$$\dot{x}(t) = \alpha(A\cos\alpha t - B\sin\alpha t)$$
$$\ddot{x}(t) = \alpha^2(-A\sin\alpha t - B\cos\alpha t)$$

so, $$m\ddot{x} = -kx$$

means $$-m\alpha^2(A\sin\alpha t + B\cos\alpha t) = -k(A\sin\alpha t + B\cos\alpha t)$$

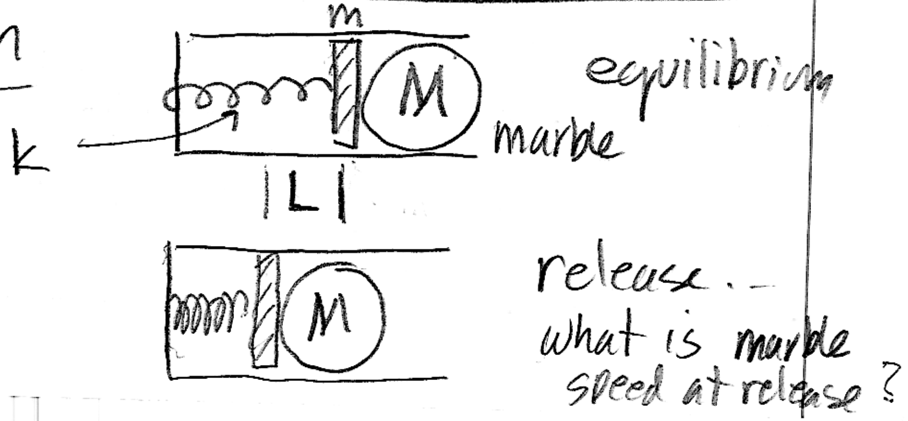
$$\alpha^2 = +\frac{k}{m}$$

$$\alpha = \pm\sqrt{\frac{k}{m}}$$ "absorb" $-$ by $A \to -A$

<u>change notation</u>: $\alpha = \omega$

solution of $$m\ddot{x} = -kx$$
is $$x(t) = A\sin(\omega t) + B\cos(\omega t)$$
with $$\omega = \sqrt{\frac{k}{m}}$$
and $A + B$ determined by initial conditions

Spring Gun



$$w = \sqrt{\frac{k}{m+M}}$$ ← total mass driven by spring

Conditions: $t = 0$

$x(0) = -L$ (displacement)

$v(0) = 0$ (released from rest)

$$x(t) = A\sin(wt) + B\cos(wt)$$

$$x(0) = \boxed{B = -L}$$

$$\dot{x}(t) = v(t) = w(A\cos wt - B\sin wt)$$

$$v(0) = wA = 0$$

$$\boxed{A = 0}$$

so
$$\boxed{\begin{aligned} x(t) &= -L\cos wt \\ v(t) &= wL\sin wt \end{aligned}}$$

$V_{max}(t)$ when $\sin wt = 1$ $(wt = \pi/2)$

then $V_{max}\left(\frac{\pi}{2w}\right) = wL = \sqrt{\frac{k}{m+M}}\,L$

later: spring pulls back
marble doesn't

Momentum

Actually, Newton did not say,

$$\sum \vec{F}_i = m \cdot \underbrace{\vec{a}}_{\frac{d\vec{v}}{dt}} = m \frac{d\vec{v}}{dt}$$

← true only when m is constant

$$\sum \vec{F}_i = \frac{d}{dt}(m\vec{v})$$

What Newton really said.

