t) = A\sin\omega t + B\cos\omega t.$$