$$= \underbrace{\frac{dm}{dt}\vec{v}}_{} + m\frac{dv}{dt}$$

only matters when $\frac{dm}{dt} \neq 0$

example: rocket ships

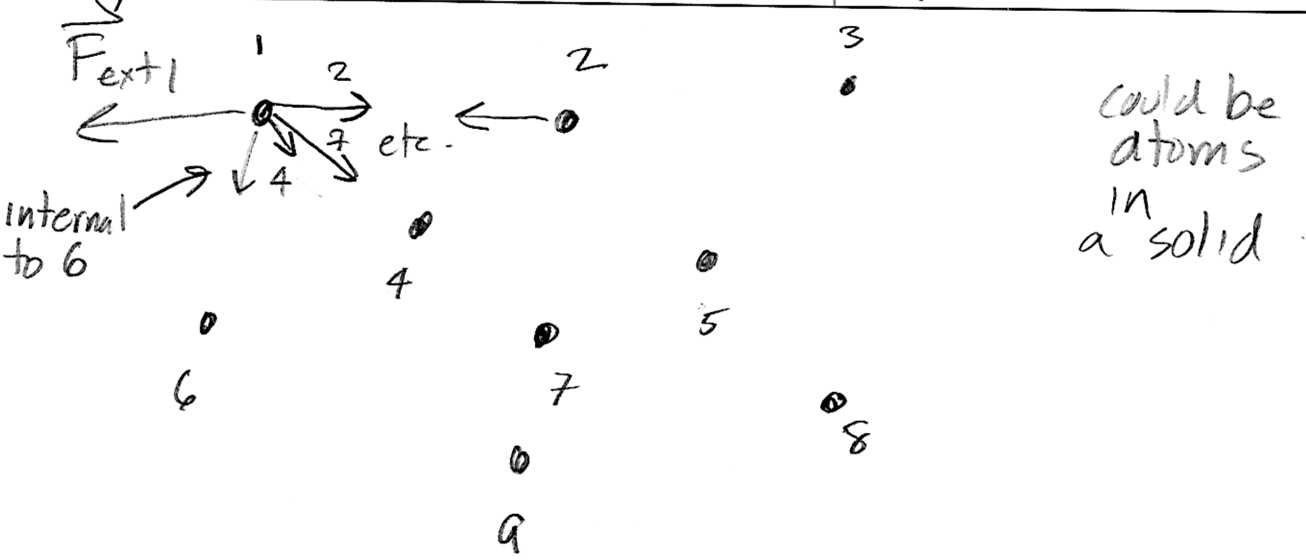
The quantity $\vec{p} \equiv m\vec{v}$ is known as the momentum of a body.

First point: when $\sum_{i=1}^{n} F_i = 0$

then $0 = \frac{d\vec{p}}{dt}$ $\quad \vec{p} = \text{constant}$

(Newton #1).

Now look at a system of bodies...



Categorize forces: internal
external

when adding up, focus on #1

$$\frac{d\vec{p}_1}{dt} = \underbrace{\sum_i \vec{F}_{i1}}_{\text{include all forces, internal + external}} = \sum \vec{F}_{\text{internal},1} + \underbrace{\vec{F}_{\text{ext},1}}_{\text{net external force.}}$$

$$\frac{d\vec{p}_2}{dt} = \sum_i \vec{F}_{i2} = \sum \vec{F}_{\text{internal},2} + \vec{F}_{\text{ext},2}$$

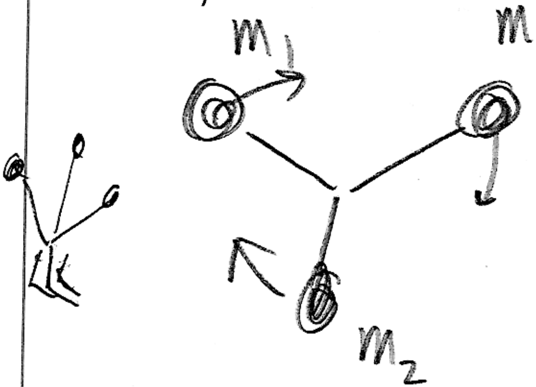
add up all ..

$$\sum \frac{d\vec{p}_j}{dt} = \sum_{i,j} \vec{F}_{ij} = \underbrace{0}_{\text{internal forces cancel in pairs by Newton 3}} + \sum_j \vec{F}_{\text{ext},j}$$

$$\boxed{\sum_i \frac{d\vec{p}_i}{dt} = \sum_j \vec{F}_{ext,j}}$$

can neglect internal forces.

Flying "bola"



$$\frac{d}{dt}(\vec{p}_1+\vec{p}_2+\vec{p}_3) = m_1\vec{g} + m_2\vec{g} + m_3\vec{g}$$

neglect string!

$$\vec{P} \equiv \vec{p}_1+\vec{p}_2+\vec{p}_3 \qquad \frac{d\vec{P}}{dt} = \underbrace{(m_1+m_2+m_3)}_{M}\vec{g} = M\vec{g}$$

motion ($\vec{p}_1(t)$) of individual is complicated

but

motion of $\underline{\vec{P} \text{ sum}}$ is that of a single particle, mass M.

Center of mass

Trajectory of precisely $\underline{\text{what}}$ follows that of something with mass M?

Find an $\vec{R}$ such that

$$\vec{F} = M\ddot{\vec{R}} = \frac{d}{dt}\sum \vec{p}_i = \underbrace{\frac{d}{dt}\sum m_i\vec{v}_i = \sum m_i\ddot{\vec{r}}_i}_{\text{assume } \frac{dm_i}{dt}=0}$$